# ELECTRICAL MEASUREMENTS

## A LABORATORY MANUAL

BY

HENRY S. CARHART, M.A., LL.D.
PROFESSOR OF PHYSICS

AND

GEORGE W. PATTERSON, JR., M.A., B.S.
ASSISTANT PROFESSOR OF PHYSICS
UNIVERSITY OF MICHIGAN



Boston
ALLYN AND BACON
1895

PRESS OF
Rockwell and Churchill
BOSTON

# PREFACE.

Progress in the methods of Electrical Measurement is quite as marked as in the applications of electricity. The perfecting of measuring instruments keeps pace with the demands imposed by scientific accuracy. Laboratory practice should not be permitted to lag behind discovery and commercial applications; obsolete methods may with propriety be relegated to historical collections, along with antiquated apparatus, so that students in electricity may learn only the latest modes of procedure.

The authors of this book have proceeded on this plan in collecting and devising methods to form a graded series of experiments for the use of several classes in electrical measurements. How well they have succeeded others must decide. Quantitative experiments only have been introduced, and they have been selected with the object of illustrating the general methods of measurement rather than the applications to specific departments of technical work, such as submarine cable testing, telegraphy and telephony, or dynamo electric machinery. It is thought to be better that these subjects should be treated in special handbooks.

It is assumed that electrodynamometers and direct reading ammeters and voltmeters of good quality are now a part of every laboratory equipment, and methods are given for their ready calibration. Much less space

has been devoted to the tangent galvanometer than has been customary in the past; but it has been retained because it is a good appliance for practice, though very inferior as an instrument of precision in comparison with later instruments for measuring current. Zero methods have been resorted to wherever it has appeared practicable to do so. The student is advised to use them as far as possible.

The experience of a number of years leads to the conclusion that the Standard Cell may be made of very great service in electrical measurements. Its construction has therefore been described with a good deal of detail, and a considerable number of experiments involving its use have been introduced. Since the Clark cell is now the legal standard of electromotive force, both in Great Britain and the United States, its use should be encouraged for this reason, aside from its convenience.

The several chapters have been introduced in what appears to the authors the order of the difficulties involved in them. Further, in each chapter the simpler experiments have been described first, and the more difficult ones later on. It is assumed that the student has completed a first course in the principles of Physics, and that he has some knowledge of analytic geometry and the calculus. It will be found of advantage if he has also had a course in the physical laboratory, comprising measurements of length, mass, periods of oscillation, moments of inertia, and the like.

It will be noticed also that we have not contented ourselves with the description of methods, but have added an explanation or a demonstration of the principle involved, and have given numerous references to original sources of information.

The subject of induction coefficients has been treated with more detail than usual on account of the increasing interest in it in connection with alternating currents and their practical applications. Dr. Karl E. Guthe, Instructor in Physics, has kindly determined by experiment the practical details of several of the methods described.

It is hoped that the examples, which for the most part have been taken from work done under the supervision of the authors, will prove a useful feature of the manual.

Thanks are due to Nalder Brothers & Co., Queen & Co., and the Weston Electrical Instrument Co., for kindly furnishing a number of the illustrations of apparatus made by them.

University of Michigan, 1895.

# CONTENTS.

# ELECTRICAL MEASUREMENTS.

## CHAPTER I.

### DEFINITIONS OF UNITS AND THEIR DIMENSIONAL FORMULAS.

**1. Fundamental and Derived Units.** — One kind of quantity may always be expressed in terms of two or three other kinds. For example: Velocity, involving two other kinds; force, involving three other quantities.

A systematic scheme of units involves as many different ones as there are kinds of quantity to be measured; and it connects them together, at least in all dynamic science, in such a manner that they are defined in terms of three original or underived units. The three which are generally employed for this purpose are the units of length, time, and mass. These are called fundamental units, in distinction from all others, which in turn are called derived units. This particular selection is a matter of convenience rather than of necessity, and rests upon several considerations which properly determine the selection of these fundamental quantities.

**2. Dimensional Formulas.** — In all scientific investigations of a quantitative character it is of great importance to know the relations of the derived units to the fundamentals; so that whatever arbitrary units are employed as the fundamentals, it may be possible to pass

directly and with certainty from one system of arbitrary fundamentals to another. This is most conveniently done by expressing the dimensions of all units. Dimensional formulas show the powers of the fundamentals that enter into the derived units. When a given unit varies as the $n^{th}$ power of a fundamental, it is said to be of $n$ dimensions with respect to that fundamental. Thus the unit of area is of two dimensions as regards a length, while the unit of volume is of three dimensions with respect to the linear unit employed. In other words, the unit of area varies as the square of the unit of length, and the unit of volume as the third power of the same.

"Every expression for a quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity to be expressed, which is taken as a standard of reference."[1] The other is merely numerical, and expresses the number of times the standard must be applied to make up the quantity measured. Thus (ten) (feet), (five) (grammes), (fifty) (seconds). The dimensions of a length are simply $L$; of time, $T$; and of mass, $M$.[2] The numerical part of an expression does not enter into the dimensional equation. It is exactly these numerical relations that we wish to determine by means of the dimensional formulas, when we have occasion to pass from one system of fundamentals to another. Thus, if we have given the numerical constants of an equation expressing the relation between any physical quantities, with the foot, the pound, and the second as the three arbitrary fundamental units, to find the numerical constants of the same relation with the centimetre, the gramme, and

[1] Maxwell's *Electricity and Magnetism*, p. 1.

[2] They are sometimes written with a square bracket and sometimes without.

the second as the arbitrary fundamentals, we need to know only the ratios between the three pairs of fundamentals and the relation of the derived units to the fundamentals, or the dimensional formulas of those derived units which express the given physical relationship.

Further, it is important to observe that the numerical parts of two expressions for the same quantity in different units are inversely as the magnitudes of the units employed. Thus, if $L\ [L]$ represents a given linear quantity in feet and $l\ [l]$ the same quantity in metres, in which the parts enclosed in brackets are the units of length, the foot and the metre respectively, then

$$l\ [l] = L\ [L], \text{ or}$$
$$l : L :: [L] : [l].$$

Since $[l] = 3.280856\ [L]$ (one metre $= 3.280856$ feet) it follows that

$$L = 3.280856\ l.$$

**3. Examples of the Use of Dimensional Formulas.**—*First.* A pendulum with a mass of 1 *kg.* has an equivalent length of 1 *m.* Its moment of inertia in $cm.^2 - gm.$ is

$$1000 \times 100^2 = 10^7.$$

What is it in $mm.^2 - mg.$?

$$1\ mm. = \tfrac{1}{10}\ cm.$$
$$1\ mg. = \tfrac{1}{1000}\ gm.$$

Hence $$1\ cm.^2 = 1\ mm.^2 \times 10^2$$

and $$1\ gm. = 1\ mg. \times 10^3.$$

Hence $$1\ cm.^2 - gm. = 1\ mm.^2 - mg. \times 10^5.$$

Since the numerical part of an expression for a given quantity is inversely as the magnitude of the unit of measurement, it follows that

$$10^7\ cm.^2 - gm. = 10^7 \times 10^5\ mm.^2 - mg. = 10^{12}.$$

*Second.* The period of vibration of a pendulum depends on its length and on gravity. Let us assume that it varies as the $m^{th}$ power of its length and as the $n^{th}$ power of $g$.

Then since gravity is an acceleration, which is the rate of change of velocity, and velocity is a length divided by a time, it follows that acceleration is a length divided by the second power of a time. We may therefore write the dimensional equation for the period of vibration of a pendulum in accordance with the assumed relationship, thus:

$$T = L^m (LT^{-2})^n = L^{m+n} T^{-2n}.$$

But the dimensions of the terms in both members of the equation must be identical. On one side we have $T$, and on the other $T^{-2n}$.

Hence $$1 = -2n$$

or $$n = -\frac{1}{2}.$$

Also $$0 = m + n = m - \frac{1}{2}, \text{ and } m = \frac{1}{2}.$$

Hence the time of vibration of a pendulum varies directly as the square root of its length, and inversely as the square root of gravity,

or $$T = const. \sqrt{\frac{l}{g}}$$

**4. The Unit of Length.** — Nearly all the quantities with which physical science deals are measured in units which in practice are referred to the three fundamental units of length, mass, and time, irrespective of the particular system to which these three units belong. But

it is eminently desirable to so choose these standards as fundamentals that we shall have a systematic arrangement, avoiding numerous and fractional ratios. The variety of weights and measures employed commercially in the United States and England illustrates an unsystematic arrangement. The metric system, on the other hand, is an example of a logical and simple systematic arrangement and relationship of the various units employed. Hence the metric system is now almost exclusively used in science.

Theoretically the metre was intended to be the ten-millionth part of the earth-quadrant passing through Paris from the equator to the north pole. Practically the metre is the distance between the ends of a bar of platinum when at 0° C., preserved in the national archives at Paris, and known as the *Mètre des Archives*. This bar was made by Borda. It was constructed in accordance with a decree of the French Republic, passed in 1795, on the recommendation of a committee of the Academy of Sciences, consisting of Laplace, Delambre, Borda, and others. The arc of a meridian between Dunkirk and Barcelona was measured by Delambre and Méchain, and the length of the metre was derived from this measurement. An earth-quadrant is now known to be about

$$10{,}002{,}015 \text{ metres.}$$

The relation between the foot and the metre is

$$1 \text{ metre} = 3.280856 \text{ ft.}$$

By Act of Congress of the United States, in 1866, the metre was defined to be 39.37 inches. The unit of length employed in magnetic and electrical measurements is the $\frac{1}{100}$ part of a metre, called a centimetre.

The choice of the centimetre was made by the British Association Committee on Electrical Standards and Measurements.

**5. The Unit of Mass.** — It is important to distinguish between mass and weight. Mass is the quantity of matter contained in a body. It is entirely independent of gravity, though gravity is usually employed to compare masses. Weight, on the other hand, means the downward force of gravity on a body, and is measured by gravity. Weight depends upon the situation of a body on the earth, and is the product of mass and gravity. Hence the weight of a given mass of matter varies with the variation of gravity from place to place.

Theoretically the unit of mass in the C.G.S. system is the gramme, or the mass of a centimetre cube of distilled water at the temperature of maximum density, or 4° C. Practically it is the $\frac{1}{1000}$ part of a standard mass of platinum preserved in the archives at Paris, and called the *Kilogramme des Archives*. This, also, was made by Borda in accordance with the decree of 1795. The theoretical and practical definitions prove not to be absolutely identical.

From Kupffer's observations Miller deduces the absolute density of water as 1.000013.[1] Hence the practical kilogramme is defined not as the mass of a cubic decimetre of distilled water at 4° C., but as the kilogramme of Borda, though the two are very approximately equal.

The gramme was recommended as the unit of mass by the British Association Committee because of its convenience, since it is nearly the mass of unit volume of

---

[1] According to the observations of Trallis, reduced by Broch, it is 0.99988. — *Everett, C.G.S. System of Units*, p. 34.

water at maximum density; and as water is usually taken as the standard in determining specific gravity, it follows that densities and specific gravities become numerically equal.

**6. The Unit of Time.** — The unit of time universally employed in scientific investigations is the second of mean solar time. An apparent solar day is the interval between two successive transits of the sun's centre across the meridian of any place. But since the apparent solar day varies in length from day to day by reason of the unequal velocity of the earth in its orbit, the mean or average length of all the apparent solar days throughout the year is taken and divided into 86,400 equal parts, each of which is a second of mean solar time.

**7. Dimensions of Mechanical Units.** — *Area.* Since area is a length multiplied by a length, its dimensional formula is $L^2$.

*Volume.* Since volume is a length or space of three dimensions, its dimensional formula is $L^3$.

*Velocity.* Velocity is a length divided by a time, or generally $\frac{dl}{dt}$.

Hence its dimensions are $\frac{L}{T} = LT^{-1}$.

*Acceleration.* Acceleration is the time-rate of change of velocity, or $\frac{dv}{dt}$. Its dimensional formula is therefore

$$LT^{-1} \div T = LT^{-2}.$$

*Force.* The magnitude of a force is the product of

mass by acceleration. Hence the dimensional equation for force is

$$F = M \times LT^{-2} = LMT^{-2}.$$

If, therefore, the unit of time should be changed from the second to the minute, the unit of force would be reduced to $1/60^2$ or $1/3600$.

*Momentum.* Momentum is the product of mass and velocity. Its dimensional formula is

$$M \times LT^{-1} = MLT^{-1}.$$

Force, according to Gauss, is measured by the time-rate of change of momentum. Its dimensions should then be

$$MLT^{-1} \div T = MLT^{-2},$$

the same as before.

The unit of force in the C.G.S. system is that force which acting on a gramme mass for one second imparts to it a velocity of one cm. per second. This is called the *dyne.* A force of one dyne produces unit acceleration of unit mass.

*Work.* Work is said to be done by a force when it produces mass motion in the direction in which the force acts. It is numerically equal to the product of the force and the component of the displacement produced while the force acts, and in the direction in which it acts. The dimensions of work are, therefore, a force multiplied by a length or

$$MLT^{-2} \times L = ML^2T^{-2}.$$

The unit of work in the C.G.S. system is the work done by a dyne through one cm. This is called the *erg.* In practical electricity a unit of work, called the *joule*, and equal to $10^7$ ergs, is frequently used.

*Activity.* Activity or power is the time-rate of doing work. The horse-power in the gravitational system of units is a rate of working equal to 33,000 foot-pounds per minute, or 550 foot-pounds per second.

Unit activity in the C.G.S. system is work at the rate of one erg per second. The *watt*, a practical unit of activity in electricity, is equal to $10^7$ ergs per second. One horse-power is equivalent to 746 watts.

Since activity is the work done in unit time, its dimensional formula is

$$ML^2T^{-2} \div T = ML^2T^{-3}.$$

Energy is measured by the work done. Its dimensional formula is therefore the same as that of work.

**8. Magnetic and Electrical Units.** — *Strength of Pole.* The two ends of a long slender magnet possess opposite properties. These ends are called *poles*, and the magnet is said to possess *polarity*. Poles of the same name, sign, or properties repel each other, while those possessing opposite properties attract. The strength of a pole is accordingly defined as proportional to the force it is capable of exerting on another pole.

If $m$ and $m'$ represent the strengths of two poles, and $d$ is the distance between them, then since magnetic attraction and repulsion vary as the inverse square of the distance, the force may be expressed as proportional to $mm'/d^2$. In the C.G.S. system the constant in the expression for $f$ becomes unity. Unit pole, therefore, has unit strength when it repels an equal and similar pole at a distance of one cm. with a force of one dyne. It produces unit magnetic field at a distance of one cm. from it.

We may then write generally

$$fd^2 = mm' = const. \times mm.$$

But since constants do not enter into dimensional equations,

$$m^2 = fd^2$$

or $$m = f^{\frac{1}{2}} d,$$

and $$m = (LMT^{-2})^{\frac{1}{2}} \times L = M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1}.$$

**9. Magnetic Field.** — Any region within which a magnetic pole is acted upon by magnetic force is called a *magnetic field*. It is a region pervaded by lines of magnetic force, or one in which the ether is in a state of strain.

A magnetic field is completely specified by expressing the value and direction of the magnetic force at every point. The direction of the force is the line along which a positive or north-seeking magnetic pole tends to move, and the force is the force sustained by unit pole. If this force is called $\mathcal{H}$, then the force acting upon any pole of strength $m$ is $\mathcal{H}m$, or

$$f = \mathcal{H}m.$$

Hence $$\mathcal{H} = \frac{f}{m}.$$

The dimensions of $\mathcal{H}$ are therefore

$$MLT^{-2} \div M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} = M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1}.$$

*Unit magnetic field* is one in which a unit magnetic pole is acted on by a force of one dyne.

**10. Magnetic Moment.** — The product of the strength of pole and the length of the magnet is called its *magnetic moment*. When a thin magnet of length $l$ is placed in a field of strength $\mathcal{H}$, so that it is at right

angles to the direction of the field, the moment of the couple acting on it, tending to turn it so that its magnetic axis shall correspond with the field, is $\mathcal{H}ml$. When the field is unity, this couple becomes $ml$. Its dimensional formula is

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1} \times L = M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}.$$

**11. Intensity of Magnetization.** — Intensity of magnetization is the quotient of the magnetic moment of a magnet by its volume, or its magnetic moment per cubic centimetre. Hence the dimensions of magnetization are

$$M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1} \div L^{3} = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}.$$

**12. Two Systems of Electrical Units.** — A system of units for the measurement of any physical quantity must be founded upon some phenomenon exhibited by the physical agent involved. The two systems of electrical units in use are founded respectively upon the repulsion exhibited by like charges of electricity and the magnetic field produced by an electric current. The one is therefore called the electrostatic and the other the electromagnetic system of units. There is no obvious relation between the two, but the dimensional formulas of the several units show that the ratio of like units in the two systems is either a velocity, the square of a velocity, or the reciprocal of the one or the other. Many series of investigations have been undertaken with a view to determine the value of this velocity $v$. According to Maxwell's electromagnetic theory of light, it is numerically equal to the velocity of light. At least six different methods have been employed with reasonably concurrent

results. The appended table gives a few of the most recent values of the ratio $v$ and of the velocity of light:

| Date. | Ratio of Units. | | Date. | Velocity of Light. | |
|---|---|---|---|---|---|
| | Experimenter. | $v$ in cms. per sec. | | Experimenter. | Vel. of light in cms. per sec. |
| 1883 . . | J. J. Thomson, | $2.963 \times 10^{10}$ | | | |
| 1888 . . | Himstedt . . . | $3.009 \times 10^{10}$ | 1879 . . | Michelson . . | $2.9991 \times 10^{10}$ |
| 1889 . . | Rowland . . . | $2.9815 \times 10^{10}$ | 1882 . . | Michelson . . | $2.9985 \times 10^{10}$ |
| 1889 . . | Rosa . . . . . | $3.0004 \times 10^{10}$ | 1882 . . | Newcomb . . | $2.9981 \times 10^{10}$ |
| 1889 . . | W. Thomson, | $3.004 \times 10^{10}$ | | | |
| 1890 . . | J. J. Thomson and Searle . . | $2.9955 \times 10^{10}$ | | | |

We shall consider generally only the electromagnetic system, founded upon the discovery of Oersted in 1820, that a magnetic needle is deflected by an electric current; or, in other words, that a current of electricity produces a magnetic field.

**13. Strength of Current.**— A current flowing through a loop of wire is equivalent to a magnetic shell, which may be considered as composed of a great many short filamentary magnets placed side by side, with all the north-seeking poles forming one surface of the shell, and all the south-seeking poles the other surface. The magnetic field at any point produced by a current in an element of the conductor is proportional to the strength of the current, to the length of the element, and to the inverse square of the distance of the point from the element. If we conceive a conductor 1 cm. in length, bent into an arc of 1 cm. radius, the current through it will have unit strength when it produces unit magnetic field at the centre of the arc; that is, a unit pole placed at the centre will be acted on by a force

of one dyne at right angles to the plane of the circle. If the conductor forms a complete circle of one cm. radius, the strength of field at the centre due to unit current will be $2\pi$.

The dimensions of unit current may be derived from the consideration that the magnetic field produced by a current at the centre of a circular conductor equals the strength of the current multiplied by the length of the conductor and divided by the square of the radius. Let $I$ equal the intensity, or strength, of current. Then

$$\text{intensity of field} = \frac{IL}{L^2} = \mathcal{H},$$

or,
$$I = \mathcal{H}L.$$

Hence, $I = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} \times L = M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$.

**14. Quantity.** — The unit of quantity is the quantity conveyed by unit current in one second. Its dimensional formula may, therefore, be found as follows:

$$\begin{aligned} \text{Quantity} &= \text{current} \times \text{time} \\ &= M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1} \times T \\ &= M^{\frac{1}{2}}L^{\frac{1}{2}}. \end{aligned}$$

The unit of quantity is, therefore, independent of the unit of time, and depends only on the units of mass and length.

**15. Electromotive Force.** — The word force is used in this connection in a somewhat figurative way, and not in a mechanical sense.

Force is that which produces or tends to produce motion or change of motion of matter. But electromotive force (E.M.F.) produces, or tends to produce, a

flow of electricity. It is analogous to hydrostatic pressure, and is often called electric pressure. It must not be confused with electric force — a force electrical in origin, and producing motion of matter.

The numerical value of the E.M.F. between two points of a circuit, when there is no source of E.M.F. in this part of the circuit, equals the difference of potential between the same points. Difference of potential between two points, $A$ and $B$, is defined as the work required to be done in carrying a unit quantity of electricity from the one point to the other. Hence the work required to convey a quantity $Q$ from $A$ to $B$ is

$$W = Q\,(V_2 - V_1),$$

in which $V_1$ and $V_2$ are the potentials of the points $A$ and $B$ respectively. The electric potential at a point is the work required to carry unit electricity from the boundary of the field to that point. But since potential difference is numerically equal to E.M.F., we have

$$\text{E.M.F.} = W \div Q.$$

Hence the dimensional formula of E.M.F. is

$$ML^2T^{-2} \div M^{\frac{1}{2}}L^{\frac{1}{2}} = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}.$$

Unit difference of potential exists between two points when one erg of work is expended in conveying unit quantity from the one point to the other.

**16. Resistance.** — Every conductor of electricity offers greater or less obstruction to its passage. The researches of Dewar and Fleming[1] on the resistance of metals at the temperature of boiling oxygen go to show that the resistance of all pure metals is zero at $-274°$ C.,

[1] *Phil. Mag.*, Oct., 1892, p. 327; Sept., 1893, p. 271.

or the "absolute zero." The resistance of pure metals is, therefore, very nearly proportional to the absolute temperature.

Ohm's law expresses the relation subsisting between E.M.F., resistance, and current strength. Thus

$$I = \frac{E}{R},$$

where $E$ expresses the algebraic sum of all the E.M.F.'s in the circuit, and $R$ the total resistance.

From this
$$R = \frac{E}{I},$$

or that property of a conductor by virtue of which a part of the energy of the current is converted into heat is equal to the ratio of the effective E.M.F., producing a current, to the current itself.

A portion $A$, $B$ of a conductor offers unit resistance when the difference of potential between the points $A$, $B$ is numerically equal to the current produced.

From the expression for resistance its dimensional formula is

$$\begin{aligned} R &= E \div I \\ &= M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-2} \div M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \\ &= L T^{-1} = \frac{L}{T}. \end{aligned}$$

Resistance is, therefore, expressed in terms of a length and a time as a velocity.

**17. Capacity.** — A conductor possesses unit capacity when it is charged by unit quantity to unit difference of potential. Since the potential varies directly as the charge, we have

$$\begin{aligned} C &= Q \div \text{P.D.} \\ &= M^{\frac{1}{2}}L^{\frac{1}{2}} \div M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2} \\ &= L^{-1}T^{2} = \frac{T^2}{L}. \end{aligned}$$

Capacity is, therefore, the reciprocal of an acceleration.

**18. The Practical Electrical Units of the Paris Congress of 1881.**[1] — At the Paris Congress of Electricians in 1881, the members of which were officially delegated by the governments represented, the following conclusions were reached:

1. For electrical measurements the fundamental units, the centimetre, the mass of a gramme, and the second (C.G.S.) shall be adopted.
2. The practical units, the ohm and the volt, shall retain their present definitions, $10^9$ for the ohm, and $10^8$ for the volt.
3. The unit of resistance (ohm) shall be represented by a column of mercury of a square millimetre section at the temperature of zero degrees centigrade.
4. An international committee shall be charged with the determination, by new experiments, for practice of the length of a column of mercury of a square millimetre section at the temperature of zero degrees centigrade, which shall represent the value of the ohm.
5. The current produced by a volt in an ohm shall be called the *ampere*.
6. The quantity of electricity defined by the condition that an ampere gives a coulomb per second shall be called the *coulomb*.
7. The capacity defined by the condition that a coulomb in a farad gives a volt shall be called the *farad*.

**19. The Practical Units of the Chicago Congress of 1893.** — A conference was held at the British Association meeting in Edinburgh in 1892 in connection with

[1] *Congrès International des Électriciens*, p. 249.

the B. A. Committee on Electrical Standards. In addition to members of the committee there were present, among others, Professor von Helmholtz, of Germany, and M. Guilleaume, of France. At this conference it was resolved to adopt the length 106.3 centimetres for the mercurial column, and to express the mass of the column of constant cross-section instead of the cross-sectional area of one square millimetre. These recommendations the committee on the part of the Board of Trade in turn recommended for official adoption by the British government. Final official action was, however, delayed to await the action of the Chamber of Delegates of the International Congress of Electricians, which convened in Chicago, August 21, 1893.[1]

The following resolutions met the unanimous approval of the Chamber:

*Resolved*, That the several governments represented by the delegates of this International Congress of Electricians be, and they are hereby, recommended to formally adopt as legal units of electrical measure the following:

1. As a unit of resistance, the *international ohm*, which is based upon the ohm equal to $10^9$ units of resistance of the C.G.S. system of electromagnetic units, and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grammes in mass, of a constant cross-sectional area and of the length 106.3 centimetres.

2. As a unit of current, the *international ampere*, which is one-tenth of the unit of current of the C.G.S. system of electromagnetic units, and which is represented sufficiently well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water, in accordance

[1] *Proceedings of the International Electrical Congress*, Chicago, 1893 (Amer. Inst. Elec. Engineers).

with accompanying specification, deposits silver at the rate of 0.001118 gramme per second.

3. As a unit of electromotive force, the *international volt*, which is the E.M.F. that, steadily applied to a conductor whose resistance is one international ohm, will produce a current of one international ampere, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of the E.M.F. between the poles or electrodes of the voltaic cell known as Clark's Cell, at a temperature of 15° C., and prepared in the manner described in the accompanying specification.

4. As the unit of quantity, the *international coulomb*, which is the quantity of electricity transferred by a current of one international ampere in one second.

5. As the unit of capacity, the *international farad*, which is the capacity of a conductor charged to a potential of one international volt by one international coulomb of electricity.

6. As the unit of work, the *joule*, which is $10^7$ units of work in the C.G.S. system, and which is represented sufficiently well for practical use by the energy expended in one second by an international ampere in an international ohm.

7. As the unit of power, the *watt*, which is equal to $10^7$ units of power in the C.G.S. system, and which is represented sufficiently well for practical use by the work done at the rate of one joule per second.

8. As the unit of induction, the *henry*, which is the induction in the circuit when the E.M.F. induced in this circuit is one international volt, while the inducing current varies at the rate of one international ampere per second.

The adoption of these units was approved for publication by the Treasury Department of the United States government, December 27, 1893. They were made legal by Act of Congress, approved by the President, July 12, 1894.

**20. Relation between the B.A. Units and the International Units.** — The Electrical Standards Committee of the British Association for the Advancement of

Science has agreed that the following relations exist between the B.A. unit and the international ohm:

1 B.A. unit = 0.9866 international ohm.
1 international ohm = 1.01358 B.A. units.

Since the unit of E.M.F. is defined in terms of the ampere and the ohm, and since the ampere is independently determined, it follows that the unit of E.M.F. varies directly as the unit of resistance. Hence:

1 B.A. volt = 0.9866 international volt.
1 international volt = 1.01358 B.A. volts.

The numeric of any given E.M.F., however, being inversely as the value of the unit employed, will have reciprocal relations to the above. Thus, if the E.M.F. of the Clark normal cell with excess of zinc sulphate crystals is 1.434 volts, in B.A. units it is

$$1.434 \times 1.01358 = 1.453.$$

The "legal ohm," which was adopted in 1882 as a temporary unit by the international committee, to which the subject had been committed by the Congress of 1881, was represented by the resistance of a column of mercury, described as above, but 106 centimetres in length. Hence the legal volt and ohm are $\frac{1060}{1063}$ of the corresponding international units.

# CHAPTER II.

## RESISTANCE.

**21. The Laws of Resistance.** — *First.* Let $AB$,


Fig. 1.

$BC$, $CD$, be three resistances, $R_1$, $R_2$, $R_3$, respectively (Fig. 1), and let their total resistance in series be $R$. Then is

$$R = R_1 + R_2 + R_3.$$

Let the potentials of the several points be $V_1$, $V_2$, $V_3$, $V_4$. Then if $I$ is the current flowing

$$V_1 - V_2 = R_1 I$$
$$V_2 - V_3 = R_2 I$$
$$V_3 - V_4 = R_3 I$$
$$V_1 - V_4 = RI.$$

These equations are derived from Ohm's law, and are true because the current $I$ is the same in each section of the conductor.

By addition of the first three equations,

$$V_1 - V_4 = (R_1 + R_2 + R_3)\, I.$$

Combining this with the fourth equation,

$$IR = I\,(R_1 + R_2 + R_3),$$

or $$R = R_1 + R_2 + R_3\,.$$

Hence the resistance of the three conductors placed end to end, or in series, is the sum of the resistances of the several conductors. If these conductors are parts of a uniform wire, it follows that *the resistance of a uniform conductor is proportional to its length.* This may be called the first law of resistance.

*Second.* The second law may be derived from a discussion of the resistance of parallel circuits.

Let two conductors of resistance, $R_1$, $R_2$, join two points of a circuit $A$, $B$. They are then said to be con-

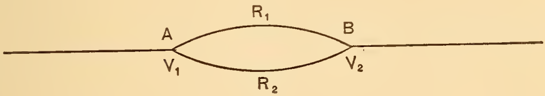



Fig. 2.

nected in parallel or in multiple. Let the potentials of the points $A$ and $B$ be $V_1$ and $V_2$, and let the currents through the two branches be $I_1$ and $I_2$, the total current being $I$.

Then by Ohm's law

$$I_1 = \frac{V_1 - V_2}{R_1}\,; \; I_2 = \frac{V_1 - V_2}{R_2}.$$

Also if $R$ is the combined resistance of the two conductors in parallel

$$I = \frac{V_1 - V_2}{R}.$$

Hence,
$$\frac{V_1 - V_2}{R} = \frac{V_1 - V_2}{R_1} + \frac{V_1 - V_2}{R_2}$$

or
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

The reciprocal of resistance is called *conductivity*. The conductivity of two conductors in parallel is, therefore, the sum of their separate conductivities. From the last equation

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

This is the expression for the combined resistance of the two conductors in parallel. The same reasoning may be extended to several conductors in parallel. The conductivities of any number of conductors in parallel is the sum of their separate conductivities. The resistance of three conductors in parallel is

$$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}.$$

If now these resistances are equal to one another, then

$$R = \frac{R_1^3}{3R_1^2} = \frac{R_1}{3}.$$

These conductors may be considered as elements of a single conductor. It follows therefore that the resistance of a uniform conductor varies inversely as its cross-section.

*Third.* The *specific resistance* of a conductor is the electrical resistance of a centimetre cube of it when the current flows through from any face to the one opposite. This is the resistance of a prism of the conductor, measured from end to end, when the cross-section of the prism is a square cm. and the length one cm. Specific resistance depends entirely upon the nature of the conductor.

Let specific resistance be denoted by $s$, and let $l$ be

the length of a uniform conductor and $a$ its cross-sectional area. Then its resistance is

$$r = \frac{sl}{a},$$

or conversely,
$$s = r\frac{a}{l}.$$

**22. The Resistance Temperature Coefficient.** — The resistance of metallic conductors in general increases with rise of temperature. If $R_0$ is the resistance of a conductor at 0° C., and $R_t$ at $t°$, then

$$R_t = R_0\,(1 + at)$$

as a first approximation. In this equation $a$ is the temperature coefficient, a constant depending upon the nature of the conductor. In the case of pure copper the extended experiments of Kennelly and Fessenden[1] demonstrate a linear relation between the resistance and temperature between the limits of 20° C. and 250° C., indicating a uniform temperature coefficient of 0.00406 per degree C. throughout the range. The maximum observed value at any point was 0.004097 and the minimum 0.00399. It is altogether likely that the discrepancies existing among the results obtained by many observers should be attributed to the presence of small percentages of other metals.

The temperature coefficient of alloys is in general smaller than that of the pure metals comprising them. Thus the coefficient of German silver[2] composed of 60 per cent copper, 25.4 per cent zinc, 14.6 per cent nickel, is 0.00036, and of platinum-silver, 0.00030.

---

[1] *The Physical Review*, Vol. I., p. 260.

[2] Dr. Lindeck, *Report of the Electrical Standards Committee of the British Association*, 1892, p. 9.

The alloy platinoid, consisting of German silver with a very small addition of tungsten, has a coefficient of only 0.00022, or about half that of common German silver (0.00044).

The new alloy, *manganin*, composed of 12 per cent of manganese, 84 per cent of copper, and about 4 per cent of nickel, has a temperature coefficient but slightly in

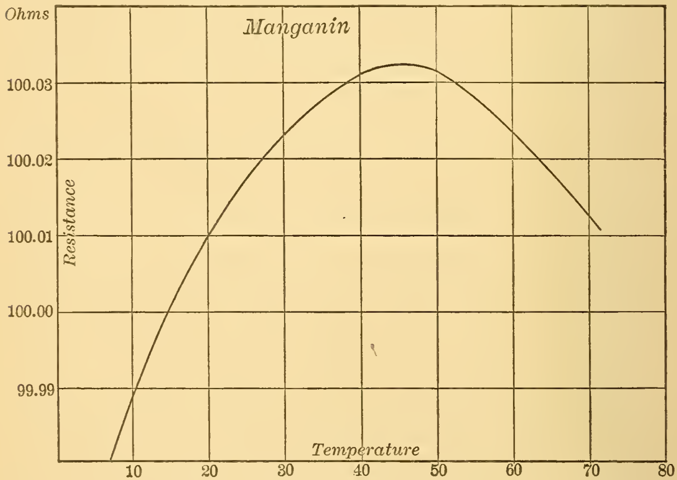



Fig. 3.

excess of zero; and at a definite temperature, which varies with different specimens, its coefficient is zero.

The general character of the resistance-variations of manganin with temperature may be ascertained from the diagram (Fig. 3), in which temperatures are plotted as abscissas, and corresponding resistances of a hundred-ohm standard as ordinates.[1] In this case the temperature

[1] Dr. Lindeck, *Report of the Electrical Standards Committee of the British Association*, 1892, p. 12; *Proceedings of the International Electrical Congress*, 1893, p. 165.

coefficient is positive up to 40° C., the absolute value, however, being very small, as the following table of the mean linear coefficients between the given temperatures shows:

TABLE.

| Range of Temperature. | Mean Linear Temperature Coefficient. | Range of Temperature. | Mean Linear Temperature Coefficient. |
|---|---|---|---|
| 10° to 20° | $25 \times 10^{-6}$ | 45° to 50° | $-1 \times 10^{-6}$ |
| 20° to 30° | $14 \times 10^{-6}$ | 50° to 55° | $-2 \times 10^{-6}$ |
| 30° to 35° | $4 \times 10^{-6}$ | 55° to 60° | $-4 \times 10^{-6}$ |
| 35° to 40° | $3 \times 10^{-6}$ | 60° to 65° | $-5 \times 10^{-6}$ |
| 40° to 45° | $1 \times 10^{-6}$ | | |

For most purposes the variability of the resistance of manganin with temperature may be quite neglected. At about 45° the resistance of the specimen under consideration passes its maximum, and the curve beyond this temperature shows a negative coefficient.

**23. Resistance Boxes.** — The resistance of conductors is commonly measured by comparison with other resistances the values of which are known with some precision. They are generally coils of insulated wire wound non-inductively on bobbins, and their values are so arranged that they can be used in any convenient combination. Collectively they make what is called a resistance box.

Each bobbin is made non-inductive by the following method of bifilar winding: A length of wire sufficient to give more than the required resistance is cut off, bent double at its middle point, and wound double on its spool or form. This is done for the purpose of avoiding self-induction on starting or stopping the current. If

the coil is wound on a metal form, the form should be split longitudinally to prevent induction currents in it. The resistance of a length of wire is usually increased somewhat by bending as it is wound on its core.

Each coil is exactly adjusted and finally fixed to the under side of the hard-rubber top of the resistance box.

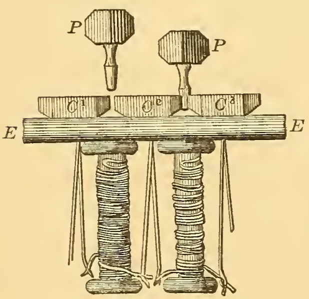


Fig. 4.

Its ends are soldered to two heavy brass or copper rods which extend through the hard rubber and are connected to massive brass blocks $C^1$, $C^2$ (Fig. 4), which offer no appreciable resistance. The coils are connected across the gap between these blocks. When any brass plug $P$ is withdrawn the current must pass through the coil bridging the gap between the disconnected blocks.

The coils are adjusted in ohms in series as follows: 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, and multiples of these. The total capacity of the preceding series is 1000 ohms. Or they may be arranged in this manner: 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500, and so on, making an aggregate of 1,110 or 11,110 ohms. For a hundred thousand ohm-box there are commonly four coils, of 10,000, 20,000, 30,000, and 40,000 ohms, respectively.

Resistance boxes are also made so that the coils may be joined in multiple. If coils of 25,000 ohms each are connected across from the block 0 to 1, 1 to 2, 2 to 3,

and so forth (Fig. 5), they may be joined in multiple or in series by the plugs so as to give a resistance between the terminal binding-posts ranging from 2,500 to 250,000 ohms.

The plugs are slightly conical, and they should fit very exactly in the conical sockets reamed out between the ends of the adjacent brass blocks. Unless the fit is exact and the plugs are clean, the resistance of the con-

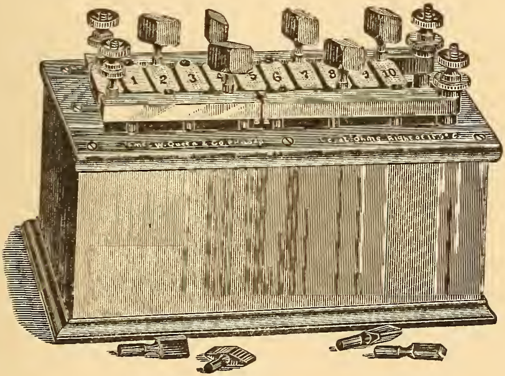



Fig. 5.

tact will not be negligible, especially with coils of small value. The plugs should be kept very clean — free from dust, oxide, and grease. They may be cleaned by rubbing with a cloth dipped in a very weak solution of oxalic acid. In pressing the plugs into their places a firm pressure should be used while the plug is slightly turned; but great care should be exercised not to seat them too rigidly or forcibly; otherwise their removal endangers their hard-rubber tops.

Each resistance box is adjusted at some convenient temperature which should be marked on the box. Cor-

rections may then be made to reduce to the resistance corresponding to the temperature of the box, which is ascertained at the time of use either by means of an attached thermometer, or by one passed through a hole provided for the purpose in the cover.

The blocks to which the coils are attached should be pierced with a tapering hole for special plugs with binding terminals, so that each coil may be put into the circuit separately for the purpose of comparing the resistances among themselves.

It is very essential that a good resistance box be kept in an outer case to protect it from dust and the light when not in use. Direct sunlight on the hard-rubber top should be carefully avoided, since the sulphur in the rubber oxidizes in the light, especially in the presence of moisture, with the production of sulphuric acid. This greatly reduces the insulation of the hard rubber.

**24. Pohl's Commutator.** — In the practice of many of the following methods of measurement, a commutator for reversing the current through any portion of the circuit, or for switching from one circuit to another, is an indispensable appliance. Pohl's commutator meets the purpose admirably.

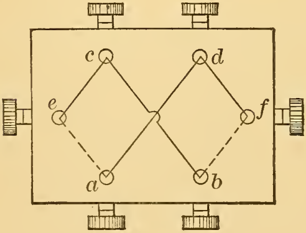


Fig. 6.

The six binding-posts (Fig. 6) make connection with the corresponding mercury cups. The points *e* and *f* are connected with the source of the current. With the connecting wires *ad*, *cb*, in place, the apparatus is adapted to reverse the

direction of flow through the circuit connected with *cd*. In the position shown, *e* is connected with *c*, and *f* with *d*. But if the movable lever is tilted over, it is easy to see that *e* will be connected with *d* and *f* with *c* through the cross-connecting wires. Of course the two conductors at the ends of the tilting-switch are joined by an insulating stem of glass or hard rubber. If now the cross-conductors are removed, then when the switch is in the position shown, the points *e* and *f* are joined to *c* and *d* respectively; but if the lever is thrown over, *e* and *f* are put in connection with another circuit from *a* round to *b*.

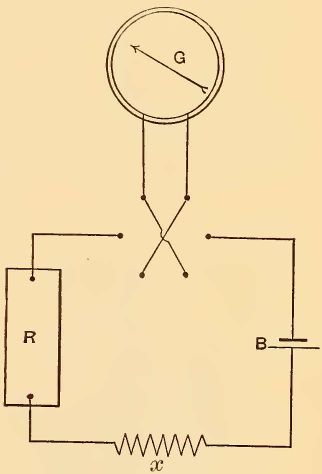


Fig. 7.

**25. Measurement of Resistance by Means of a Tangent Galvanometer.**[1] — Connect the galvanometer, the resistance $x$ to be measured, a battery of constant E.M.F., and a resistance box in series (Fig. 7). Then if $\theta$ is the deflection and $E$ the E.M.F. of the battery,

$$I = \frac{E}{B + G + R + x} = A \text{ tang } \theta = \frac{A}{\cot \theta}.$$

In order to measure $x$ by means of one observation only it would be necessary to know $B$, the battery resistance, $G$, the galvanometer resistance, $E$, and the constant $A$.

[1] For description of the tangent galvanometer, see Article 62.

But $x$ may be determined without knowing any of the above quantities, as follows:

Make two sets of observations without $x$, and with resistances $R_1$ and $R_2$ in the box, of such value that the two deflections $\theta_1$ and $\theta_2$ shall be respectively about 30° and 60°.

Then

$$\frac{E}{B+G+R_1}=A \text{ tang } \theta_1, \text{ or } \frac{E}{A} \cot \theta_1 = B + G + R_1; \quad (1)$$

$$\frac{E}{B+G+R_2}=A \text{ tang } \theta_2, \text{ or } \frac{E}{A} \cot \theta_2 = B + G + R_2. \quad (2)$$

Subtract (2) from (1) and

$$\frac{E}{A}(\cot \theta_1 - \cot \theta_2) = R_1 - R_2. \quad . \quad . \quad . \quad (3)$$

Then with $x$ in circuit and a resistance $R$ such that the deflection $\theta$ may be intermediate between $\theta_1$ and $\theta_2$, we have

$$\frac{E}{A} \cot \theta = B + G + x + R. \quad . \quad . \quad . \quad . \quad (4)$$

Subtract (2) from (4) and

$$\frac{E}{A}(\cot \theta - \cot \theta_2) = x + R - R_2. \quad . \quad . \quad (5)$$

From (3) and (5)

$$\frac{x+R-R_2}{R_1-R_2} = \frac{\cot \theta - \cot \theta_2}{\cot \theta_1 - \cot \theta_2},$$

and $$x = R_2 - R + (R_1 - R_2)\frac{\cot \theta - \cot \theta_2}{\cot \theta_1 - \cot \theta_2}.$$

**Example.**

The tangent galvanometer gave the following deflections with the resistances indicated:

| Ohms. | Deflections. | | | Cotangents. |
|---|---|---|---|---|
| | Right. | Left. | Average. | |
| 12 | 31.5° | 31.5° | 31.5° | 1.632 |
| 3 | 62.5 | 61. | 61.75 | 0.537 |
| $x$ | 44.7 | 44.5 | 44.6 | 1.014 |

Therefore,

$$x = 3 + (12 - 3)\,\frac{1.014 - 0.537}{1.632 - 0.537} = 6\;92 \text{ ohms.}$$

In this case $R$ was zero.

**26. The Reflecting Galvanometer.** — For the purpose of observing a very small deflection of the needle of a galvanometer, a light mirror is attached to the movable system, and a beam of light reflected from this serves as a long pointer *without weight*. Such a galvanometer of the "tripod" pattern is shown in Fig. 8; the mirror may be seen at the centre of the coil. The instrument is surmounted with a long rod, on which the curved magnet may slide up and down. It is held in place by friction. This magnet is employed to vary the sensitiveness of the instrument. To increase its deflection for a given small current, the plane of the mirror, which contains the magnetic needle at its back in the form of several pieces of very thin watch-spring, is first made to coincide as

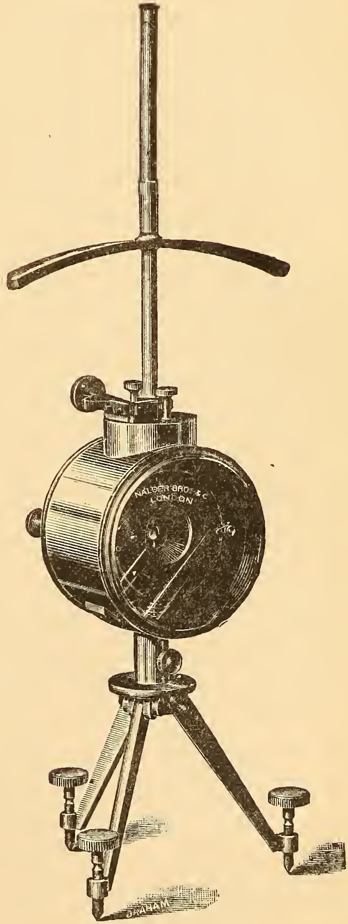


Fig. 8.

nearly as possible with the magnetic meridian. The north-seeking pole of the control magnet is then *turned toward the north.* It must be remembered that the magnetism of the northern hemisphere of the earth corresponds to that of a south-seeking pole; that is, it produces at the needle of the galvanometer a magnetic field equivalent to that which would be produced by a permanent magnet with its south-seeking pole turned toward the north. Now, the object of the control magnet is to neutralize or compensate a part of this magnetic field if increased sensibility is desired. This it can do only when its north-seeking pole is turned toward the north. To make the sensibility a maximum, the magnet is slowly lowered; this lengthens the period of oscillation of the needle. If the control magnet is placed too low, it reverses the magnetic field at the needle, and the needle then turns completely around, with its south-seeking pole toward the north. The magnet must then be slowly withdrawn till the needle again returns to its normal position. The control magnet can be turned around slowly by means of the tangent screw on the top of the galvanometer. This is necessary for the purpose of placing the needle in the magnetic meridian after the control magnet is in position.

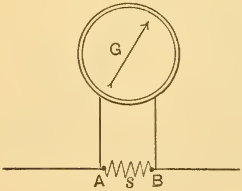


Fig. 9.

**27. The Multiplying Power of a Shunt.** — Let $g$ and $s$ be the resistances of the galvanometer and shunt respectively, measured between the two points $A$ and $B$ (Fig. 9); and let $I_g$ and $I_s$ be the currents through the two paths. Let $V$ be the potential difference (P.D.) between $A$ and $B$.

Then $$I_g = \frac{V}{g}, \text{ and } I_s = \frac{V}{s}.$$

Also if the total current is $I$,

$$I = \frac{V}{g} + \frac{V}{s}.$$

But $\frac{I_g}{I_s} = \frac{s}{g}$, and therefore $\frac{I_g}{I_g + I_s} = \frac{s}{s+g} = \frac{I_g}{I}$.

Therefore

$$I = I_g \frac{s+g}{s}.$$

The fraction $\frac{s+g}{s}$ is called the "multiplying power of the shunt." It is the factor by which the current flowing through the galvanometer must be multiplied in order to find the total current. Also from the above equation

$$I_g = I \frac{s}{s+g}.$$

If it is desired that $I_g$ shall be $\frac{1}{10}$ of $I$, then

$$\frac{s}{s+g} = \frac{1}{10}, \text{ or } 10s = s + g, \text{ and } g = 9s.$$

Whence $$s = \tfrac{1}{9}g.$$

If $I_g$ is to be $\frac{1}{100}$ of $I$, then

$$s = \tfrac{1}{99}g.$$

If $I_g$ is to be $\frac{1}{1000}$ of $I$, then

$$s = \tfrac{1}{999}g.$$

These are the three relative values usually given to shunts in order to avoid inconvenient factors. Such shunts are applicable only to the galvanometers for which they are made. The plan of the top of such a

shunt-box is shown in Fig. 10. One end of all three coils is connected with the block *A*; the other ends to the blocks *C*, *D*, *E*. The central block is connected to *B*.

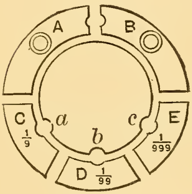


Fig. 10.

Shunts are also made for a current of $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ through the galvanometer, while the total resistance in the circuit remains constant. The entire current *I* thus remains the same whichever shunt is used.

**28. Two Methods of reading a Mirror Galvanometer.** — The deflection is read by means of a scale of equal parts, preferably millimetres, numbered continuously from one end to the other. Let *BAB′* (Fig. 11) be the scale, and let *C* be the mirror; and let the scale be so placed that it shall be parallel to the galvanometer mirror when no current is passing. Then if the magnet and mirror have been turned through an angle $\theta$,

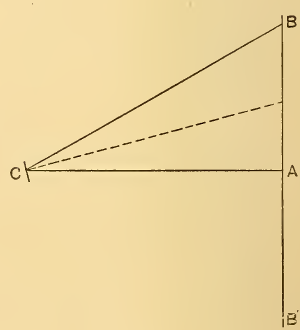


Fig. 11.

$$ACB = 2\theta,$$

since the reflected ray of light is always turned through twice the angle of the deviation of the mirror. Also

$$\frac{AB}{AC} = \text{tang } 2\theta.$$

The two methods of observing the distance *AB* are known as the "lamp and scale" method and the "tele-

scope and scale" method. The device for lamp and scale is shown in Fig. 12. The light of the lamp passes through an opening across which is stretched a fine wire corresponding to the point *A* of Fig. 11. After reflection from the mirror, the image of the wire falls on the dimly illuminated scale. In order to obtain a good image, a converging lens may be placed some distance in front of the wire in such a position that the wire and the scale are conjugate foci for a beam reflected from the mirror, which in this case must be plane. But if a concave mirror, with a radius of curvature of about one metre, be used in the galvanometer, then the image of the wire will be focused on the scale when the wire is placed just below centre of curvature of the mirror. A translucent scale is much to be preferred.

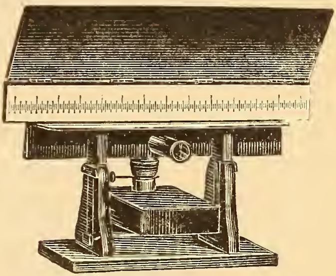


Fig. 12.

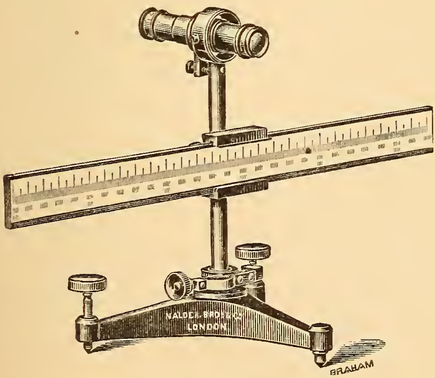


Fig. 13.

The observer is then on the side of the scale away from the galvanometer, and the reading is much more convenient. A gas-jet at one side may be used in place of the lamp; and in this case a mirror at the back of the scale

reflects the light through the opening containing the wire.

In the other or subjective method of observing the deflection a telescope takes the place of the lamp and slit or wire. Such a reading telescope with attached scale is shown in Fig. 13. It is set up so that an image of the middle point of the scale is obtained by reflection from the galvanometer mirror when at rest with no current passing. If now the mirror is deflected the scale appears to swing across the field of view of the telescope, and when it comes to rest the observer reads the division of the scale coinciding with the vertical cross-wire in the eye-piece. Instead of the usual spider webs for cross-wires, fine quartz fibres may be substituted with most satisfactory results. If the galvanometer is to be used merely as a galvanoscope for detecting the passage of a current, then it is necessary only to observe whether the scale appears to move when the key is pressed.

The telescope and scale possess the advantage that they can be used in a light room; and this method admits of greater accuracy than that of the lamp and scale, because the magnification of the telescope allows the divisions to be read to tenths.

Let $n_1$ and $n_2$ be the readings of the scale when no current is passing and when deflected by a current respectively. Let $a$ be the distance between the mirror and the scale and $d$ the "deflection." Then

$$d = n_2 - n_1,$$

and

$$\theta = \frac{1}{2} \tan^{-1} \frac{d}{a}.$$

For small angles we may write approximately

$$\theta = \tan \theta = \sin \theta = \frac{d}{2a}.$$

If $\delta = \frac{d}{a}$ the following equations express the expansions of the several quantities in terms of the tangent of twice the angle:

$$\theta = \frac{\delta}{2}\left\{1 - \frac{1}{3}\delta^2 + \frac{1}{5}\delta^4 - \frac{1}{7}\delta^6 + \ldots\ldots\right\}$$

$$\tan\theta = \frac{\delta}{2}\left\{1 - \frac{1}{4}\delta^2 + \frac{1}{8}\delta^4 - \frac{5}{64}\delta^6 + \ldots\ldots\right\}$$

$$\sin\theta = \frac{\delta}{2}\left\{1 - \frac{3}{8}\delta^2 + \frac{31}{128}\delta^4 - \ldots\ldots\right\}$$

$$2\sin\frac{\theta}{2} = \frac{\delta}{2}\left\{1 - \frac{11}{32}\delta^2 + \frac{431}{2048}\delta^4 - \ldots\ldots\right\}$$

If the deflection does not exceed 6° the first term of the correction is usually sufficient.

Table I. in the Appendix gives the correction factors for the above four quantities from $\delta = 0.01$ to $0.2$.

Table II. gives the number to be subtracted from the deflection $d$ to make it proportional to the tangent of the angle instead of the tangent of twice the angle, or to $\tan\theta$ instead of $\tan 2\theta$.

**29. Determination of the Figure of Merit of a Galvanometer.** — The figure of merit of a galvanometer is the constant current which will produce a deflection of one scale division, or what is practically the same thing for small angular deflections, the ratio of the current to the deflection in scale divisions. If this ratio is not a constant for different values of the current, the galvanometer should be calibrated and the figure of merit calculated from the corrected readings.

A convenient method of determining the figure of merit is to connect the galvanometer in series with a

battery of known electromotive force $E$, and a known resistance $R$, which should be as large as possible and still give a suitable deflection. Note the deflection $d$ of the galvanometer and calculate the current. For the latter it is necessary to know the resistance $G$ of the galvanometer and $B$ of the battery, unless they are negligible in comparison with $R$. If they are not negligible and are unknown, they may be measured by means of methods described in articles 38 and 55. The figure of merit $F$ is expressed by the following relation:

$$F = \frac{E}{(R + G + B)\, d}.$$

In the case of a very sensitive galvanometer, it sometimes happens that the deflection is excessive, even with the highest resistance at hand in series with the galvanometer.

In this case the galvanometer may be shunted by a coil of known resistance, preferably $\frac{1}{9}$, $\frac{1}{99}$, or $\frac{1}{999}$ of that of the galvanometer. If the resistance of the galvanometer is $n$ times that of the shunt, $\frac{1}{n+1}$ of the whole current passes through the galvanometer. The figure of merit is then expressed by the following relation:

$$F = \frac{E}{\left(R + \frac{G}{n+1} + B\right)(n+1)\, d}.$$

As the deflection of the galvanometer depends on the distance of the scale from the mirror, it is customary to mention the distance at which the figure of merit is determined. The figure of merit of galvanometers carrying a compensating magnet may be varied between wide

limits by varying the strength of the magnetic field in which the suspended needle swings.

**30. Comparison of Resistances by Means of Potential Differences.** — Connect the unknown resistance $x$ and a known resistance $R$ of about the same value in series with a battery $B$ of constant E.M.F. (Fig. 14).

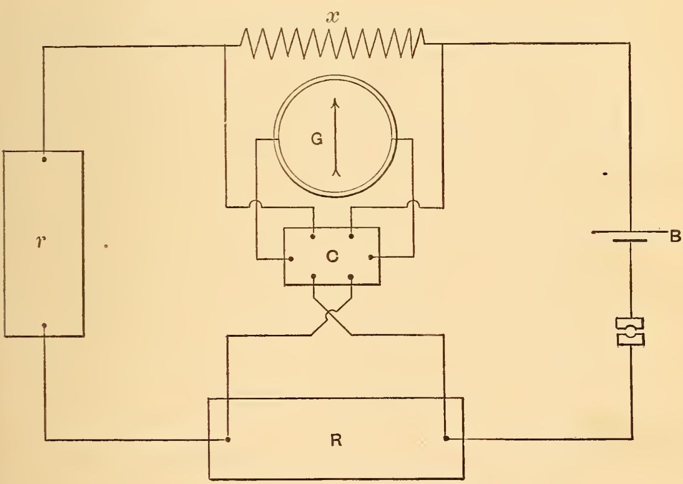



Fig. 14.

It may be necessary to use also another resistance $r$, which need not be known, but which may be necessary for the purpose of adjusting the current to the proper value, so as to secure a convenient deflection of the galvanometer. By means of a Pohl's commutator $C$, the high resistance galvanometer $G$ is connected first to the terminals of the known resistance $R$, and then to those of $x$, in such a way that the deflections shall be in the same direction. This operation should be repeated a

number of times till constant results are obtained. Then if $d_1$ and $d_2$ are the deflections in the two cases, which should be as nearly as possible the same and not too large, we have

$$R : x :: d_1 : d_2,$$

or

$$x = R\frac{d_2}{d_1}.$$

The method proceeds on the assumption that the fall of potential is proportional to the resistance, and that the galvanometer deflections are proportional to the currents flowing through the instrument, and therefore proportional to potential differences.

**Example.**

The following observations were made:

| Resistance. | Reading. | Zero Reading. | Deflection. |
|---|---|---|---|
| 0.3 | 873 | 500 | 373 |
| $x$ | 853 | 500 | 353 |

Therefore

$$x = \frac{0.3 \times 353}{373} = 0.284 \text{ ohm}.$$

**31. Measurement of Resistance by Means of the Differential Galvanometer.** — A differential galvanometer is wound with two coils of approximately equal resistance and equal magnetic field at the centre of the coils. The connections are made, as shown in the diagram (Fig. 15), the two parts into which the current divides going in

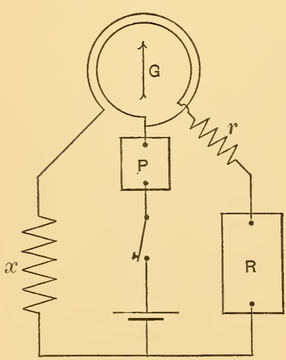



Fig. 15.

opposite directions round the two coils. The observations consist in adjusting the resistance $R$ until the galvanometer shows no deflection on closing the circuit. In case an exact balance cannot be obtained, the fraction of the smallest division of $R$, usually one ohm, necessary to produce a balance, can be determined by means of deflections in both directions and interpolating. If $d_1$ is the deflection with $R$ ohms, and $d_2$ the opposite deflection with $R + 1$ ohms, then the resistance to balance is

$$R + \frac{d_1}{d_1 + d_2}.$$

It is essential to determine whether the two coils are of equal resistance, and whether the same current through each produces the same magnetic field at the centre. For this purpose connect the two coils in series, but so that they shall produce opposing magnetic fields at the needle. If the needle shows no deflection, the coils are balanced magnetically. If there is a deflection, a balance may be secured if one coil is movable, as in the Edelmann galvanometer, by varying its distance from the needle; or it may be secured by passing one-half of the current through a coil properly placed under the galvanometer, or in its base. Such an adjustment, however, is usually troublesome.

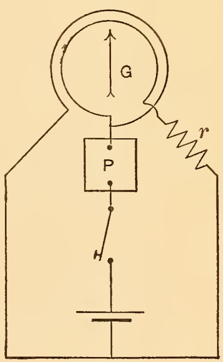


Fig. 16.

A much better method is the following: If necessary insert a resistance $r$ in one branch, as shown in the diagram (Fig. 16), in order to effect a balance. This

resistance may be simply a small increase in one of the lead wires, or it may be a good many ohms. It is advisable to introduce a resistance $P$ in the battery branch to diminish the current. Let $A$ and $B$ be the resistances of the two windings, including the connecting wires and resistance $r$, between the points of division of the circuit. Then let the resistances $R$ and $x$ be inserted as in Fig. 15, and let a balance be obtained by deflections in the two directions and by interpolation if necessary. Next exchange $R$ and $x$ and balance again. Let $R_1$ and $R_2$ be the resistances to balance in the two cases.

Then $A : B :: R_1 : x$, for the first balance,

and $A : B :: x : R_2$, for the second balance.

Whence $$x = \sqrt{R_1 . R_2}.$$

**Example.**

I. To determine the resistance of one B.A. unit in ohms:

*Apparatus.* — Edelmann's mirror galvanometer with high resistance coils.

A B.A. unit box for the unknown resistance ($x$).

An international ohm box for known resistance ($R$).

*Cond. I.* — The influence of both coils traversed by the same current, but in opposite direction, should be equal for a magnetic balance.

Current through $A$ alone deflects to smaller numbers.

Current through $B$ alone deflects to larger numbers.

Current through both coils deflects to larger numbers.

$B$ was moved 4.5 mm. away from the needle; then there was no deflection.

*Cond. II.* — Resistance of both coils should be equal for electrical balance.

Current flowing through both coils in parallel deflects to larger numbers.

Resistance put in series with $B$ until no deflection was observed.

Resistances $x$ and $R$ inserted.

$x = 1{,}000$ . . . $R = 986$; no deflection.

The galvanometer was not sensitive enough to estimate $R$ to tenths.

Hence $$x = 0.986\ R,$$

or one B.A. unit equals 0.986 of an international ohm.

II. To determine the resistance of one B.A. unit, in ohms, by the second method:

*Apparatus.* — A Thomson astatic mirror galvanometer.

Resistance of B.A. box, right at 16° C., as unknown resistance ($x$).

Resistance box, in ohms, right at 17° C., as known resistance ($R$).

*Formula:* $$x = \sqrt{R_1 \cdot R_2}.$$

*Adjustment of Apparatus:*

A current through coil $A$ deflects to smaller numbers.

A current through coil $B$ deflects to larger numbers.

A current through both coils deflects to larger numbers.

In order to get no deflection 1170 ohms ($r$) were added to coil $B$, with $A$ and $B$ in parallel.

*Observations:*

*First.* $x$ in series with $A$; $R$ with $B$ ($R$ does not include the 1170 ohms).

($a$) $x' = 500$ B.A. . . . $R = 363.1$; no deflection.
($b$) $x'' = 600$ B.A. . . . $R = 435.65$; no deflection.
($c$) $x''' = 800$ B.A. . . . $R = 580.71$; no deflection.

*Second.* Resistances $x$ and $R$ exchanged.

($d$) $x' = 500$ B.A. . . . $R = 671.7$; no deflection.
($e$) $x'' = 600$ B.A. . . . $R = 804.83$; no deflection.
($f$) $x''' = 800$ B.A. . . . $R = 1072.83$; no deflection.

*Calculation:*

From ($a$) and ($d$), $x = \sqrt{363.1 \times 671.7} = 493.86$; $\dfrac{x}{x'} = 0.98772.$

From ($b$) and ($e$), $x = \sqrt{435.65 \times 804.83} = 592.14$; $\dfrac{x}{x''} = 0.98690.$

From ($c$) and ($f$), $x = \sqrt{580.71 \times 1072.83} = 789.29$; $\dfrac{x}{x'''} = 0.98661.$

Mean . . . . . . . . . . . 0.98708.

*Correction for temperature:*

Temperature of both boxes, 20.5° C. Temperature coefficient for both, 0.00044.

1 B.A. unit at 20.5° C. $= 1 + (0.00044 \times 4.5) = 1.00198$.
1 ohm unit at 20.5° C. $= 1 + (0.00044 \times 3.5) = 1.00154$.
Therefore 1.00198 B.A. units $= x \times 1.00154$ ohms.

Whence 1 B.A. unit $= \dfrac{1.00154}{1.00198} \times 0.98708 = 0.98664$ ohm.

**32. Heaviside's Modification of the Differential Galvanometer.**[1] — Instead of dividing the current from the battery between the two coils, join the coils so that the same current passes through both of them, and by reversing one of the coils $g'$ (Fig. 17), prevent the current from influencing the needle. The rheostat $R$ is connected in parallel with one coil $g$ and the resistance $x$ to be measured in parallel with the other $g'$. When $R$ equals $x$ it is easily seen that the currents in $g$ and $g'$ are equal provided $g$ and $g'$ are equal to each other. But this method may be used exactly as in the last article. Let $R_1$ be the resistance to balance $x$ in the relative positions shown in the figure. Then exchange the rheostat and the unknown resistance and balance again, interpolating, if necessary, and let $R_2$ be the resistance in the rheostat. Then

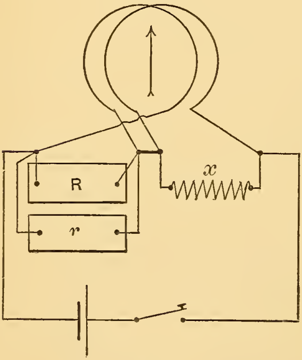


Fig. 17.

$$R_1 : x :: g : g',$$

and

$$x : R_2 :: g : g'.$$

Whence

$$x = \sqrt{R_1 \cdot R_2}.$$

[1] *Electrical Papers*, Vol. I., p. 13.

This method assumes that the galvanometer is magnetically balanced. If the galvanometer is not magnetically balanced, the stronger coil may be shunted with a resistance $r$ (Fig. 17), such that when the two galvanometer coils (one shunted and the other not) are placed in series, no deflection is obtained. When $x$ is greater than $g$ the other method is to be preferred. But for values of $x$ less than $g$, the present method gives greater sensibility. If, for instance, the battery have a resistance of 10 ohms, each coil of the galvanometer 500 ohms, and $x$ is 10 ohms, then the Heaviside method is seven times as sensitive as the first method.

**33. Wheatstone's Bridge.** — Wheatstone's Bridge is a combination of resistances most commonly employed to measure all except a very high resistance or a very low one. It consists of six conductors connecting four points, in one of which is a source of electromotive force, which need not be constant; and another branch contains a galvanometer.

Let $ABCD$ (Fig. 18) be the four points connected by six conductors. Then since the fall of potential by the two paths between $A$ and $D$ is the same, there must be a point $B$ on the path $ABD$ which has the same potential as another point on the path $ACD$. If these points are joined by a conductor, including a galvanometer, no current will flow through it, and we have the relation

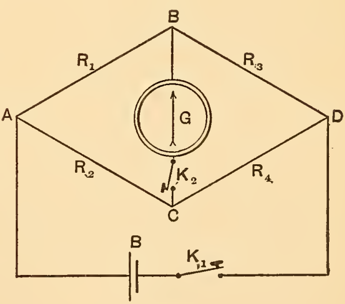



Fig. 18.

$$R_1 : R_2 :: R_3 : R_4.$$

For let $I_1$ be the current through $R_1$. It will also be the current through $R_3$, since none flows across through the galvanometer. Also let $I_2$ be the current through the other branch $ACD$. Then since the potential difference between $A$ and $B$ is the same as between $A$ and $C$,

$$R_1I_1 = R_2I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly, $$R_3I_1 = R_4I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Dividing (1) by (2), $\dfrac{R_1}{R_3} = \dfrac{R_2}{R_4}$.

This may also be written,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

or $$R_1 : R_2 :: R_3 : R_4.$$

The last equation might have been obtained by balancing with the galvanometer connecting $AD$ and the battery applied to the points $BC$. The conditions for a balance are, therefore, the same after the galvanometer and battery have exchanged places as before, and depend only upon the proportionality of the four resistances.

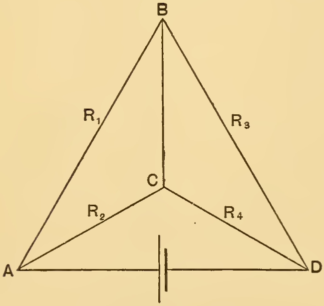


Fig. 19.

If the six conductors are arranged as shown in Fig. 19, and if

$$R_1 : R_2 :: R_3 : R_4,$$

so that no current flows through the galvanometer, then any change of E.M.F. in $AD$ will not produce a potential

difference between $B$ and $C$; the converse is, therefore, true, so that the battery and galvanometer may exchange places without disturbing the balance. The balance is in no way dependent upon the resistance of $BC$ and $AD$, though the sensibility of the arrangement is dependent upon these relative resistances. $AD$ and $BC$ are said to be *conjugate*; that is, they are connected by this mutual relation of independence. So, also, when the

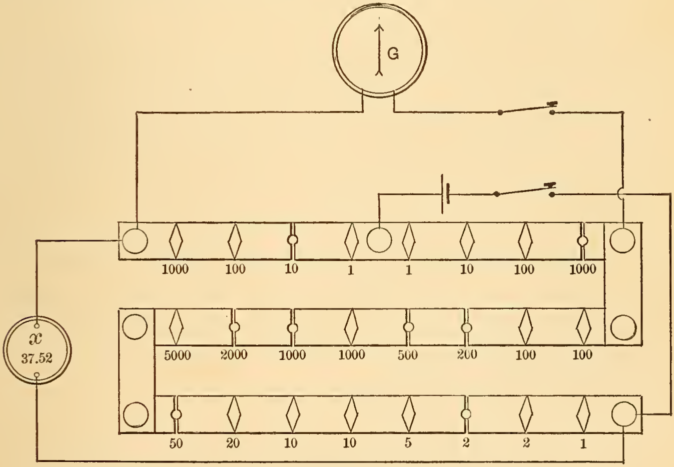



Fig. 20.

corresponding resistances are proportional, $AB$ and $DC$, $BD$ and $AC$ are conjugate.

To use the Wheatstone's bridge for the measurement of a resistance, three known resistances are taken, having such a relation to the unknown $x$ that a balance is obtained with the galvanometer. In practice two resistances, $R_1$ and $R_2$, are chosen, and $R_3$ is made to vary till a balance is secured. Then

$$x = R_3 \frac{R_2}{R_1}.$$

Maxwell gives the following rule for the connection of the battery and the galvanometer to the four resistances:[1] "Of the two resistances — that of the battery and that of the galvanometer — connect the greater resistance so as to join the two greatest to the two least of the four other resistances."

If, for example,

$$R_1 = 1000,\ R_2 = 10,\ R_3 = 3752,\ x = 37.52,$$

then the battery should join the point between the two proportional coils to the junction of $R_3$ and $x$, as shown in the diagram (Fig. 20), if the resistance of the galvanometer is greater than that of the battery, which is usually the case.

The battery circuit should be closed first, and then the galvanometer circuit, so as to avoid the effect of any self-induction in the coils of the resistances. A double successive contact key is very convenient for this purpose. It opens the two circuits in the inverse order to that in which they are closed.

**34. The Post-Office Resistance Box.** — One of the most convenient arrangements for the use of the Wheatstone's bridge method is the *Post-Office Resistance Box*, so called because of its employment in the telegraph department of the British post-office. Fig. 21 is a plan of the top of this box.

The arms $AB$ and $AC$ consist of two sets of proportional coils — two 10's, two 100's, and two 1000's. Any pair of these represent the resistances $R_1$ and $R_2$ of Fig. 18, which is lettered to correspond with the plan of the

[1] *Electricity and Magnetism*, Vol. I., p. 438.

post-office box. These proportional coils may contain, also, a pair of 1's or a pair of 10,000's. The ratio, $\frac{R_1}{R_2}$ is then either 1, 10, 100, 1000 or 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$. The unknown resistance may be measured directly to $\frac{1}{100}$ or $\frac{1}{1000}$ of the smallest coil included in the rheostat arm

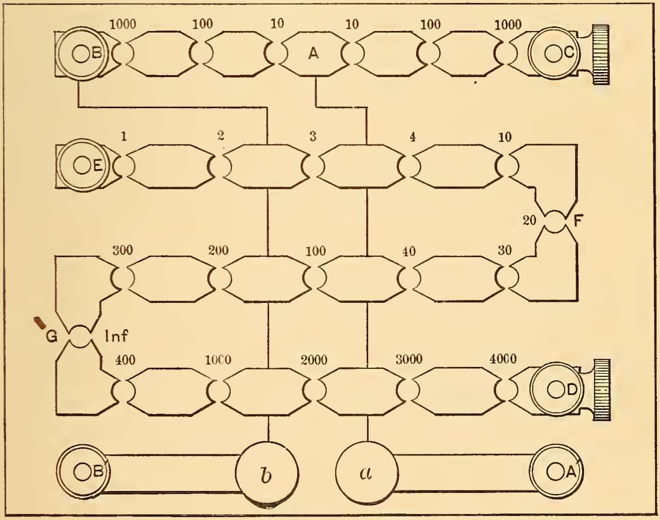



Fig. 21.

*EFGD*. Thus, if $R_1$ is 1000, $R_2$ 10, and $R_3$ 253 ohms, then $x$ is $\frac{1}{100}$ of 253 or 2.53 ohms. No binding-post is provided at *A*, but *A* is joined by a wire under the hard-rubber top of the box to a stud at *a*, so that it is put in connection with *A′* by pressing the key *A′a*. In the same way the terminal *B′* is put in connection with *B* by pressing the key *B′b*. Connection is made between *B* and *E* by a heavy copper strap not shown. This is

screwed down tightly by the binding-screws *B* and *E*. Since two wires must be connected at both *C* and *D*, these points are provided with double binding-posts. At the point marked *Inf.* is the *infinity plug*. When this plug is out, the circuit through the rheostat arm is completely broken. It will be observed that the series of resistances shown are 1, 2, 3, 4, 10, and multiples of these.

If it is impossible to obtain a balance with the smallest coil in the rheostat arm, then the fraction required to balance may be determined by observing the deflections of the galvanometer, first in one direction and then in the other, and the true value of $x$ may be found by interpolation. For example, let the following be the resistances and deflections in divisions of the scale:

| | Deflections. | |
|---|---|---|
| $R_3$. | Left. | Right. |
| 1206 . . . . . . . . . . . . . . | | 6 |
| 1205 . . . . . . . . . . . . . . | 14 | |

Then one ohm causes a change in the deflection of 20 divisions. Hence the value of $R_3$, which would give an exact balance, is $1205 + \frac{14}{20}$, or 1205.7.

**Example.**

Ratio of proportional coils $R_1$ and $R_2$, 1000 : 1000. Galvanometer used with $\frac{1}{1000}$ shunt.

| $R_3$ | Deflection. |
|---|---|
| 0 ohms. | To higher numbers. |
| 100 | " lower " |
| 40 | " " " |
| 20 | " " " |
| 10 | " " " |
| 4 | " higher " |
| 6 | " lower " |
| 5 | Almost none, slightly to lower numbers. |

Changing the ratio of $R_1$ and $R_2$ to 1000 : 10 and removing the galvanometer shunt, the following observations were obtained:

| $R_3$ | DEFLECTION. |
|---|---|
| 500 | To lower numbers. |
| 495 | " higher " |
| 497 | 10 mm. to higher numbers. |
| 498 | 33 mm. to lower " |

Therefore to give no deflection $R_3$ should be $497\frac{10}{43} = 497.23$, or $x = 4.9723$.

From this must be subtracted the resistance of the lead wires, which was obtained as follows:

Ratio of $R_1$ and $R_2$, 1000 : 10:

| $R_3$ | DEFLECTION. |
|---|---|
| 8 | To lower numbers. |
| 1 | 75 mm. to higher numbers. |
| 2 | 16 mm. to lower " |

Therefore to give no deflection $R_3$ should be $1\frac{75}{91} = 1.82$, or the resistance of the lead wires was 0.0182, giving for the resistance of $x$, $4.9723 - 0.0182 = 4.9541$ ohms.

The temperature of the box was 20° C.; and as it was right at 17° C. and had 0.00023 for its temperature coefficient, the final corrected value for $x$ was $x = 4.9541\ [1 + 0.00023\ (20 - 17)] =$ 4.9576 ohms at 20° C.

**35. The Slide Wire Bridge.** — Since it is necessary to know only the ratio of $R_1$ to $R_2$, and not their absolute values, the resistances of two adjacent portions of a uniform wire may be employed in place of adjusted coils.

With the openings at 1 and 2 (Fig. 22) closed by heavy copper straps, obtain a balance by moving the contact $C$ along the wire. Then

$$\frac{x}{R_3} = \frac{a}{b}, \text{ or } x = R_3\frac{a}{b}.$$

The resistance of the two parts of the wire $a$ and $b$ are here supposed to be proportional to their lengths.

A single determination of a resistance by this method does not admit of very great exactness, since the position of $C$ may not be read with precision, and the wire may not be of the same resistance for each unit of length.

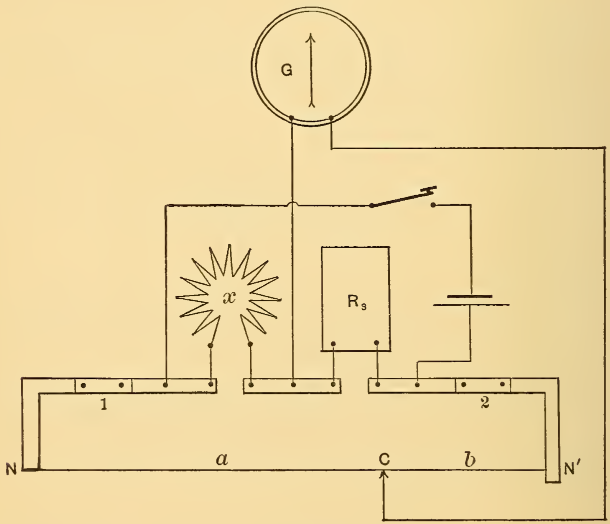



Fig. 22.

**36. Effect of Errors of Observation.** — An error in reading the position of $C$ produces the smallest effect on the result when $C$ is at the middle point of the wire. This may be demonstrated as follows: We have from the preceding

$$x = R\frac{a}{b} = R\frac{a}{c-a}, \quad . \quad . \quad . \quad . \quad (1)$$

when $c$ is the entire length of the wire

Suppose now an error $f$ has been made in reading the position of the contact $C$ on the bridge wire. Then the value of $x$ is $x+F$, in which

$$x+F=R\frac{a+f}{c-a-f}. \quad . \quad . \quad . \quad . \quad (2)$$

The general formula to apply in determining the conditions for the least error may be derived as follows:

Let $x$ be the observed quantity.

Let $X$ be the derived quantity.

Also let $f$ be the error in the observed quantity, and let $F$ be the resulting error in $X$.

The error $F$ arises from the use of $x+f$ instead of $x$ in the equation connecting $x$ and $X$. Then the relation of the four quantities is expressed by the equation

$$F=f\frac{\delta X}{\delta x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$F$ and $X$ are quantities of the same kind; also $f$ and $x$. The partial differential coefficient $\frac{\delta X}{\delta x}$ expresses the rate of variation of $X$ with respect to $x$, other variables for the time being considered constants. This rate, multiplied by the error in the observation, gives the total error in the result, or $F$.

Applying this formula to the present case, we have from (1)

$$\frac{\delta X}{\delta x}=\frac{\delta x}{\delta a}=R\frac{c}{(c-a)^2}, \quad . \quad . \quad . \quad (4)$$

since $a$ is the observed quantity and $x$ the derived resistance.

Whence

$$F=fR\frac{c}{(c-a)^2} \text{ and } \frac{F}{x}=\frac{fc}{a(c-a)}. \quad (5)$$

This ratio will be a minimum when $a(c-a)$ is a maximum. But the product of two quantities whose sum is a constant ($c$) is a maximum when they are equal to each other, or when $a=c-a$. In that case $2a=c$ or $a=\frac{c}{2}$; or the contact $C$ is at the middle point of the wire. $R$ and $x$ should therefore be made as nearly equal as possible.

**37. Use of the Slide Wire Bridge—First Method.**[1] — Referring to the figure of Art. 35, it will be seen that the resistance of the copper bars, straps, and contacts from $N$ to $x$ and from $N'$ to $R$ are measured in with $a$ and $b$ respectively. It may further happen that the index line of the slide is not exactly over the metal edge making contact with the bridge wire. Let $f$ be this error, so that the true bridge reading is $a_1+f$. Let $r_1$ be the resistance of the bridge between $N$ and $x$, and $r_2$ that between $N'$ and $R$. It is necessary to observe that $r_1$ and $r_2$ are here expressed in terms of the resistance of unit length of the bridge wire. Then

$$\frac{x}{R}=\frac{a_1+f+r_1}{1000-(a_1+f)+r_2} \quad . \quad . \quad . \quad (1)$$

if the bridge wire is divided into 1000 parts.

Let now the positions of $x$ and $R$ be reversed. Then

$$\frac{x}{R}=\frac{1000-(a_2+f)+r_2}{a_2+f+r_1} \quad . \quad . \quad (2)$$

where $a_2$ is the new bridge reading to balance.

Adding numerators and denominators, we have

$$\frac{x}{R}=\frac{1000+r_1+r_2+(a_1-a_2)}{1000+r_1+r_2-(a_1-a_2)} \quad . \quad . \quad . \quad (3)$$

---

[1] Stewart and Gee's *Practical Physics*, Part II., p. 148.

The error $f$ is thus eliminated. Moreover, the equation contains the small quantity $r_1 + r_2$ added to a large number in both numerator and denominator.

If the resistances $r_1$ and $r_2$ are disregarded, then the formula becomes

$$\frac{x}{R} = \frac{1000 + (a_1 - a_2)}{1000 - (a_1 - a_2)}. \quad . \quad . \quad . \quad (4)$$

If we consider formula (3), it will be evident that $r_1 + r_2$ would make no difference in the ratio if $x$ and $R$ were equal to each other, for their addition to numerator and denominator would be the addition of equals to equals, the ratio remaining unity. But under these circumstances $a_1 - a_2$ equals zero; and the larger the numerical value of $a_1 - a_2$, the greater will be the error introduced by neglecting the resistance $r_1 + r_2$. Hence $R$ should be adjusted so as to be as nearly equal to $x$ as possible.

**Example.**

It was desired to determine the resistance of a coil marked 1000 B.A. units. 1000 ohms in a box made by Nalder Bros. was used as the known resistance.

| | |
|---|---|
| Reading on the bridge wire . . . . . . . . . . | 497 |
| Reading after exchanging $x$ and $R$ . . . . . . . | 505 |

Here $\qquad a_1 - a_2 = -8,$

and $\qquad x = \frac{1000 - 8}{1000 + 8},$

or $\qquad x = 984.1$ ohms.

The temperature of the boxes was 23° and the known resistance was right at 15°. Its temperature coefficient was 0.00044; therefore the corrected value of $x$ was

$$x = 984.1\ [1 + 0.00044\ (23 - 15)]$$
$$= 987.6 \text{ at } 23°.$$

**38. Galvanometer Resistance by Thomson's Method.** — Connect the galvanometer, whose resistance is to be measured, in one of the proportional branches, $AB$, of a Wheatstone's bridge (Fig. 23). A second branch, $BC$, should consist of a resistance $R_3$, as nearly equal to the resistance of the galvanometer as convenient. The other two proportional branches, $R_1$ and $R_2$, are obtained on

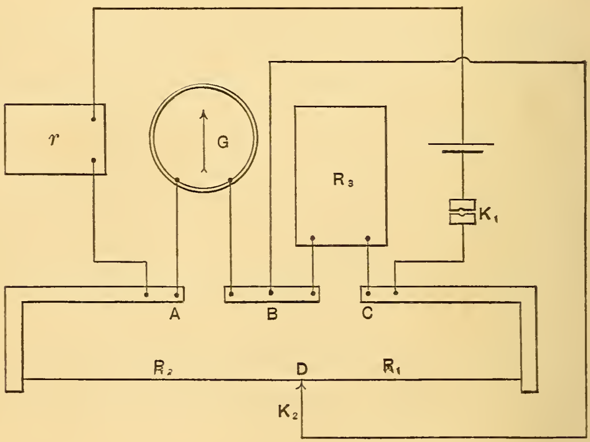



Fig. 23.

the wire of a slide metre bridge. The battery branch, which should be made up of a Daniell or other closed circuit cell, a resistance $r$, and a plug-key $K_1$ should join $A$ and $C$. The last branch should consist of wires of low resistance and a key $K_2$.

We should now close the plug-key $K_1$ in the battery branch and adjust the resistance $r$ until the galvanometer gives a large steady deflection. If the deflection goes beyond the end of the scale, the scale may be moved until a reading is obtained. The actual value of the

reading is not important. So long as $K_2$ remains open there should be no change in the deflection, no matter where on the slide wire the point $D$ may be taken; and if a point on this wire is found at which the potential is the same as that at $B$, key $K_2$ may be closed and there will still be no change in the deflection. In this case

$$R_1 : R_2 :: R_3 : G.$$

If a slide wire bridge or its equivalent is not obtainable, two resistance boxes may be used for $R_1$ and $R_2$. It will be found most convenient to keep the sum of their resistances constant, otherwise there will be different galvanometer readings with each different value of their sum, even before $K_2$ is closed.

For galvanometers of the d'Arsonval type (Art. 70) the slide wire of low resistance is much more convenient than the resistance boxes, as it acts like a low resistance shunt to bring the galvanometer to rest; however, with the resistance boxes a shunt of low resistance may be used in addition, which will practically accomplish the same thing.

Instead of one cell of battery and a resistance $r$, we may use two cells of slightly different E.M.F.'s in opposition to each other. Their difference will in general give sufficient E.M.F.

It is not well to exchange the battery and the key $K_2$, although a balance may be obtained in this way; for each change in the position of $D$ would then give a different galvanometer reading, which would make the experiment very tedious, as it would be necessary to wait for the galvanometer to come to rest after each change in the ratio.

It is necessary in this, as in other experiments with

the slide wire bridge, to exchange the positions of $G$ and $R_3$ and find the new position of $D$ to give a balance. It is also advisable to have a commutator in the circuit to reverse the direction of the current, although errors due to differences of temperature are practically eliminated by exchanging $G$ and $R_3$.

In the practice of this method it will be found convenient to make a trial measurement of $G$ with any convenient value for $R_3$, and determine the value of $G$ roughly. For this it is not necessary to exchange $G$ and $R_3$. Next make $R_3$ as near the value of $G$ as convenient, say to the nearest ohm; then proceed as above to make the more exact determination. The reason for making $R_3$ as nearly equal to $G$ as possible is that the resultant error is a minimum when $D$ is at the middle of the slide wire.

#### Example.

*First*, $R_3 = 100$ ohms; $R_1 = 611.4$; $R_2 = 388.6$; $\therefore G = 63.56$ ohms.

*Second*, make $R_3 = 64$ ohms. Then $R_1 = 503.2$; $R_2 = 496.8$. Exchanging $R_3$ and $G$, $R_1 = 498.8$; $R_2 = 501.2$.

Therefore $$G = 64\,\frac{1000 - 4.4}{1000 + 4.4} = 63.44 \text{ ohms}.$$

In both cases changing the direction of the current had no effect on the values of the readings.

### 39. Use of Slide Wire Bridge — Second Method.

— The bridge can be made more sensitive by inserting two resistances, $R_1$, $R_2$, in the openings at 1 and 2 (Fig. 24). These resistances should also be nearly equal to each other, or, more strictly, should have the same ratio as $x$ and $R$. If the resistance of unit length of the bridge wire is $\rho$, and $a$ and $b$ are the two parts of the

wire on either side of the slide when a balance has been secured, then

$$\frac{x}{R} = \frac{R_1 + a\rho}{R_2 + b\rho}.$$

The value of $x$ is thus known if $\rho$ has been determined. Since the resistance of $a$ and $b$ now form only a small part of the total resistance of their respective branches, any error in reading the position of the slider must pro-

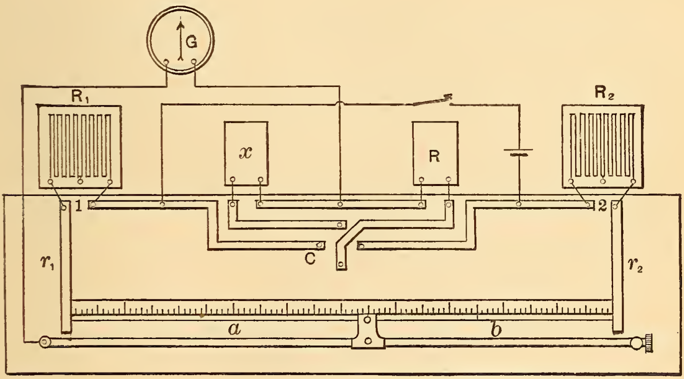



Fig. 24.

duce a smaller effect in the resulting value of $x$ than when $R_1$ and $R_2$ are not used. These auxiliary resistances may be considered simply as extensions of the two ends of the bridge wire.

If we introduce $r_1$ and $r_2$ as before, and suppose $R_1$ and $R_2$ expressed in terms of a division of the bridge wire, then

$$\frac{x}{R} = \frac{R_1 + r_1 + a_1}{R_2 + r_2 + c - a_1}. \quad . \quad . \quad . \quad (1)$$

Reversing,

$$\frac{x}{R} = \frac{R_2 + r_2 + c - a_2}{R_1 + r_1 + a_2}. \quad . \quad . \quad . \quad (2)$$

Here $c$ represents the entire length of the wire.

Adding (1) and (2) by addition of numerators and denominators,

$$\frac{x}{R} = \frac{R_1 + R_2 + r_1 + r_2 + c + (a_1 - a_2)}{R_1 + R_2 + r_1 + r_2 + c - (a_1 - a_2)}. \quad . \quad . \quad (3)$$

Put $R_1 + R_2 + r_1 + r_2 + c = r$, and $a_1 - a_2 = d$. Then

$$\frac{x}{R} = \frac{r+d}{r-d}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If $d$ is small compared to $r$, we may neglect small quantities of the second order and write,

$$\frac{x}{R} = 1 + \frac{2d}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the bridge is a metre long divided into millimetres, then the greatest value that $d$ can have is 1000, and the least may be perhaps .2 mm.

Let $r = 5000$; then from (4)

$$\frac{x}{R} = \frac{5000 + 1000}{5000 - 1000} = \frac{3}{2}.$$

This gives the maximum ratio of $x$ to $R$ to which the method is applicable.

From (5) $$\frac{x}{R} = 1 + \frac{0.4}{5000} = 1.00008.$$

This is the smallest ratio of $x$ to $R$ for which the bridge can be used with the assumed extensions, $R_1$ and $R_2$, each resistance twice that of the bridge wire.

The effect of increasing $r$ is to make the ratio of the resistance of the bridge wire to the whole resistance of the wire and extensions or auxiliary resistances, $R_1$ and $R_2$, smaller; this reduces the range of the bridge.

Fig. 25 is a bridge in which the connections are conveniently arranged to exchange $x$ and $R$ by means of

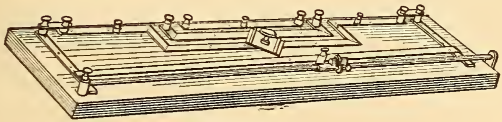


Fig. 25.

a single commutator. Fig. 24 shows the connections with end resistances attached. The contact maker is carried on a long brass rod by means of a sleeve, which can be clamped at any point, and the final adjustment is made by means of the attached slow-motion screw. The scale is divided into millimetres, and a vernier reads to tenths. Fig. 26 is a section of the contact device

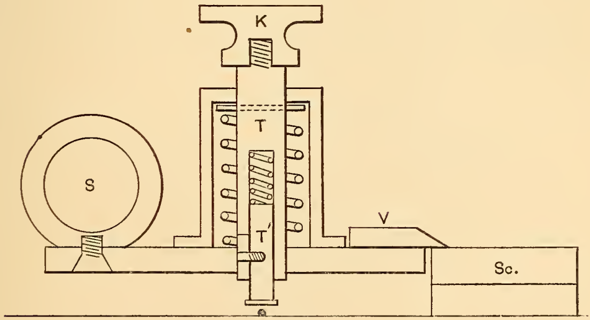



Fig. 26.

designed to allow a pressure on the bridge wire not exceeding a limited amount, which is governed by the small spring above the inner piston $T'$. The button $K$ is depressed against the force of the larger outer spiral spring. The descent of $T$ carries $T'$ with it till contact

is made with the wire. The piston $T'$ then enters $T$ against the pressure exerted by the small spiral. This device prevents any injury to the bridge wire by careless and excessive pressure on $K$. $S$ is the sleeve which slides on the long brass rod, $Sc$ is the scale, and $V$ the vernier. A short piece of the wire used on the bridge is soldered to the bottom of $T$, so as to make contact on the bridge wire at right angles. The rod $T$ is prevented from turning by a square shoulder at the top where it passes through the outer housing which encloses the larger spring. This device, made by our mechanician, R. H. Miller, has proved very satisfactory.

**Example.**

*Apparatus:* New bridge (least reading 0.1 mm.).

To measure resistance of manganin coil in oil.

Two nearly equal resistances of about 5 ohms used for lengthening the bridge wire — $R_1$ and $R_2$.

A. *Observation I.:* $R$ on side with $R_2$, and $x$ with $R_1$.

Then
$$\frac{x}{R} = \frac{R_1 + a_1}{R_2 + c - a_1}.$$

| $R$ in Ohms. | Reading of Bridge. |
|---|---|
| 4.6 | 554.0 |
| 4.7 | 275.6 |

*Observation II.:* $R$ and $x$ exchanged.

Then
$$\frac{x}{R} = \frac{R_2 + c - a_2}{R_1 + a_2}.$$

| $R$ in Ohms. | Reading of Bridge. |
|---|---|
| 4.6 | 463.3 |
| 4.7 | 739.5 |

B. *Determination of* $R_1$ (Art. 40):

(*a*)
$$\frac{2000 \text{ ohms}}{50 \text{ ohms}} = \frac{40}{1} = \frac{R_1 + 362.2}{637.8}.$$

$$\therefore\ R_1 = 25149.4.$$

(b) $$\frac{2000 \text{ ohms}}{40 \text{ ohms}} = \frac{50}{1} = \frac{R_1 + 487.1}{512.9}.$$

$$\therefore R_1 = 25157.9.$$

Mean value for $R_1 = 25153.6$ parts of the bridge wire.

*Determination of $R_2$:*

(a) $$\frac{2000 \text{ ohms}}{50 \text{ ohms}} = \frac{40}{1} = \frac{R_2 + 367.2}{632.8},$$

$$\therefore R_2 = 24944.8.$$

(b) $$\frac{2000 \text{ ohms}}{40 \text{ ohms}} = \frac{50}{1} = \frac{R_2 + 492.0}{508},$$

$$\therefore R_2 = 24908.$$

Mean value of $R_2 = 24926.4$ parts of the bridge wire.

*Calculation:*

Formula, $$\frac{x}{R} = \frac{r+d}{r-d},$$

in which $r = R_1 + R_2 + c$, and $d = a_1 - a_2$.

Therefore, $r = 25753.6 + 24926.4 + 1000 = 51080$,

and $d = 90.7$ for $R = 4.6$, and $-463.9$ for $R = 4.7$ ohms.

Hence, $$\frac{x}{4.6} = \frac{51080 + 90.7}{51080 - 90.7} = 1.00356;$$

and $$x = 4.6 \times 1.000356 = 4.615 \text{ ohms}.$$

Also, $$\frac{x}{4.7} = \frac{51080 - 463.9}{51080 + 463.9} = .982,$$

and $$x = 4.7 \times .982 = 4.615 \text{ ohms}.$$

**40. To find $R_1$ and $R_2$ in Terms of the Divisions of the Bridge Wire.** — If the auxiliary resistances $R_1$ and $R_2$ are used, the resistances $r_1$ and $r_2$ with a good bridge will be small in comparison, and they may safely be disregarded. Close the opening 2 with the heavy copper strap provided for the purpose, and put $R_1$ in the opening 1. Then with two known resistances, $P$ and

$Q$ (Fig. 27), obtain a balance and let the bridge reading be $a$. It is evident that $P$ should be larger than $Q$, or the point on which a balance may be obtained may lie

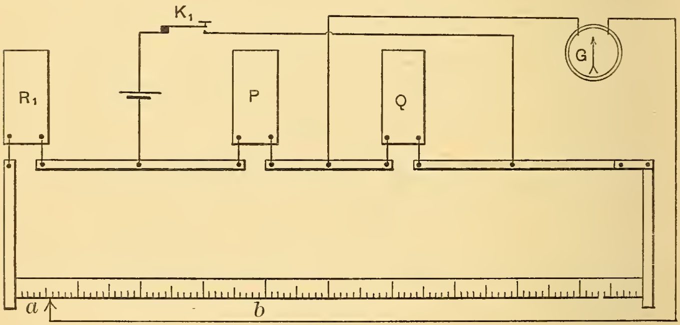



Fig. 27.

beyond the limits of the actual wire of the bridge, since $R_1$ is an extension of this wire. Then

$$\frac{P}{Q} = \frac{R_1 + a}{c - a}.$$

Whence $$R_1 = \frac{P}{Q}(c - a) - a.$$

$R_2$ may be determined in the same way.

**Example.**

$$\frac{P}{Q} = 5, \ a = 304.$$

Then $$R_1 = 5\,(1000 - 304) - 304$$
$$= 3176.$$

This result should be checked by measuring the resistance of the bridge wire and $R_1$ independently.

**41. The Carey Foster Method of comparing Resistances.**[1] — This method is especially useful for the

[1] *Philosophical Magazine*, May, 1884; Glazebrook and Shaw's *Practical Physics*, 2d ed., p. 561.

purpose of determining the difference between two nearly equal resistances of from one to ten ohms. The method is as follows:

Let $S_1$ and $S_2$ (Fig. 28) be the two nearly equal resistances to be compared, and let $R_1$ and $R_2$ be two nearly equal auxiliary resistances, which should not differ much from $S_1$ and $S_2$. Let $r_1$ and $r_2$ be the resistances of $NM$ and $N'M'$ respectively. Then if $\rho$ be the resistance of unit length of the bridge wire,

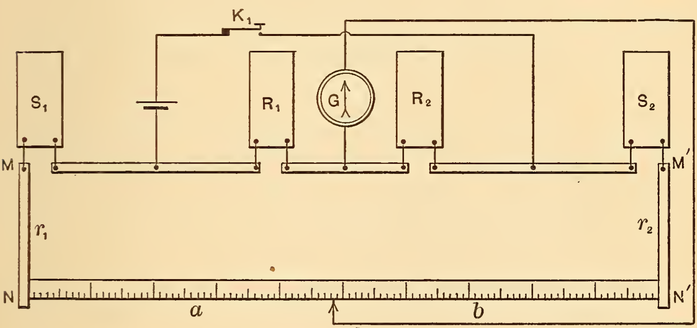



Fig. 28.

$$\frac{R_1}{R_2} = \frac{S_1 + r_1 + \rho a_1}{S_2 + r_2 + \rho b_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let now $S_1$ and $S_2$ exchange places, and let $a_2$ be the reading on the bridge wire for the new balance.

Then
$$\frac{R_1}{R_2} = \frac{S_2 + r_1 + \rho a_2}{S_1 + r_2 + \rho b_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Adding unity to both sides of (1) and (2), we have

$$\frac{R_1 + R_2}{R_2} = \frac{S_1 + r_1 + \rho a_1 + S_2 + r_2 + \rho b_1}{S_2 + r_2 + \rho b_1} =$$

$$\frac{S_2 + r_1 + \rho a_2 + S_1 + r_2 + \rho b_2}{S_1 + r_2 + \rho b_2}.$$

Since $a_1 + b_1 = c = a_2 + b_2$, the numerators of these fractions are equal; hence the denominators are also equal, or

$$S_1 + r_2 + \rho b_2 = S_2 + r_2 + \rho b_1.$$

Therefore $S_1 - S_2 = \rho\,(b_1 - b_2) = \rho\,(a_2 - a_1)$.

The difference in the resistance of the two coils, $S_1$ and $S_2$, is therefore equal to the resistance of that part of the bridge wire between the points at which the slide rests for the balance in the two positions of the coils $S_1$ and $S_2$.

**Example.**

$S_1$ = coil No. 273, 0.99795 of an ohm at 15.4° C. Temperature coefficient 0.00023.

$S_2$ = coil No. 194. Resistance to be determined.

$\rho = 0.00095459$ at 20° C. (Art. 42).

$$\left.\begin{array}{l} S_1 \text{ left, } S_2 \text{ right, reading } 508.1 \\ S_2 \text{ left, } S_1 \text{ right, reading } 497.25 \end{array}\right\} \begin{array}{l} \text{Temperature of } S_1 \text{ and } S_2 \\ 19.3^\circ \text{ C., of bridge } 20^\circ \text{ C.} \end{array}$$

$\therefore S_2 = 0.99795\,[1 + .00023\,(19.3 - 15.4)] - 0.00095459\,(508.1 - 497.25)$.

$S_2 = 0.98849$ ohm at 19.3° C.

**42. The Determination of $\rho$.** — The methods to be pursued in the determination of the resistance of unit length of the bridge wire will depend to a considerable extent upon the value of this resistance and the length of the wire. Since

$$S_1 - S_2 = \rho\,(a_2 - a_1),\ \rho = \frac{S_1 - S_2}{a_2 - a_1}.$$

Hence, if the difference between the resistance of the two coils $S_1$ and $S_2$ is known, $\rho$ can be found by determining by two successive balances the length of the bridge wire corresponding to this known difference. For this purpose three standard coils may be used, two

1-ohm coils and one 10-ohm. The 10-ohm coil and one of the units are placed in multiple on one side, and the other unit on the other. The resistance $S_2$ of the two in parallel is

$$\frac{1 \times 10}{1 + 10} = \frac{10}{11}.$$

Hence $$S_1 - S_2 = 1 - \frac{10}{11} = \frac{1}{11} = .09091,$$

and $$\rho = \frac{.09091}{a_2 - a_1}.$$

If the entire resistance of the bridge wire is considerably in excess of one ohm, then $\rho$ may be found by the aid of a single standard ohm and a heavy copper link, the resistance of which may be neglected. Then

$$\rho = \frac{1}{a_2 - a_1}.$$

With 1 and 100 ohms in parallel the difference between 1 and the two others in parallel is .009901.

A third method may be used when only one standard coil (and that of greater resistance than the bridge wire) is available. In the particular case considered the bridge wire really had a resistance of about 20 ohms; but, to obtain greater sensitiveness, it was used with a coil of 1 ohm resistance in shunt. The equivalent resistance of the combination was then about $\frac{20}{21}$ of an ohm, and the difference of readings on the bridge wire was increased about twenty times. The standard coil used, marked "No. 273—1 'legal' ohm at 12.8° C.," called coil *A* in what follows, had a resistance of 0.99795 of an ohm at 15.4° C. The two other coils were taken as unknown quantities. Coil *B* was a standard coil marked "No.

194 — 1 B.A. unit at 15° C." This value, however, was somewhat in error. The third coil $C$ was of about $\frac{3}{4}$ of an ohm resistance. By making the resistance of $C$ a mean between that of $A$ and of $A$ and $B$ in parallel, the effect of errors of observation was reduced to a minimum.

In the first arrangement coils $A$ and $B$ were placed on opposite sides of the bridge, and their difference measured in terms of $\rho$. In this, as in the following arrangements, the coils were in water baths of practically the same temperature as that of the room. It is necessary for this experiment that $A$ should be of exactly the same temperature as $B$, though that of $C$ may be different.

To obtain this equality of temperature the water in the two water-baths should be well mixed, repeating the operation several times if need be. If the coils and the water are practically at the temperature of the room, the whole will rapidly reach a temperature which will remain constant for the experiment. Should the temperature vary, it will be found in general better to repeat the observations than to correct for the variations, though, of course, the latter is possible.

If the bridge wire is used with a shunt of relatively low resistance, the temperature of the shunt is of more importance than that of the bridge wire. In fact, if the bridge wire has $n$ times the resistance of the shunt, a change of one degree in the temperature of the latter will produce $n$ times as great a change in the value of $\rho$ as would be produced by a change of one degree in the temperature of the former.

In the second arrangement $A$ and $B$ were placed in parallel on one side, and $C$ on the other. The difference between $A$ and $B$ in parallel and $C$ was measured in terms of $\rho$.

In the third arrangement $B$ was removed, and the difference between $A$ and $C$ measured in terms of $\rho$. Let the bridge reading in these three arrangements be $a$, $a'$; $b$, $b'$; $c$, $c'$. Expressed in the form of equations, these three arrangements give the following relations:

$$A - B = (a - a')\,\rho = m\rho, \quad . \quad . \quad . \quad (1)$$

$$C - \frac{AB}{A+B} = (b - b')\,\rho = n\rho, \quad . \quad (2)$$

$$A - C = (c - c')\,\rho = p\rho\,; \quad . \quad . \quad . \quad (3)$$

adding (2) and (3),

$$A - \frac{AB}{A+B} = \frac{A^2}{A+B} = (n+p)\,\rho. \quad . \quad . \quad (4)$$

Eliminating $B$ between (1) and (4), we obtain

$$\rho = \frac{A}{n + p \pm \sqrt{(n+p)(n+p-m)}}. \quad . \quad (5)$$

To find which sign of the $\pm$ is to be taken, substitute this value of $\rho$ in (4). We obtain

$$\frac{A}{A+B} = \frac{1}{1 \pm \sqrt{1 - \dfrac{m}{n+p}}}.$$

From this it is evident that the plus sign should be taken, as otherwise $B$ must be a minus quantity, which would be absurd.

Consequently,

$$\rho = \frac{A}{n + p + \sqrt{(n+p)(n+p-m)}}. \quad . \quad (6)$$

**Example.**

$A = 0.99795\ [1 + 0.00023\ (19.3 - 15.4)]$.

$a = 508.1$. Coils $A$ and $B$ were at 19.3° C.

$a' = 497.25$. The bridge wire was at 20° C.

Whence, $A - B = 10.85\rho = m\rho$.

$b = 634.4.$ Temperatures as before.
$b' = 369.0.$

Whence, $$C - \frac{AB}{A+B} = 265.4\rho = n\rho.$$

$c = 632.6.$ Temperatures as before.
$c' = 372.1.$

Whence, $$A - C = 260.5\rho = p\rho.$$

Therefore,

$$\rho = \frac{0.99795\ [1 + 0.00023\ (19.3 - 15.4)]}{525.9 + \sqrt{525.9 \times 515.05}} = 0.00095459 \text{ at } 20^\circ \text{ C.}$$

**43. Apparatus for exchanging the Two Coils to be compared.** — Since the coils to be compared should be placed in water or oil baths, it is inconvenient to exchange their position from one side of the bridge to the other. A convenient and reliable device for this purpose is a necessity. Fig. 29 shows one form which may be used in connection with a slide wire bridge by connecting with two binding-screws at one opening of the bridge. The connections are shown through the two commutators. If now both commutators are given a quarter turn, the circuits will be by the dotted lines, and it will be evident on tracing them that the two coils $S_1$ and $S_2$ have exchanged sides on the bridge.

An essential condition of such a commutating device is that the two sides shall be as perfectly symmetrical as possible, so that when the coils are exchanged unequal resistances are not exchanged along with them. An inspection of the diagram will show that the device is symmetrical.

Connections are made by means of mercury cups. These should be of copper, with flat inside bottoms; and the copper rods composing the terminals of the coils compared, as well as the ends of the heavy copper links

of the commutators, should be well amalgamated, and they should be kept firmly pressed against the bottoms of the cups. Care should be taken to keep the amalgamated ends of the rods clean.

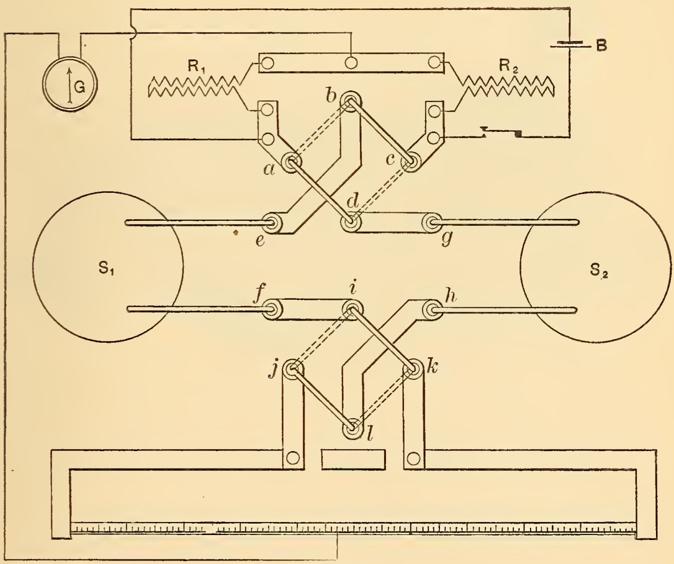



Fig. 29.

The complete apparatus, shown in Fig. 30, contains the auxiliary coils *S* wound together non-inductively. They can be easily removed and others can be substituted for them. The battery is connected to the binding-posts marked *Ba*. There are four mercury cups on either side for the purpose of placing two standard coils in parallel. Copper binding-posts are also provided for measurements not requiring the highest accuracy. The rods in each commutator are loosely mounted in a

hard-rubber platform. They then adjust themselves to the bottom of the mercury cups, and good contact is secured. This apparatus may be used with any form of bridge.

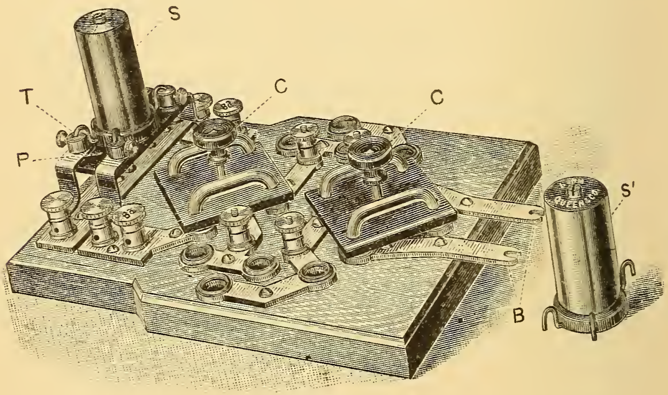



Fig. 30.

It is desirable to employ in the battery circuit another commutator, so as to reverse the circuit when the coils are exchanged, for the purpose of eliminating any possible thermal currents, or electromotive forces of thermal origin.

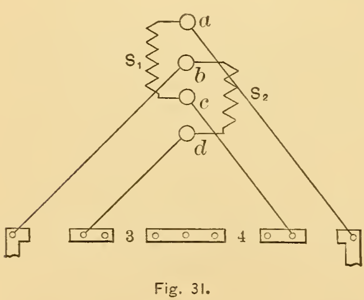



Fig. 31.

Fig. 31 shows the exchanging device employed by Mr. Glazebrook in comparing the standard coils of the British Association. It is only necessary to move one coil up and the other down one step in order to have them exchange sides.

**44. The Calibration of the Bridge Wire — First Method.** — The Carey Foster method itself may be applied to the calibration of the bridge wire. The calibration consists in laying off on the wire a series of exactly equal resistances. The process corrects not only for inequalities in the wire, but for errors of the scale as well. These inequalities and errors have thus far been neglected; but they are always appreciable, though the error arising from neglecting them may be very small.

It is evident that if the balance point for a given pair of coils $S_1$ and $S_2$ can be shifted along the wire of the bridge by successive steps, and the readings $a_1$ and $a_2$ taken, the process will result in laying off equal resistances on the wire, each equal to $S_1 - S_2$. For this purpose take two resistance boxes of good adjustment for the auxiliary resistances $R_1$ and $R_2$. Let the difference between the two coils $S_1$ and $S_2$ be small enough to give convenient steps along the bridge wire. Adjust the auxiliary resistances, which should be as large as the sensibility of the galvanometer will permit, till the balance point $a_1$ falls toward the zero end of the bridge wire. Since generally only a portion of the bridge wire near the centre will be used in the Carey Foster method, it is not necessary to calibrate it throughout its entire length. Find now by the exchange of the coils $S_1$ and $S_2$ the length of bridge wire having a resistance equal to their difference. Call this length $l_1$. Next shift resistance from $R_1$ to $R_2$ till with $S_1$ and $S_2$ in the first position the point of balance nearly coincides with the last point. It is not necessary to make these points agree exactly, though if they do the tabulation of the results is a little simpler. We shall assume for the present that the points do coincide, or that the distances

$l_1$, $l_2$, etc., are end to end measurements. Now exchange $S_1$ and $S_2$, and by balancing again find $l_2$, or a second length of the wire having a resistance equal to $S_1 - S_2$. Reverse the coils, shift resistance from $R_1$ to $R_2$ again till the beginning of the length of calibration $l_3$ corresponds with the end of $l_2$. Then exchange coils and balance again to find $l_3$. Continue the process till the required length of the bridge wire has been traversed. The balance first obtained should be tested over again occasionally to be assured that $S_1 - S_2$ has not changed by reason of a change in temperature. These coils should be kept in a water bath to avoid changes of temperature as far as possible. It is equally important that the temperature of the bridge should remain constant. If any change in the length $l_1$ occurs, the other values of $l$ must be corrected in consequence.

Now let the beginning of $l_1$ on the scale read $x$, and the end of the $n^{th}$ length read $y$.

Then $l_1 + l_2 + l_3 + \ . \ . \ . + l_n = y - x$, and $\dfrac{y-x}{n} = l$,

the mean length of calibration.

Let

$$l - l_1 = \delta_1$$
$$2l - (l_1 + l_2) = \delta_2$$
$$3l - (l_1 + l_2 + l_3) = \delta_3$$
$$4l - (l_1 + \ . \ . \ . + l_4) = \delta_4.$$
$$. \quad . \quad . \quad . \quad . \quad .$$
$$(n-1)\, l - (l_1 + \ . \ . \ . + l_{n-1}) = \delta_{n-1}$$
$$nl - (l_1 + \ . \ . \ . + l_n) = \delta_n = 0.$$

$\delta_n$ is necessarily zero as $l = \dfrac{l_1 + \ . \ . \ . + l_n}{n}$. These quantities, $\delta_1$, $\delta_2$, $\delta_3$, etc., are the corrections for the readings of the bridge wire. They are the amount which

must be added algebraically to the readings to obtain the corrected readings. The correction for any length of the wire is the difference between the corrections at the ends of the length. The quantities $\delta_1$, $\delta_2$, $\delta_3$, may be either positive or negative. The plus and minus signs are used here in their algebraic sense.

**Example I.**

| Resistances. | | Bridge Readings. | Lengths. | Corrected Readings. | Corrections. |
|---|---|---|---|---|---|
| | | 361.15 | | 361.15 | 0.0 |
| 818 | 1341 | | 12.0 | | |
| | | 373.15 | | 373.113 | — 0.04 |
| 816 | 1277 | | 12.0 | | |
| | | 385.15 | | 385.076 | — 0.07 |
| 853 | 1275 | | 12.05 | | |
| | | 397.2 | | 397.039 | — 0.16 |
| 848 | 1210 | | 12.05 | | |
| | | 409.25 | | 409.002 | — 0.25 |
| 885 | 1207 | | 12.15 | | |
| | | 421.4 | | 420.965 | — 0.44 |
| 885 | 1151 | | 12.0 | | |
| | | 433.4 | | 432.928 | — 0.47 |
| 924 | 1152 | | 11.9 | | |
| | | 445.3 | | 444.891 | — 0.41 |
| 923 | 1101 | | 11.9 | | |
| | | 457.2 | | 456.854 | — 0.35 |
| 963 | 1099 | | 11.8 | | |
| | | 469.0 | | 468.817 | — 0.18 |
| 962 | 1051 | | 12.0 | | |
| | | 481.0 | | 480.780 | — 0.22 |
| 1001 | 1047 | | 12.0 | | |
| | | 493.0 | | 492.743 | — 0.26 |
| 1000 | 1000 | | 12.0 | | |
| | | 505.0 | | 504.706 | — 0.29 |
| 1045 | 1000 | | 12.0 | | |
| | | 517.0 | | 516.669 | — 0.33 |
| 1046 | 958 | | 11.9 | | |

| Resistances. | | Bridge Readings. | Lengths. | Corrected Readings. | Corrections. |
|---|---|---|---|---|---|
| | | 528.9 | | 528.633 | — 0.27 |
| 1093 | 958 | | 11.8 | | |
| | | 540.7 | | 540.596 | — 0.10 |
| 1091 | 915 | | 11.9 | | |
| | | 552.6 | | 552.559 | — 0.04 |
| 1139 | 914 | | 11.9 | | |
| | | 564.5 | | 564.522 | + 0.02 |
| 1140 | 875 | | 11.9 | | |
| | | 576.4 | | 576.485 | + 0.08 |
| 1192 | 875 | | 12.0 | | |
| | | 588.4 | | 588.448 | + 0.05 |
| 1193 | 837 | | 11.9 | | |
| | | 600.3 | | 600.411 | + 0.11 |
| 1246 | 835 | | 12.05 | | |
| | | 612.35 | | 612.374 | + 0.02 |
| 1243 | 796 | | 11.95 | | |
| | | 624.3 | | 624.337 | + 0.04 |
| 1281 | 783 | | 12.0 | | |
| | | 636.3 | | 636.3 | 0.0 |

Next let us suppose that the distances $l_1$, $l_2$, $l_3$, etc., overlap a little on the wire. As before let

$$l = \frac{l_1 + l_2 + \quad . \quad . \quad . \quad + l_n}{n},$$

and let

$$\begin{aligned} l - l_1 &= \delta_1 \\ 2l - (l_1 + l_2) &= \delta_2 \\ 3l - (l_1 + l_2 + l_3) &= \delta_3 \\ 4l - (l_1 + \quad . \quad . \quad . \quad + l_4) &= \delta_4, \end{aligned}$$

etc.

Also let

$$l' = \frac{y - x}{n}.$$

Then $\delta_1$, $\delta_2$, $\delta_3$, etc., are again the amounts which must be added to $l_1$, $l_1 + l_2$, $l_1 + l_2 + l_3$, etc., to make them

equal to $l$, $2l$, $3l$, etc.; and supposing the overlap to be an insignificant part of each length, we may consider $\delta_1$, $\delta_2$, etc., to be the corrections from one end of the calibrated portion of the wire up to the point considered. Strictly speaking, we should reduce these values $\delta_1$, $\delta_2$, $\delta_3$, etc., in proportion to the amount of overlap.

**Example II.**

| Bridge Readings. | Lengths. | Corrections. |
|---|---|---|
| 2.30 | | 0.0 |
| 40.05 | 37.75 | |
| 39.95 | | +0.10 |
| 77.95 | 38. | |
| 77.85 | | — 0.04 |
| 115.85 | 38. | |
| 115.75 | | — 0.19 |
| 153.65 | 37.90 | |
| 153.55 | | — 0.23 |
| 191.45 | 37.90 | |
| 191.15 | | — 0.28 |
| 229.00 | 37.85 | |
| 228.90 | | — 0.28 |
| 266.85 | 37.95 | |
| 266.75 | | — 0.37 |
| 304.75 | 38.00 | |
| 304.45 | | — 0.52 |
| 342.30 | 37.85 | |
| 342.20 | | — 0.51 |
| 380.00 | 37.80 | |
| 379.9 | | — 0.46 |
| 417.85 | 37.95 | |
| 417.8 | | — 0.55 |
| 455.5 | 37.70 | |
| 455.4 | | — 0.40 |
| 493.2 | 37.80 | |
| 493.1 | | — 0.35 |
| 530.85 | 37.75 | |
| 530.75 | | — 0.24 |

| Bridge Readings. | Lengths. | Corrections. |
|---|---|---|
| 568.25 | 37.50 | |
| 568.15 | | + 0.11 |
| 606. | 37.85 | |
| 605.9 | | + 0.12 |
| 643.85 | 37.95 | |
| 643.75 | | + 0.02 |
| 681.45 | 37.70 | |
| 681.30 | | + 0.17 |
| 719.30 | 38. | |
| 719.50 | | + 0.02 |
| 756.85 | 37.35 | |
| 756.8 | | + 0.53 |
| 794.9 | 38.10 | |
| 794.8 | | + 0.28 |
| 832.75 | 37.95 | |
| 832.65 | | + 0.18 |
| 870.40 | 37.75 | |
| 870.30 | | + 0.29 |
| 908.45 | 38.15 | |
| 908.35 | | − 0.01 |
| 945. | 37.65 | |
| 944.95 | | + 0.20 |
| 983. | 38.05 | 0.0 |
| Mean | 37.854 | |

The successive points at which the correction should be applied are $l'$, $2l'$, $3l'$, etc.

**45. Calibration of Bridge Wire — Second Method.**[1] — Make as many approximately equal resistances as there are steps in the desired calibration. Let this number be $n$. Fig. 32 shows ten such resistances. Let them connect the mercury cups 1, 2, 3, etc. To insure good contact each small resistance should be soldered to a short

[1] Carl Barus, *Bulletin U.S. Geological Survey*, No. 14.

heavy rod of copper. If $L$ is the length of $AC$ to be calibrated, and $l'$ the interval of calibration

$$\frac{L}{n} = l'.$$

Find a point $M_1$ on $AB$ having the same potential as $N_1$, and $M_2$ the same as $N_2$. This is done by means of the sensitive galvanometer $G$.

Then exchange wires Nos. I. and II. Find points on $AC$ having the same potential as $N_2$, $N_3$, respectively. Call these points $M_2$, $M'_3$. The resistance of I. should

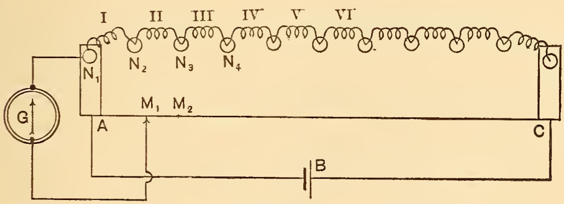



Fig. 32.

be such that the reading for $M'_2$, $M'_3$, etc., shall be a little smaller than for $M_2$, $M_3$, etc. That is, the calibration distances set off should overlap a little.

Then exchange I. and III. and perform the same operations as before. Continue the process till the conductor I. has been carried along the entire series and finally takes the place of the last one. The result is to lay off along the bridge wire distances such that the P.D. between their ends is the same as between the ends of conductor I. If the current remains absolutely constant, all these potential differences are equal to each other, and therefore the resistances of the successive lengths laid off are also equal. They will equal one another if the current does not remain constant, provided the rela-

tive resistance of conductor I. to this part of the divided circuit remain the same; for any decrease in the current will cause a decrease in the P.D. between $A$ and $C$, and this P.D. is the same in going from $A$ to $C$ by either path. So long therefore as conductor I. bears the same ratio to the entire resistance of the path of which it forms a part, the resistance between the points $M_1$, $M_2$, $M_2'$, $M_3$, etc., will be the same. The effect is then to lay off a series of equal resistance lengths on $AC$, and these lengths overlap somewhat.

Then we have as before

$$\frac{l_1 + l_2 + l_3 + \quad \cdot \quad \cdot \quad \cdot \quad l_n}{n} = l,$$

and the results are treated in the same way as by the other method. The corrections will be

$$\begin{array}{l} \text{At } \ l', + \delta_1 \\ \text{" } \ 2l', + \delta_2 \\ \text{" } \ 3l', + \delta_3 \\ \text{etc., etc.} \end{array}$$

**46. Measurement of the Temperature Coefficient.** — The Carey Foster method of comparing resistances is especially adapted to the measurement of the variation of the resistance of a conductor with temperature. The process consists in comparing the resistances of two coils, one of which is maintained at an unvarying temperature, while that of the other is changed. The resistance which is maintained constant may be a standard coil, and the other is made of the wire or conductor to be investigated. Both of them must be immersed in a bath; the one in order that the temperature may remain invariable, and the other that its temperature may be varied and

accurately measured. The equation expressing the resistance of a conductor at any temperature is, to a first approximation,

$$R_t = R_0 (1 + at).$$

If now the resistances of the conductor under test at temperatures $t_1$ and $t_2$ are $R_1$ and $R_2$, then

$$R_1 = R_0 (1 + at_1)$$

and $$R_2 = R_0 (1 + at_2).$$

Subtracting, $R_1 - R_2 = R_0 a (t_1 - t_2)$

and $$a = \frac{R_1 - R_2}{R_0} \cdot \frac{1}{t_1 - t_2}.$$

$R_0$ does not need to be known with great accuracy, for $a$ and $R_1 - R_2$ are very small; and when the numerator of a fraction is relatively small, a small change in the denominator produces only an inappreciable change in the value of the fraction. A first or approximate value of $a$ may be found, and this value may be used to find the value of $R_0$ with sufficient accuracy. A second approximation will then give a nearer value of $a$.

**Example.**

*Temperature Coefficient of a Coil of Platinoid Wire.*

| Standard Coil ($S$). | | Bridge Wire. | | | | | $X - S$. | Tested Coil ($X$). | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Temperature (C). | Resistance (ohms). | Temperature (C). | Readings. $a_1$ | Readings. $a_2$ | Readings. $a_1 - a_2$ | $\rho \times 10^4$ | $\rho (a_1 - a_2)$ | Resistance (ohms). | Temperature (C). | Increase per Degree. |
| 15.2 | 9.8666 | 18.1 | 557.0 | 442.5 | 114.5 | 9.538 | 0.1090 | 9.9756 | 16.7 | ........ |
| 15.2 | 9.8666 | 18.3 | 563.4 | 435.6 | 127.8 | 9.538 | 0.1218 | 9.9884 | 23.1 | 0.00200 |
| 15.2 | 9.8666 | 18.4 | 570.1 | 429.0 | 141.1 | 9.539 | 0.1345 | 10.0011 | 29.1 | 0.00216 |
| 15.2 | 9.8666 | 18.4 | 576.3 | 423.3 | 153.0 | 9.539 | 0.1459 | 10.0125 | 34.9 | 0.00196 |
| 15.2 | 9.8666 | 18.5 | 581.2 | 418.0 | 163.2 | 9.540 | 0.1557 | 10.0223 | 40.3 | 0.00181 |
| 15.4 | 9.8672 | 18.6 | 586.5 | 412.6 | 173.9 | 9.540 | 0.1659 | 10.0331 | 45.2 | 0.00220 |
| 15.4 | 9.8672 | 18.7 | 589.5 | 409.4 | 180.1 | 9.540 | 0.1718 | 10.0390 | 48.1 | 0 00203 |
| 15.7 | 9.8681 | 18.8 | 595.0 | 403.5 | 191.5 | 9.540 | 0.1827 | 10.0508 | 54.0 | 0.00200 |

Total increase in $X = 10.0508 - 9.9756 = 0.0752$ ohm.

Increase per degree $= \dfrac{0.0752}{54 - 16.7} = 0.002016.$

Resistance of $X$ at 0° C. $= 9.9756 - 16.7 \times 0.002016 = 9.9409$ ohms.

Therefore, $\qquad a = \dfrac{0.002016}{9.9409} = 0.000203.$

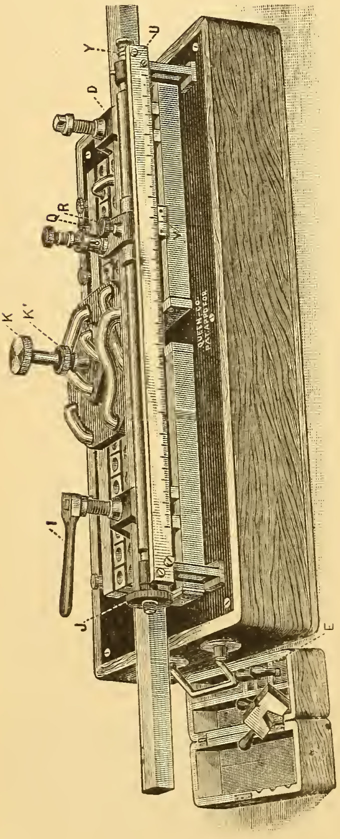


Fig. 33.

**47. The Conductivity Bridge.** — The Carey Foster method furnishes an elegant means of measuring very small resistances, such as the resistance of metal bars, rods, and the like. For this purpose a special piece of apparatus is required. Its principle is precisely the same as that employed in finding $\rho$ from a known difference of resistance. The rod or bar to be measured takes the place of the bridge wire, and its conductivity is found by laying off a length of the rod equal in resistance to a known resistance represented by accurately adjusted coils in parallel. The bar to be measured is held securely by clamps $D$ (Fig. 33). It is parallel

to a scale *U*, which is read by a vernier to 1-20th mm. The sliding contact may be clamped by the screw *R* to the rod which carries it, and a slow motion may then be given to it by the nut *J* working against the spring *Y*. The commutator *K* commutes both the known resistances and the battery, either simultaneously or separately. The adjusted coils are inserted in parallel by means of heavy copper links dipping into suitable mercury cups in large masses of copper. The battery and galvanometer are connected by means of binding-posts at the back of the instrument. The method of operation is precisely the same as in the Carey Foster method. A known difference of resistance is laid off on the bar to be tested, and the length of the bar between the two contacts is measured by means of the scale and vernier. The measurement is independent of the contacts on the bar.

**48. Insulation Resistance by Known Potential Differences.**[1] — This method of measuring a high resist-

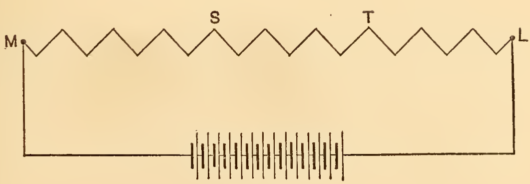



Fig. 34.

ance consists in comparing the current sent by a given P.D. through it with that sent through a known resistance by a fraction of this same P.D. A potential

[1] Ayrton's *Practical Electricity*, p. 278.

difference may be subdivided into known fractions by causing a steady current to flow through a very high resistance with known subdivisions. Then the P.D. between any two points $ST$ (Fig. 34) bears to the P.D. between the points $ML$ at the extremities of the high resistance the same ratio that the resistance of the part $ST$ bears to the whole resistance $ML$.

Let the entire P.D. between $L$ and $M$ be employed to

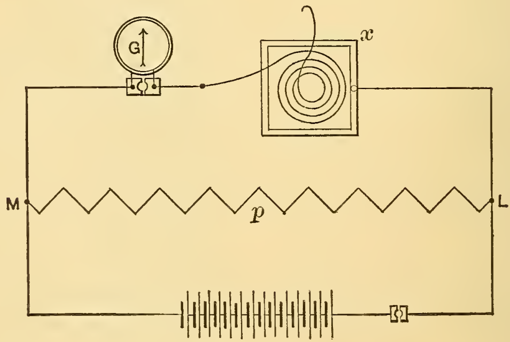



Fig. 35.

send a current $I_1$ through the unknown high resistance $x$ and the galvanometer $G$ (Fig. 35). The galvanometer must be one of the highest sensibility. Next let the P.D. between $L$ and $T$ (Fig. 36) be employed to send a current through the known resistance $r$, and the galvanometer shunted with resistance $s$; $r$ must be large with respect to $q$. Let the current through the galvanometer be $I_2$.

Then, $$\frac{I_1}{I_2}=\frac{p}{q}\cdot\frac{r+\dfrac{sg}{s+g}}{x+g}\cdot\frac{s+g}{s}.$$

Whence, $$x + g = \frac{I_2}{I_1} \cdot \frac{p}{q}\left(r + \frac{sg}{s+g}\right)\frac{s+g}{s},$$

and $$x = \frac{I_2}{I_1} \cdot \frac{p}{q}\left(r + \frac{sg}{s+g}\right)\frac{s+g}{s} - g.$$

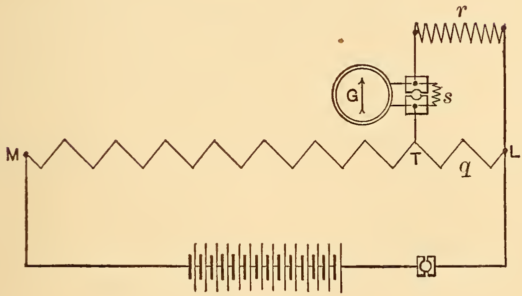



Fig. 36.

If $\frac{sg}{s+g}$ may be neglected in comparison with $r$, and $g$ in comparison with $x$, then

$$x = \frac{I_2}{I_1} \cdot \frac{p}{q} \cdot r\,\frac{s+g}{s};$$

or, if $I_1$ and $I_2$ are proportional to the deflections of the galvanometer in the two cases,

$$x = \frac{d_2}{d_1} \cdot \frac{p}{q} \cdot r\,\frac{s+g}{s}.$$

**Example.**

$r = 250{,}000$ ohms; $p = 10{,}200$ ohms; $d_1 = 48.2$;

$s = \frac{g}{9}$; $q = 200$ ohms; $d_2 = 38.0$.

$$x = \frac{10{,}200 \times 38.0 \times 250{,}000 \times 10}{200 \times 48.2} = 100.5 \times 10^6 \text{ ohms}$$

$$= 100.5 \text{ megohms}.$$

**49. Insulation Resistance by Direct Deflection.** — When the constancy of the battery cannot be relied on, it may be found advantageous to proceed as follows: First find the figure of merit of the galvanometer (Art. 29), *i.e.*, the current which will produce a deflection of one division of the scale. The galvanometer then becomes an ammeter, and may be used in connection

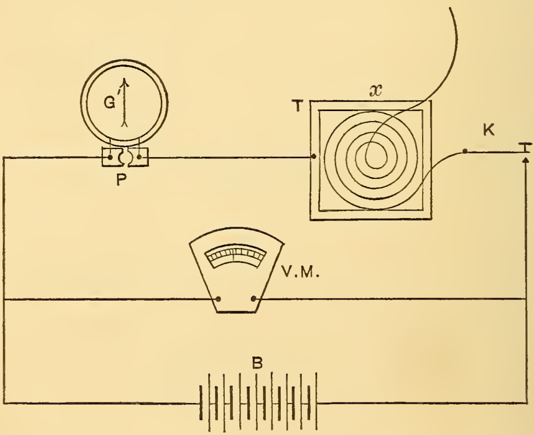



Fig. 37.

with a voltmeter V.M. (Fig. 37) to measure the unknown resistance $x$. If the $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ shunt is used with the galvanometer, its figure of merit is correspondingly increased. Let $F$ equal the figure of merit, $d$ the deflection with $x$, as in the figure, $V$ the number of volts shown by the voltmeter, and $g$ the resistance of the galvanometer. Then the current is $Fd$, and by Ohm's law

$$g + x = \frac{V}{Fd}, \text{ or } x = \frac{V}{Fd} - g.$$

**Example.**

*Test of a Piece of Common Line Wire.*

Diameter over insulation 8.2 mm.
Diameter of bare wire 4.13 mm.
Length under water 90 ft.

$r = 250{,}000$ ohms. E.M.F. of Clark cell 1.434 volts. $s = \frac{g}{99}$.
$d_1 = 143.4$ mm.

Figure of merit (with shunt) $= \frac{1.434}{143.4 \times 250{,}000} = 0.000{,}000{,}04$ ampere per mm. = 0.04 micro-ampere per mm.

Figure of merit (without shunt) = 0.0004 micro-ampere per mm.

| Time after immersion. h. m. | Volts shown by voltmeter. | Deflections in millimetres. | Current in micro-amperes. | Insulation resistance in megohms. | Resistance in megohms per mile. |
|---|---|---|---|---|---|
| 0 : 00 | 50.2 | 176 | 0.0704 | 713 | 12.2 |
| 05 | 50.2 | 165 | 0.066 | 761 | 13.0 |
| 10 | 50.2 | 160 | 0.064 | 784 | 13.4 |
| 15 | 50.2 | 160 | 0.064 | 784 | 13.4 |
| 27 | 50.2 | 177 | 0.0708 | 709 | 12.0 |
| 30 | 50.2 | 184 | 0.0736 | 682 | 11.6 |
| . . . . . . . . | . . . . . . | . . . . . . | . . . . . . . . | . . . . . . | . . . . . . |
| 1 : 00 | 50.2 | 232 | 0.0928 | 541 | 9.2 |
| 2 : 35 | 50.2 | 670 | 0.268 | 187 | 3.2 |
| 5 : 00 | 50.5 | 1925 | 0.77 | 65 | 1.12 |
| 27 : 00 | 66.5 | 38000 | 15.2 | 4.37 | 0.075 |

In the column of "Deflections in millimetres," the larger numbers are the products of the deflections and the multiplying power of the shunt.

**50. Insulation Resistance by Leakage.**[1] — The method consists in charging the cable as a condenser, letting it leak for a few observed seconds, and then charging to the full potential again by connecting through the galvanometer.

[1] *Electrical Engineer*, May 20, 1891, p. 565.

*First.* To find the constant of the ballistic galvanometer $G$ (Art. 97). This may be done in two ways. The first consists in charging a condenser of known capacity by a known E.M.F., and then discharging through the galvanometer. Let the apparatus be set up as shown in Fig. 38, in which $K$ is a charge and discharge key, $C$ is the condenser, and $G$ the galvanometer. The battery $B$ may be a standard cell, the E.M.F. of which is known. Then if $Q$ is the quantity of electricity discharged through the galvanometer, $C$ the capacity of the condenser, and $E_1$ the E.M.F. of the cell,

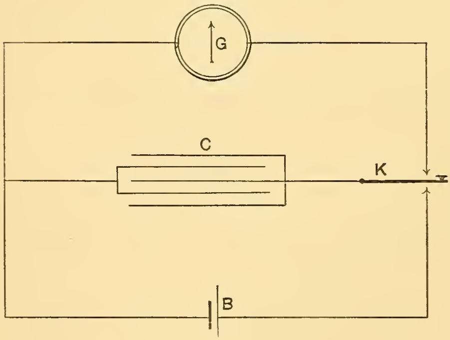



Fig. 38.

$$Q = CE_1.$$

If the deflection is $d_1$,

$$CE_1 = kd_1,$$

and

$$k = \frac{CE_1}{d_1}.$$

The other method[1] involves the exact measurement of a current. A long magnetizing coil is uniformly wound on a wooden cylinder or other non-metallic core, the diameter of which is accurately known. Over this

[1] Ewing's *Magnetic Induction in Iron and other Metals*, p. 62.

primary, at the middle of its length, a short secondary coil is wound and put in circuit with a ballistic galvanometer.

Let $A$ be the mean area of cross-section of the primary coil, and let $n$ be the number of turns in it per cm. length. Then if a current of $I$ amperes be made to pass through the coil, the magnetic flux or induction within it near the middle is $\frac{4\pi In}{10}$ per square centimetre,[1] and the total number of lines of induction within the coil is $\frac{4\pi InA}{10}$.

If $N$ is the number of turns in the secondary and $r$ the resistance in the circuit of the galvanometer, then the quantity of electricity in coulombs passing during the flow of the transient current in the secondary, when the primary circuit is made or broken, is

$$Q = \frac{4\pi InAN}{r \times 10^9} = kd_1.$$

Whence $$k = \frac{4\pi InAN}{rd_1 \times 10^9}.$$

The first method requires a knowledge of capacity and E.M.F.; the second requires a knowledge of current and resistance in addition to the dimensions of the coil.

*Second.* The operation with the cable as a condenser. The apparatus must be set up as indicated in Fig. 37. The coil is immersed in water contained in a tank $T$, lined with sheet copper. $P$ is a short-circuiting key. The entire circuit should be as well insulated as possible; but in any case particular care should be taken to insulate the

[1] Stewart and Gee's *Practical Physics*, Part II., p. 328.

ends of the coil. The end which is not used should be sealed, and there should be enough of the coil out of water at both ends to avoid leakage along the surface. If an additional wire is used to connect the coil to the key, great care must be taken to insulate it. It may be suspended by a silk thread. The insulation of the key when open should also be very good. A charge and discharge key is satisfactory for this purpose.

Then with the switch at $P$ closed, charge the coil as a condenser by pressing the key $K$. Since a part of the charge is absorbed, constant results will not be obtained unless the key be kept closed for a long time, several hours at least. If the usual rule of one minute be adopted, the insulation resistance will appear to be lower than it really is. However, on the first test of an insulated wire it is not advisable to attempt to obtain constant results from the start, as poor insulation may completely fail before such a condition is reached. Therefore, if a first test is being made, charge the coil for a short time with $P$ closed; next open the circuit for an observed number of seconds, and meanwhile open $P$. Then again close $K$, thus causing the quantity of electricity $Q$, required to replace the part of the charge which is lost by leakage or is absorbed, to pass in through the galvanometer. Let $d_2$ be the deflection produced by $Q$, and $E_2$ the E.M.F. of the charging battery. If we make no allowance for the part absorbed, the integral of the leakage current $I$ for the time $t$ must equal $Q$.

Then $$Q = \int I dt = \int \frac{E_2}{R} dt,$$

in which $R$ is the insulation resistance sought. If

during the time of leakage the difference of potential has fallen a negligible amount only, then

$$Q = \frac{E_2}{R} t = kd_2,$$

or
$$R = \frac{E_2}{k} \cdot \frac{t}{d_2}.$$

Substituting the value of $k$ from the first method, and

$$R = \frac{E_2}{E_1} \cdot \frac{d_1}{d_2} \cdot \frac{t}{C}.$$

If we use the value of $k$ obtained by the second method,

$$R = \frac{E_2}{12.566} \cdot \frac{d_1}{d_2} \cdot \frac{rt}{InAN} \cdot 10^9.$$

If $C$ in the first formula above is in microfarads, $R$ will be expressed in megohms.

In the second if $I$ is in amperes, $R$ will be in ohms.

**Example.**

*Test of a Piece of Grimshaw Wire.*

Diameter over insulation 5.6 mm.
Diameter of bare wire 2 mm.
Length under water 200 ft.
$C = 0.1$ microfarad, $d_1 = 129$ mm.
$E_1 = 1.44$ volts, $k = 0.00112$ micro-coulomb per mm.
$E_2 = 57$ volts throughout the test.

| Time after immersion. h. m. s. | Intervals in seconds. | Deflections in mm. | Deflections / Intervals. | Insulation resistance in megohms. |
|---|---|---|---|---|
| 0 : 30 | . . . . | . . . . | . . . . | |
| 1 : 00 | 30 | 43 | 1.433 | 35780 |
| 1 : 30 | 30 | 25 | 0.833 | 61540 |
| 2 : 30 | 60 | 34 | 0.567 | 90500 |
| 3 : 30 | 60 | 24 | 0.400 | 128100 |
| 4 : 30 | 60 | 18 | 0.300 | 170800 |
| 5 : 30 | 60 | 15 | 0.250 | 205100 |
| 6 : 30 | 60 | 14 | 0.233 | 219800 |
| 7 : 30 | 60 | 11 | 0.183 | 279700 |
| . . . . | . . . . | . . . . | . . . . | |
| 13 : 30 | . . . . | . . . . | . . . . | |
| 15 : 30 | 120 | 15 | 0.125 | 410200 |
| 17 : 30 | 120 | 14 | 0.117 | 439500 |
| 19 : 30 | 120 | 12 | 0.100 | 512400 |
| . . . . | . . . . | . . . . | . . . . | |
| 58 : 00 | . . . . | . . . . | . . . . | |
| 1 : 00 : 00 | 120 | 8 | 0.067 | 769200 |

The charging of the cable was begun thirty seconds after immersion.

This example gives a good illustration of the absorption of the charge by an insulated wire. This absorption will sometimes continue for hours; and if the insulation is really waterproof, the highest value — which is the real value — will be obtained only by electrifying the wire until the absorption ceases.

**51. Second Method of Insulation Resistance by Leakage.**[1] — This method is particularly applicable to a resistance having capacity, such as a cable immersed in water. Let this capacity be $C$ microfarads. Let $V$ be the P.D. between the two surfaces at the instant when the charge is $Q$. Then

$$Q = CV \text{ and } \frac{dQ}{dt} = C\frac{dV}{dt}.$$

[1] Gray's *Absolute Measurements in Electricity and Magnetism*, p. 253.

But $-\dfrac{dQ}{dt} = \dfrac{V}{R} = I$, where $R$ is the unknown resistance through which the charge leaks. Therefore,

$$C\frac{dV}{dt} + \frac{V}{R} = 0; \text{ or, } \frac{dV}{V} + \frac{dt}{CR} = 0.$$

Integrating, $\quad \log_e V + \dfrac{t}{CR} = constant.$

If the P.D. $= V_0$ when $t = 0$, then

$$\log_e V_0 = constant,$$

and

$$\log_e V_0 - \log_e V = \frac{t}{CR},$$

or

$$R = \frac{t}{C} \cdot \frac{1}{\log_e \frac{V_0}{V}}.$$ [1]

To determine the ratio of $V_0$ and $V$, the coil or cable is charged as a condenser, and then immediately discharged through a ballistic galvanometer, and the deflection is noted (Fig. 39). The coil is again charged to the same potential as before, and is then insulated and allowed to leak for an observed number

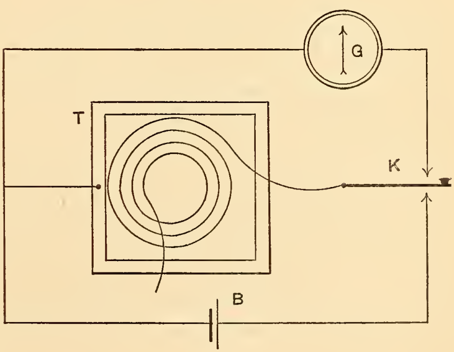


Fig. 39.

[1] This equation may be put into the form $V = V_0 e^{-\frac{t}{RC}}$, and this last expresses the law according to which the potential of a condenser varies with the time.

of seconds; and finally it is discharged through the galvanometer. The deflections, if moderately small, are taken proportional to the P.D.'s of the coil at the times of discharge. If the capacity is expressed in microfarads, and common logarithms are used in the reduction, then

$$R = 10^6 \cdot \frac{t}{C} \cdot \frac{1}{\log_{10}\frac{V_0}{V} \times 2.303},$$

where $R$ is expressed in ohms. But if it is desired to express $R$ in megohms, then the multiplier $10^6$ is omitted.

The chief difficulty with this method arises from the absorption of the charge by the dielectric. The second deflection may in consequence be larger than the first. This difficulty may be avoided in part by first charging the cable and allowing it to leak for say twenty seconds, and then discharging through the galvanometer. Then charge again and allow the leakage to extend over a longer period — say forty seconds — and then discharge again. The ratio of the deflections may then be taken as the ratio of the potential differences $V_0$ and $V$, the time $t$ being the difference in seconds of the two periods of leakage.

**Example.**

*Observations:* A coil of 1000 ft. of insulated wire was charged with one cell, and the discharge through the galvanometer gave a deflection of 123 mm.

The coil was again charged, and after leaking 120 seconds the deflection was 115.8 mm. (as a mean of five observations).

The capacity of the coil was 0.082 microfarads (Art. 97).

*Calculation:*

$$R = \frac{120}{0.082} \cdot \frac{1}{\log_e \frac{123}{115.8}} = \frac{120}{0.082} \cdot \frac{1}{\log_{10}\frac{123}{115.8} \times 2.303};$$

or $$R = 2.4251 \times 10^4 \text{ megohms.}$$

Therefore the resistance per mile is $24251 \div 5.28 = 4593$ megohms.

**52. To measure a Resistance by the Fall of Potential.** — Let $AB$ be the resistance to be measured (Fig. 40), and let an ammeter $Am$ be placed in series with

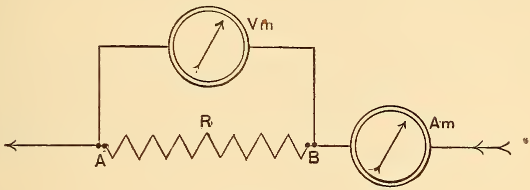



Fig. 40.

it. Let $Vm$ be a voltmeter of high resistance to measure the P.D. between $A$ and $B$. Read simultaneously the two instruments. Let $I$ be the current and $V$ the potential difference between $A$ and $B$. Then by Ohm's law

$$R = \frac{V}{I}.$$

**Example I.**

*Required the Resistance of the Secondary of a 12.6 Kilowatt Transformer.*

*Apparatus:* A milli-ammeter and a milli-voltmeter. The resistance of the milli-voltmeter was relatively high compared with the resistance to be measured. The scale read both ways from the centre. Hence to eliminate errors of the scale and zero, the milli-voltmeter was read first on one side and then on the other. Also the current was reversed through the resistance.

| | Amperes. | Volts. | |
|---|---|---|---|
| | 1.235 | .0060 | Direct. |
| | 1.249 | .0061 | Reversed. |
| | 1.245 | .0060 | Reversed. |
| | 1.255 | .0062 | Direct. |
| | 1.250 | .0060 | Direct. |
| Means, | 1.2468 | .00606 | |

$$\therefore R = \frac{.00606}{1.2468} = .00486 \text{ ohm.}$$

**Example II.**

*Measurement of the Resistance of an Edison Lamp.*

The observations of volts and amperes were made with the lamp at the given candle-power; the resistance of the lamp was then calculated for each set.

| Observed. | | | Calculated Resistances. |
| --- | --- | --- | --- |
| Candle Power. | Volts. | Amperes. | |
| .79 | 34.6 | .710 | 48.7 |
| 1.30 | 37.1 | .770 | 48.2 |
| 1.84 | 39.4 | .824 | 47.8 |
| 3.71 | 42.5 | .920 | 46.2 |
| 7.26 | 47.0 | 1.028 | 45.7 |
| 13.85 | 50.3 | 1.140 | 44.1 |
| 20.34 | 54.0 | 1.240 | 43.5 |
| 31.13 | 57.3 | 1.350 | 42.4 |
| 35.20 | 58.7 | 1.380 | 42.5 |

**53. To measure the Internal Resistance of a Battery — First Method.** — The following method of measuring the internal resistance of a battery is specially applicable when this resistance is very small, as in the case of a secondary cell, or a series of such cells. It requires a suitable voltmeter and ammeter with a resistance to give the current a convenient value.

Let $B$ be the battery (Fig. 41), $Vm$ the voltmeter, $Am$ the ammeter, $R$ the resistance in the circuit, which need not be known, and let $r$ be the internal resistance to be measured. First measure the P.D. between the terminals of the battery with the key $K$ open, and let it be represented by $E$. Then close the key, and read simultaneously and quickly both $Am$ and $Vm$, and let the current and P.D. be $I$ and $E'$. Then

$$E = E' + Ir,$$

in which $Ir$ is the loss of potential within the cell due to current $I$ passing over the resistance $r$, and $E$ is the fall of potential over the entire circuit.

Whence $$r = \frac{E - E'}{I}.$$

If the battery consists of several cells, $r$ is the sum of the internal resistances of the series.

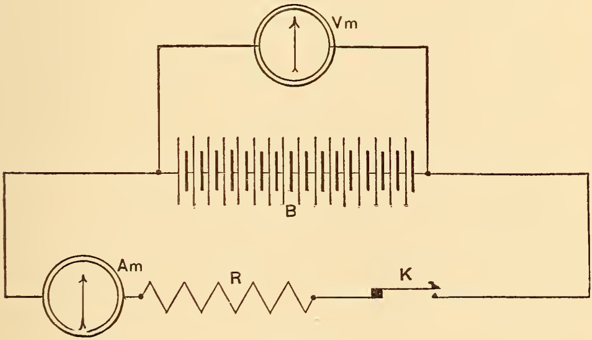



Fig. 41.

In the case of a storage battery this method may be slightly modified by measuring the charging current and the P.D. between the terminals of the battery simultaneously; and then, after opening the circuit, measuring the P.D. or E.M.F. again. Then if $E'$ is the P.D. during charging, $E$ the E.M.F. of the battery on open circuit, $I$ the charging current, and $r$ the internal resistance of the series of cells,

$$r = \frac{E' - E}{I},$$

since the difference between the two voltages is the E.M.F. required to maintain the current $I$ through the resistance of the battery.

**Example.**

It was desired to find the internal resistance of a storage battery of 36 cells. The battery was joined up in series with an ammeter and sufficient resistance to give (*a*) 5 amperes and (*b*) 10 amperes. The voltage of the battery was measured while giving these currents, and immediately afterwards on open circuit (except for the voltmeter of 19,560 ohms resistance).

| | Amperes. | Volts. | Internal resistance. |
|---|---|---|---|
| (*a*). | 5 | 71.5 | |
| | 0 | 72. | 0.10 |
| (*b*). | 10 | 70.9 | |
| | 0 | 71.8 | 0.09 |
| | | Mean, | 0.095 |

Resistance of each cell, 0.0026 ohm.

**54. Battery Resistance — Second Method.** — Form a circuit with the battery and a high resistance of 10,000 ohms or more (Fig. 42). Let a derived circuit be taken from two points on this high resistance with only a small part of the whole resistance between them; or a small additional resistance $R_1$ may be added to the high resistance, and the derived or shunt circuit may be joined up round this so as to include a d'Arsonval galvanometer $G$, as shown in the figure. If the galvanometer is a sensitive one, the resistance $R_1$ will be so small that no shunt to render

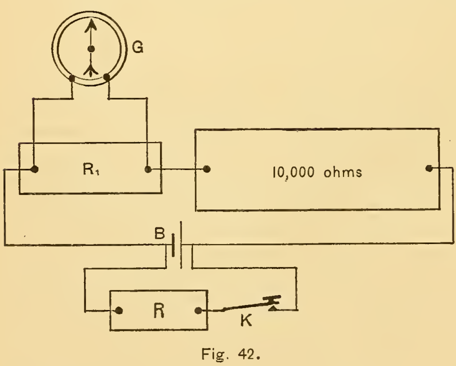


Fig. 42.

the galvanometer "dead beat" will be required. A circuit is also formed so as to close the battery through a small resistance $R$ of from one or two to five ohms.

Proceed as follows: Let $d_1$ be the deflection of the galvanometer when the circuit is closed through the high resistance, the key $K$ being left open; and let $d_2$ be the deflection when key $K$ is closed. The two deflections are proportional to the currents through the galvanometer, and therefore to the P.D.'s at the terminals of $R_1$, with $K$ open and closed respectively. Since $R_1$ bears a constant ratio to the entire resistance in circuit, the deflections $d_1$ and $d_2$ are proportional to the P.D.'s at the battery terminals in the two cases.

Hence $$d_1 : d_2 :: E : E' :: R + r : R. \quad . \quad . \quad . \quad (1)$$

When the key $K$ is open the P.D. at the battery terminals, measured by $d_1$, is the entire E.M.F. of the cell if its internal resistance is negligible in comparison with the high resistance in circuit; and when $K$ is closed the P.D. measured by $d_2$ is the fall of potential over the external resistance $R$. Now if the E.M.F. of the cell does not change immediately on closing $K$, then the fall of potential over the entire resistance $R + r$ is the E.M.F. of the cell. We may, therefore, put the two deflections proportional to the two resistances.

From (1) by subtraction,

$$d_1 - d_2 : d_2 :: r : R.$$

Whence, $$r = R\,\frac{d_1 - d_2}{d_2}.$$

It is necessary to use a "dead beat" galvanometer, or one which swings back to zero or takes a deflection corresponding to the current through it without swing-

ing back and forth, in order that the reading for $d_2$ may be taken quickly after closing $K$, and before polarization has changed the value of the E.M.F. of the cell. The d'Arsonval galvanometer is therefore recommended for this purpose.

**Example.**

| | $R$ | $d_1$ | $d_2$ | $r$ |
|---|---|---|---|---|
| Daniell cell . . . . . . | 5 | 64 | 35 | 4.14 |
| " " . . . . . . | 10 | 64 | 45 | 4.22 |
| Gassner's dry battery . . | 5 | 75 | 22 | 12.04 |
| " " " . . | 10 | 74 | 33 | 12.42 |

**55. The Condenser Method of measuring Battery Resistance.** — Let $B$ be the battery to be experimented upon (Fig. 43), $C$ a condenser, and $K$ a charge and discharge key, discharging on the upper contact. When $K$ is depressed the battery charges the condenser; when $K$ is released and makes contact on the upper point, the battery is disconnected and the condenser is discharged through the galvanometer $G$. This must be ballistic or slow-swinging, so that the first swing may be easily read; and it must have but little damping.

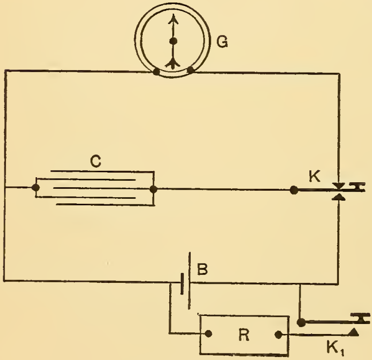



Fig. 43.

The operation consists in charging and discharging, first with the second key $K_1$ open, and then with it closed, and noting the deflections $d_1$ and $d_2$.

The deflections are taken as proportional to the quan-

tities of electricity discharged if they are not too large, and these quantities are proportional to the two P.D.'s. Hence the deflections are proportional to the P.D.'s and

$$d_1 : d_2 : : R + r : R$$

as before (Art. 54).

Also
$$r = R\,\frac{d_1 - d_2}{d_2}.$$

The key $K$ can be operated so quickly that $K_1$ need not be kept closed long enough to permit appreciable polarization.

**Example.**

| | $R$ | $d_1$ | $d_2$ | $r$ |
|---|---|---|---|---|
| Gassner's dry battery . . . | 5 | 130 | 66 | 4.85 |
| Crowdus dry battery . . . . | 1000 | 83 | 47 | 766.6 |

In the case of the Crowdus battery 5 ohms were tried at first, but no appreciable deflection was obtained for $d_2$, showing that the internal resistance was extremely large in comparison with 5 ohms. The cell was an old one nearly exhausted.

**56. Value of R for Least Error.** — To determine the conditions of highest accuracy it is necessary to consider the effect of an error in observing both $d_1$ and $d_2$.

Employing the general principle of Art. 36, find first the partial derivative of $r$ with respect to $d_2$. It will have the minus sign, because $r$ increases as $d_2$ decreases. From the equation

$$r = R\,\frac{d_1 - d_2}{d_2},$$

we have
$$F = -f\,\frac{\delta r}{\delta d_2} = Rf\,\frac{d_1}{d_2^2},$$

but
$$R = r\,\frac{d_2}{d_1 - d_2}.$$

Hence, $$F = rf\frac{d_1}{d_2(d_1 - d_2)}.$$

Finally, $$\frac{F}{r} = f\frac{d_1}{d_2(d_1 - d_2)}.$$

This is the relative error in $r$ due to an error $f$ in observing $d_2$. It is a minimum when the denominator is a maximum, since $d_1$ is now considered constant. But the denominator consists of two factors whose sum is a constant, or

$$d_2 + (d_1 - d_2) = d_1.$$

Now, when the sum of two factors is a constant their product is a maximum when they are equal to each other, or in this case, when $d_2 = d_1 - d_2$, or when $d_2 = \frac{1}{2}d_1$. This means that $R$ should be equal to $r$.

To estimate the influence of an error $f$ in $d_1$, find the derivative of $r$ with respect to $d_1$.

$$F = f\frac{\delta r}{\delta d_1} = Rf\frac{1}{d_2} = rf\frac{d_2}{d_2(d_1 - d_2)}.$$

Since this expression has the smallest value when $d_2 = 0$, or when the cell is short-circuited, the condition is inapplicable.

In case the errors in $d_1$ and $d_2$ *are equal and of the opposite sign*, then adding the corresponding values of the resulting errors,

$$\frac{F}{rf} = \frac{d_1 + d_2}{d_2(d_1 - d_2)}.$$

To find when this is a minimum, consider $d_1$ constant and differentiate the fraction with respect to $d_2$ thus:

$$\frac{\delta}{\delta d_2}\left(\frac{F}{rf}\right) = \frac{d_2(d_1 - d_2) - (d_1 + d_2)(d_1 - 2d_2)}{d_2^2(d_1 - d_2)^2} = 0.$$

Hence $$(d_1 - d_2)^2 = 2d_2^2,$$

or $$d_1 - d_2 = d_2\sqrt{2}.$$

Therefore, $$d_1 = d_2 (1 + \sqrt{2}) = 2.4142 d_2,$$

or $$d_2 = \frac{d_1}{2.4142}.$$

The resistance $R$ should then be about $\frac{5}{7}\, r$.

Finally, if the equal errors in $d_1$ and $d_2$ are of the same sign, then

$$\frac{F}{rf} = \frac{d_1 - d_2}{d_2 (d_1 - d_2)} = \frac{1}{d_2}.$$

This expression is a minimum when $d_2$ is greatest; that is, when $d_2 = d_1$, or when the external resistance is infinite. This is clearly an impossible condition.

In this particular problem an error in $d_1$ is much less likely to occur than in $d_2$. A series of readings can be taken with the battery circuit open, and the mean will be $d_1$. But $d_2$ is dependent to a considerable extent on skill in manipulation, and is affected by polarization; hence an error in it is much more likely to occur than in $d_1$. It appears better to consider $d_2$ only as the variable. The result is that $R$ should equal $r$ for highest accuracy.

The problem has been solved usually on the assumption that if the errors in $d_1$ and $d_2$ are of opposite sign, the resulting error $F$ will be a maximum; and the condition is then found for the relation between $R$ and $r$ which gives the smallest value of $F$. The result is $d_1 = \dfrac{d_2}{2.4142}$ as above.

But here a special assumption is made and a general

conclusion is drawn. There is no good reason for the assumption that the errors in $d_1$ and $d_2$ will be equal, and especially of opposite sign.

A preliminary measurement of the resistance $r$ can first be made, and then a second one, with $R$ nearly the same as the preliminary value obtained for $r$. If $r$ is quite small this should not be done, since a small external resistance will permit rapid polarization, and the error thus introduced may be greater than the one we seek to avoid.

In general, therefore, the principle can be applied only to batteries of high internal resistance, or to those which do not polarize rapidly.

**57. Variation of Internal Resistance with Current.** — The internal resistance of a voltaic cell, even at a constant temperature, has not a fixed and definite value, but depends upon the current flowing through it. The preceding methods of measuring this internal resistance enable one to determine what is the available potential difference at the battery terminals with a given resistance in the external circuit, or with a given current flowing. The resistance measured is a quantity satisfying the equation

$$r = R\frac{E-E'}{E'}, \text{ or } \frac{E}{R+r} = \frac{E'}{R} = I,$$

where $r$ is the internal resistance corresponding to a current $I$.

To determine the dependence of $r$ upon $I$, the condenser method may be employed, using different external resistances in succession. The examples following illustrate the great variation in $r$ which is sometimes found:

### Example I.

*Gassner's Dry Battery.*

$E = 1.213$ volts.

| $d_1$ | $d_2$ | $R$ | $r$ | $I$ |
|---|---|---|---|---|
| 271 | 258 | 400 | 21.5 | .0028 |
| | 249 | 200 | 17.7 | .0056 |
| | 238 | 100 | 13.86 | .0106 |
| | 227 | 50 | 9.69 | .0203 |
| 271 | 223 | 40 | 8.6 | .0249 |
| | 218 | 30 | 7.3 | .0324 |
| | 212 | 20 | 5.56 | .0473 |
| | 204 | 15 | 4.93 | .0607 |
| | 194 | 10 | 3.96 | .0868 |
| 271 | 172 | 5 | 2.87 | .1538 |
| | 164 | 4 | 2.59 | .1838 |
| 270 | 153 | 3 | 2.29 | .2289 |

### Example II.

*Daniell Cell.*

$E = 1.1$ volts.

| $d_1$ | $d_2$ | $R$ | $r$ | $I$ |
|---|---|---|---|---|
| 246.5 | 216.4 | 40 | 5.56 | .024 |
| | 208.7 | 30 | 5.43 | .031 |
| | 194.2 | 20 | 5.39 | .043 |
| | 181.8 | 15 | 5.34 | .054 |
| 246.5 | 161.8 | 10 | 5.24 | .072 |
| | 148.4 | 8 | 5.29 | .083 |
| | 132. | 6 | 5.21 | .098 |
| | 121.6 | 5 | 5.14 | .109 |
| 246.5 | 108. | 4 | 5.13 | .121 |
| | 91.8 | 3 | 5.06 | .137 |
| | 70.5 | 2 | 4.99 | .157 |
| 246.5 | 42.5 | 1 | 4.80 | .190 |

These results are plotted in Fig. 44, with internal resistances as ordinates and currents as abscissas. The Gassner cell shows a much larger decrease in the internal resistance than the Daniell cell for the same range of current. The scale of internal resistances for the Daniell is twice as large as for the Gassner.

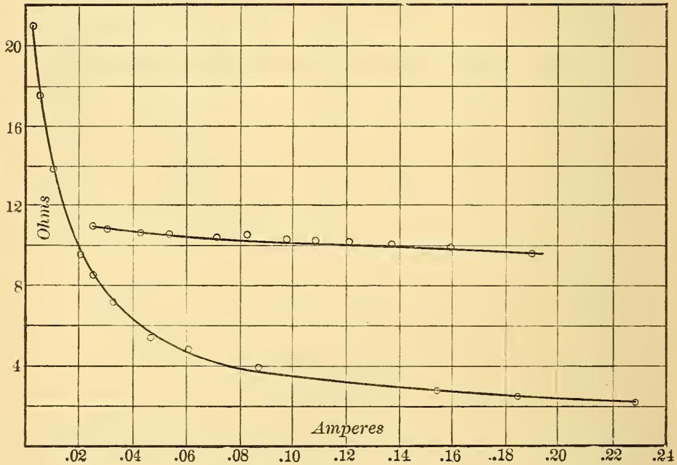



Fig. 44.

**58. Auxiliary Apparatus for the Condenser Method.** — In applying the condenser method to the measurement of internal resistance, or to the determination of polarization in an electrolyte, it is essential for quantitative comparison that some mechanical means be adopted to control the time during which the circuit is kept closed. It is perhaps equally important that the condenser should be discharged as soon as possible after charging, and before it has lost appreciably by leakage. The pendulum apparatus of Fig. 45 meets the requirements admirably. For the principle employed the authors are indebted to Dr. Milne Murray, of Edinburgh.

A rectangular frame carries at the bottom a heavy pendulum bob adjustable in height. The time of vibration of this pendulum is about one second. The bob is held in place by a detent in the position shown. When it is released it swings between two parallel circular arcs, concentric with the axis of suspension. The distance apart of these arcs is a little less than the length of the lower crossbar carrying the heavy bob. They support four keys, which can be clamped at any desired points. The keys have an upper and a lower contact like a simple discharge key. When the key lever is erect, the key makes contact on the lower point; and when the lever is thrown over by the crossbar of the pendulum as it swings forward, the key makes contact on the upper point. These keys can be set in any relation to one another which may be desired, and their operation is controlled entirely by the pendulum. Thus the time during which the battery is kept closed through the resistance $R$ may be made very short, and the condenser may be charged and discharged during this short interval of time. By this means polarization is reduced to a minimum and uniformity is secured.

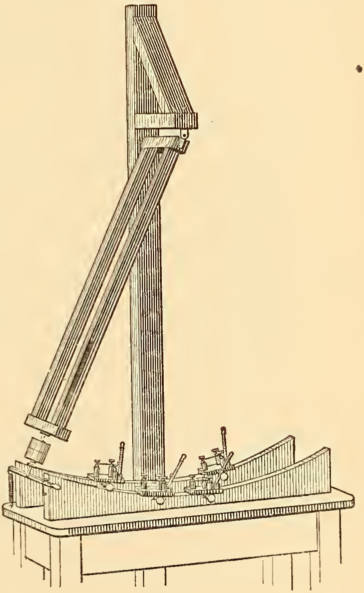


Fig. 45.

The connections for making a measurement of internal resistance are shown in Fig. 46. The pendulum is supposed to swing from left to right. When it strikes the lever or detent of key $K_1$ contact is made on the upper point, and this closes the battery circuit through a known resistance $R$. The overturning of the lever key $K_2$ puts

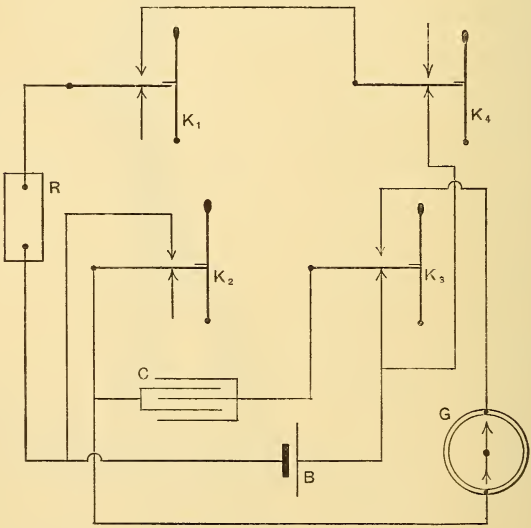



Fig. 46.

the two terminals of the battery in connection with the condenser $C$. When the pendulum reaches $K_3$ and overturns its detent lever, the battery is removed from the condenser, and contact on the upper point causes a discharge through the galvanometer $G$. Finally, on passing $K_4$ the pendulum operates this key and opens the battery circuit. To charge the condenser with the total E.M.F. of the battery, it is only necessary to leave

the levers of $K_1$ and $K_4$ thrown forward. The circuit then remains open. After each reading the pendulum is brought back to the detent at the left, and the levers are then set up *in the order* in which they are thrown over by the pendulum.

It will be observed that the battery circuit is open when the levers of keys $K_1$ and $K_4$ are both up, and when they are both thrown over as well. This arrangement may be reversed so that the circuit is closed under the same circumstances, and is open only during the interval required for the pendulum to pass from $K_1$ to $K_4$. This last arrangement is useful in getting the total E.M.F. of a cell while under test for polarization. The condenser is then charged and discharged while the battery circuit is open, and the recovery from polarization will be negligible during this short interval. It is essential that the platinum contacts of the keys should be kept strictly clean.

**59. Resistance of Electrolytes — First Method.** — All conducting liquids are electrolytes, except mercury and molten metals; that is, the passage of a current through them is accompanied by the decomposition of the liquid conductor. If the rate of decomposition exceeds the rate of diffusion of the ions or products of the electrolysis, so that they accumulate on the electrodes, the result is a counter E.M.F. of polarization. This E.M.F. interferes with the measurement of electrolytic resistances by the most simple means. The most usual method of annulling its effect is to employ rapid reversals of current or an alternating current of high frequency.

For this purpose a double commutator on one shaft is

applicable. The shaft should be capable of rapid rotation by means of a crank and a train of gears. One commutator is included in the battery circuit and the other in that of the galvanometer. They should be set so that the current is reversed through the liquid at the same time that the galvanometer is commuted. The current reversals are supposed to be so frequent that polarization is annulled. The apparatus is shown in Fig. 47.

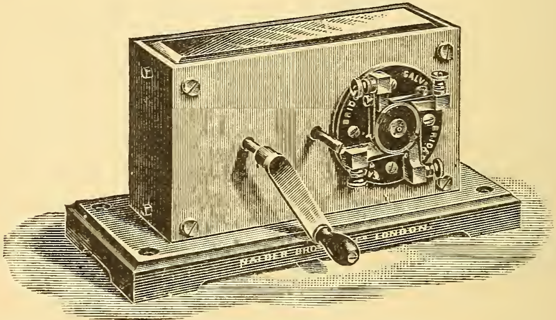


Fig. 47.

For the purpose of relative measurement of resistance or conductivity, comparison or standard solutions are needed. The following are recommended by F. Kohlrausch[1] as good conducting solutions, having a conductivity denoted by $k$ at the temperature of $t$ degrees C.:

$NaCl$, 26.4 per cent, sp. gr. 1.201.

$$k = 2015 \times 10^{-8} + 45 \times 10^{-8}(t - 18).$$

$MgSO_4$, 17.3 per cent, sp. gr. 1.187.

$$k = 460 \times 10^{-8} + 12 \times 10^{-8}(t - 18).$$

[1] *Wied. Ann.* II., p. 653, 1880; *Phys. Meas.*, p. 320.

These conductivities are relative compared with mercury at 0° C. But the specific conductivity of mercury is $1063 \times 10^{-8}$ C.G.S. units. Hence the conductivity of the above solutions in C.G.S. units may be found by multiplying the value of $k$ by $1063 \times 10^{-8}$.

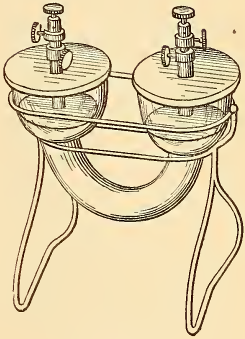


Fig. 48.

To measure the conductivity of any liquid one of the standard solutions is first placed in the appropriate vessel (Fig. 48), designed by Kohlrausch. It is well to be provided with several of these vessels, with connecting tubes of different cross-section, adapted to liquids of different conductivity.

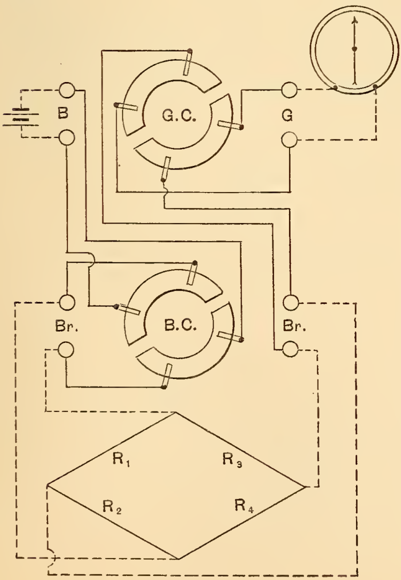



Fig. 49.

The electrodes are platinized platinum, with their lower surfaces convex. Let this liquid resistance be connected in one of the arms of the bridge, as $R_1$ (Fig. 49), and let $R_2$, $R_3$, and $R_4$ be non-inductive resistances. The continuous lines indicate permanent connections inside the commutator box, the

dotted lines temporary connections outside. Then if the commutator is rapidly rotated the circuit through the galvanometer is reversed simultaneously with that through the battery and resistances. Hence the currents through the galvanometer are rendered unidirectional.

The resistances are then adjusted to balance, and the same relation subsists between them as in the case of steady currents. Next, fill the vessel with the electrolyte to be measured and balance as before. The ratio of the two resistances will be the relative resistance of the two liquids, and their conductivities will be inversely as these resistances.

**Example.**

Standard solution: $NaCl$, spec. grav. 1.201 at 18° C.

Let $k_x$ equal the conductivity to be measured. The electrolyte was placed in one arm of the bridge, and two incandescent lamps in another. Two resistance boxes, $A$ and $B$, were in the other arms. Call the resistance of the lamps $R$. Then if $r$ and $r'$ are the resistances of the two solutions,

$$r = R\frac{A}{B}; \quad r' = R\frac{A'}{B'}.$$

Whence
$$k_x = k\frac{A}{B} \cdot \frac{B'}{A'}.$$

*Observations:*

With standard solution.

| $A$ | $B$ | $\frac{A}{B}$ | *Mean.* |
|---|---|---|---|
| 290 | 2000 | .1450 | |
| 263 | 1800 | .1461 | .1457 |
| 219 | 1500 | .1460 | |

Temperature 18.8° ; $k = 2180 \times 10^{-13}$.

| Electrolyte. | Temp. | Spec. Grav. | $A'$ | $B'$ | $\frac{AB'}{BA'}$ | Conductivity. |
|---|---|---|---|---|---|---|
| $ZnSO_4$ | 17.8° | 1.0502 at 18.3° | 1470<br>1754<br>2053 | 1000<br>1200<br>1400 | .0992 | $216 \times 10^{-13}$ |
| $ZnSO_4$ | 18.° | 1.2563 at 18.2° | 737<br>1099<br>1484 | 1000<br>1500<br>2000 | .197 | $430 \times 10^{-13}$ |
| $CuSO_4$ | 17.8° | 1.0317 at 18.8° | 1840<br>1377<br>1155 | 800<br>600<br>500 | .0633 | $138 \times 10^{-13}$ |

**60. Resistance of Electrolytes — Second Method.** — Instead of a double commutator and a galvanometer, an induction coil or a sine inductor and an electrodynamometer (Art. 67) may be employed. This is the method of Kohlrausch. If the induction coil is used it should be one with a solid iron core, to avoid the great difference in the value of the direct and inverse currents due to a wire core.[1]

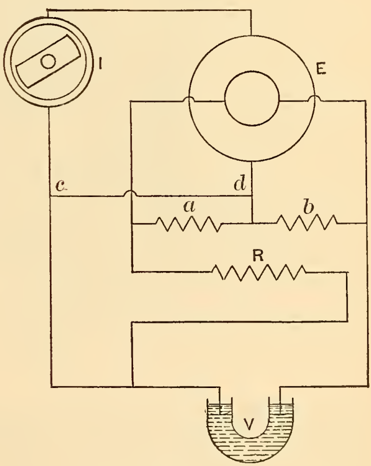



Fig. 50.

Let $I$ be the inductor (Fig. 50), $E$ the electrodynamometer, $R$ a post-office bridge, and $V$ the electrolyte. The electrolytic resistance is one arm of the bridge. The fixed coil of the electrodynamometer is in series with the main current, while the movable coil

[1] Professor Daniel, *Physical Rev.*, Vol. I., No. 4, p. 241.

is connected in place of the galvanometer to the two ends of the proportional coils, *a* and *b*. By this means resistances can be measured to several significant figures. The sensibility is increased by increasing the current and shunting the bridge by a suitable resistance *c d*.

The sine inductor may be used in place of the induction coil. It may consist of a stationary Gramme ring,

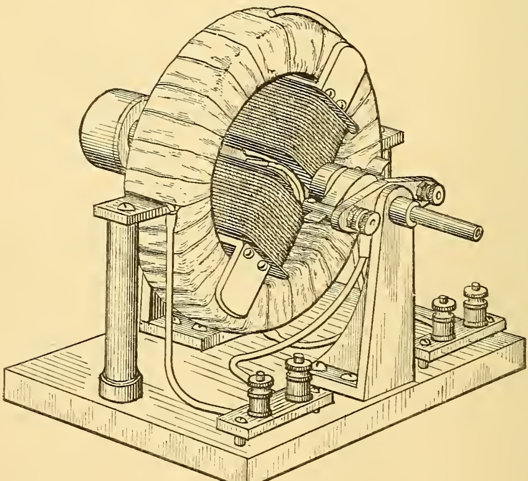


Fig. 51.

inside of which rotates a two-pole field-magnet. Connection is made with the wire of the ring at two opposite points. This constitutes a simple alternating current generator. It may be driven by a direct current motor. If conductors are led off from four equidistant points on the ring, each pair of conductors, 180° apart, compose an alternating current circuit, and the generator is then two-phased (Fig. 51).

**Example.**

Source of current, the sine inductor. E.M.F., 10 volts.

The electrodynamometer contained two fixed coils. These were joined in parallel with one another, and the whole in parallel with a Wheatstone's bridge. The movable coil was connected to the two ends of the proportional coils of the bridge.

Standard solution: $NaCl$, spec. grav. 1.201 at 18° C.

*Observations:*

With standard solution, $r = 41.47$ ohms.

Temperature, 24.4° C.; $k = 2410 \times 10^{-13}$.

| Electrolyte. | Temp. | Spec. Grav. | $r'$ | $\frac{r}{r'}$ | Conductivity. |
|---|---|---|---|---|---|
| $ZnSO_4$ | 24.1 | 1.0502 at 18° | 444.1 | .09338 | $225 \times 10^{-13}$ |
| $CuSO_4$ | 23.0 | 1.0317 at 18.8° | 662 | .06325 | $152 \times 10^{-13}$ |

The difficulty in the way of effecting a balance arises from the E.M.F. introduced by capacity and induction.

Chaperon has found that the static capacity of coils with "bifilar" winding of many turns produces a greater disturbance than the self-induction. To avoid this he winds the two wires, not side by side, but in alternate layers. It is better to wind in one direction only, and to bring each wire back parallel to the axis of the spool.

**61. Resistance of Electrolytes — Third Method.** — Professors Ayrton and Perry have proposed a method which does not require the prevention of polarization. A current is passed through the solution between two plates of platinum, $P$, $P$ (Fig. 52), till it acquires a constant value. Two platinum wires, $w$, $w$, are sealed into glass tubes and held rigidly in a fixed position between the platinum plates. The current is brought

to some definite value and measured by an electrodynamometer or other current-measuring device.

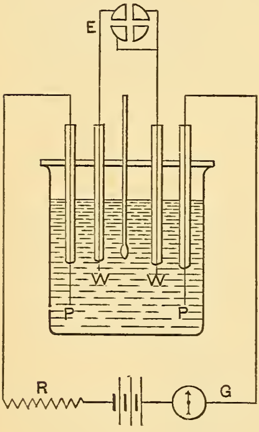


Fig. 52.

The potential difference between the platinum wires is then measured by an electrometer or static voltmeter, $E$ (Art. 95).

An observation is first made with a standard solution and then with the electrolyte to be measured, the current being brought to the same value each time. Then the resistances of the two liquids are proportional to the P.D.'s between the platinum wires in the two cases. These P.D.'s can be in arbitrary units, since it is necessary to know their ratio only.

**Example.**

Standard solution: $NaCl$, spec. grav. 1.201 at 18° C.

Let $k_x$ = the conductivity to be measured. Let $d$ and $d_1$ be the deflections of the electrometer with the standard solution and the unknown respectively.

Then $$k_x = k\frac{d}{d_1}.$$

Deflection with standard $NaCl$ solution . . . = 4.3
Temperature of solution . . . . . . . . . . . = 20.1° C.
Specific conductivity ($k$) . . . . . . . . . . = $2243 \times 10^{-13}$.

*Observations:*

| Electrolyte. | Temp. | Spec. Grav. | Deflection. | $\frac{d}{d_1}$ | Conductivity. |
|---|---|---|---|---|---|
| $ZnSO_4$ | 19° | 1.0502 at 18.3° | 46.2 | .093 | $208.6 \times 10^{-13}$ |
| $CuSO_4$ | 18.5° | 1.0319 at 18.8° | 70.75 | .062 | $139 \times 10^{-13}$ |

A serious objection to the method is the change in the density of the solution due to electrolysis. The deposit of zinc or copper on the platinum electrode reduces the density and the conductivity of the solution.

The temperature coefficient of the $ZnSO_4$ at 18° is 0.0226, and of the $CuSO_4$, 0.0215. When corrected for temperature, the three results for $CuSO_4$ agree quite closely. The last two determinations of the $ZnSO_4$ also agree fairly well, but the first is considerably higher.

# CHAPTER III.

## MEASUREMENT OF CURRENT.

**62. The Tangent Galvanometer.** — The tangent galvanometer has lost most of its former importance, but it is useful in a laboratory, and will be described because of its historical importance, if for no other reason. In its simplest form a tangent galvanometer consists of a circular conductor, supported vertically in the magnetic meridian, and having at its centre a magnetized needle capable of turning around a vertical axis. The length of the needle must be short in comparison with the radius of the coil. This is essential, so that when the needle is deflected by a current the movement shall not place the poles in a field of magnetic strength different from that at the centre of the coil. A small deflection of a long needle would move its poles from the uniform field in the plane of the coil to a relatively weaker one on either side of this plane. The lines of force due to a current circulating around a circular coil, or the lines along which a magnetic pole is urged, coincide with the axis of the coil at its centre. Near the centre they are very nearly parallel lines. If, therefore, a short needle, in length from one-twelfth to one-tenth the diameter of the coil, has its magnetic axis in the plane of the coil when no current is passing, then when it is deflected by a current, the direction of the deflecting force acting on its poles in the new position

will be perpendicular to the plane of the coil, and the orienting force due to the earth's magnetism will be exactly at right angles to the deflecting force.

Let $NS$ (Fig. 53) be the magnetic meridian, and let the plane of the coil be in it, with its centre at $O$. Let the needle be deflected through an angle $\theta$. Then two forces act upon each pole. One is $\mathcal{H}m$, due to the horizontal component of the earth's field; the other, due to the current $I$, is at right angles to the plane of the coil, and equals $\frac{2\pi mI}{r}$, where $r$ is the mean radius of the coil, consisting of a single turn of the conductor, and $m$ is the strength of pole of the needle. For equilibrium the moments of these two forces, or the moments of the two couples, due to the pairs of equal forces acting on the two ends of the needle, must be equal to each other.

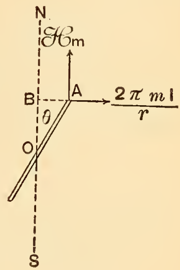



Fig. 53.

The moment of the orienting magnetic force due to the earth's field is $\mathcal{H}ml \sin \theta$, where $l$ is the half length of the needle. The moment of the deflecting force is $\frac{2\pi mI}{r} l \cos \theta$. Hence,

$$\frac{2\pi \cancel{m}I}{r} l \cos \theta = \mathcal{H}\cancel{ml} \sin \theta. \quad \frac{2\pi I}{r} \cos\theta = H\sin\ldots$$

The $m$ and $l$ cancel out. The deflection is therefore independent of the strength of pole; but the length is limited for a reason already given.

From this equation

$$I = \mathcal{H}\frac{r}{2\pi} \tan \theta.$$

For $n$ turns of wire, where $n$ is only a very small number, and where the $n$ turns may be considered coincident,

$$I = \mathcal{H}\frac{r}{2\pi n}\tan\theta.$$

The fraction $\frac{2\pi}{r}$, or $\frac{2\pi n}{r}$, is called the constant of the galvanometer.

It depends upon the dimensions and the number of turns of wire in the coil, and equals the strength of field produced at the centre by unit current flowing through the coil. If this constant be denoted by $G$, then

$$I = \frac{\mathcal{H}}{G}\tan\theta. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The equation may be written simply

$$I = A\tan\theta. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The current is measured in C.G.S. units. The number of amperes is 10 times as great. When the constant of the galvanometer is determined from its dimensions, equation (1) must be used; when it is determined by silver or copper electrolysis, equation (2) is more convenient.

**63. Influence of Errors of Observation.** — The effect of an error in reading the deflection of the needle of a tangent galvanometer will be least relative to $I$ when $\theta = 45°$. This may be demonstrated by applying the formula of Article 36 to the tangent galvanometer, the equation of which is

$$I = A\tan\theta.$$

Then

$$F = f\frac{dI}{d\theta}.$$

But

$$\frac{dI}{d\theta} = \frac{A}{\cos^2\theta}.$$

Therefore

$$F = A\frac{f}{\cos^2\theta} = I\frac{f\cos\theta}{\cos^2\theta\sin\theta}$$

by the substitution of $\frac{I}{\tan\theta}$ for the constant $A$.

But

$$\frac{f\cos\theta}{\cos^2\theta\sin\theta} = \frac{f}{\sin\theta\cos\theta} = \frac{2f}{\sin 2\theta}.$$

Hence, $\frac{F}{I}$ will be a minimum for any error of observation $f$ when $\frac{2f}{\sin 2\theta}$ is a minimum, or when $\sin 2\theta$ is a maximum. The sine of an angle is a maximum when the angle equals 90°; and $\sin 2\theta$ is therefore a maximum when $\theta = 45°$.

**64. Plotting Currents measured by a Tangent Galvanometer.** — Let $R$ equal the resistances in the circuit of the galvanometer except that of the battery, and let $r$ be the resistance of the battery. Then by Ohm's law

$$I = \frac{E}{R+r}.$$

If now a constant E.M.F. be employed, and if the internal resistance of the battery does not change,

$$I \propto \tan\theta \propto \frac{1}{R+r} \propto \frac{1}{\cot\theta}.$$

Hence $\cot\theta \propto R + r$.

If, therefore, we plot a curve with the tangents of the several observed deflections as ordinates and the corresponding resistances $R$ as abscissas, we shall obtain the curve $I$ (Fig. 54), which is an hyperbola. Plotting cotangents of $\theta$ as ordinates and resistances $R$ as abscissas, on the other hand, gives the straight line II. The two

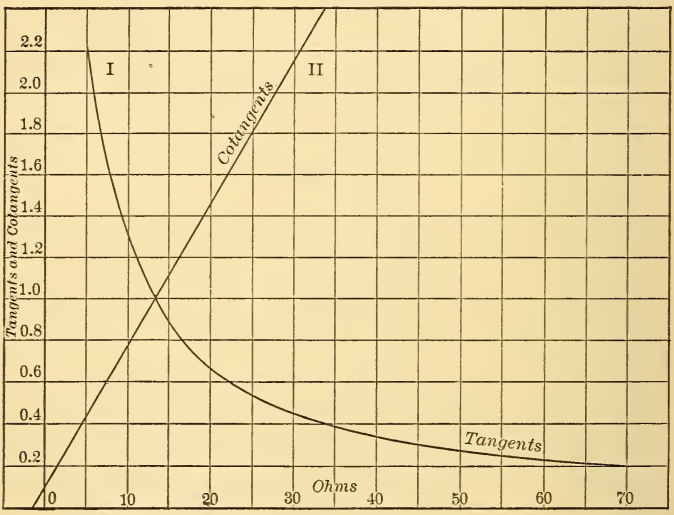



Fig. 54.

curves intersect at a point corresponding to a deflection of 45°. The cotangents' line does not intersect the axis of resistances at the origin, but at a distance to the left equal to the internal resistance of the battery $r$.

**65. To find the Magnetic Field at any Point on the Axis of the Coil.** — Let $DE$ (Fig. 55) be the trace of the plane of the coil, and let $OC$ be its axis. It is

required to find the deflecting force at any point $A$ on the axis. Let $AB$ represent the force on unit pole at $A$ due to the current in a small element $ds$ of the circle at $E$. It will equal $\frac{Ids}{l^2}$, and its direction will be perpendicular to $EA$ in the plane of the paper. The deflecting force at a point is always perpendicular to a plane containing the element of the conductor and a line drawn from the middle point of the element to the given point. The effective component $AC$ is

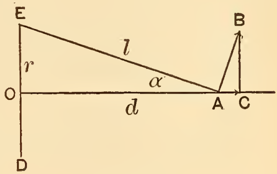


Fig. 55.

$$\frac{Ids}{l^2}\sin a = \frac{Ids}{l^2}\cdot\frac{r}{l} = \frac{Irds}{(r^2+d^2)^{\frac{3}{2}}}.$$

Hence for one turn of wire

$$\mathcal{F} = \frac{2\pi r^2 I}{(r^2+d^2)^{\frac{3}{2}}} = \frac{2SI}{(r^2+d^2)^{\frac{3}{2}}},$$

where $S$ is the area of the circle $\pi r^2$. This expression represents the total force, since the components $CB$ balance one another all around the circle. Each element of the circle has a symmetrical one at the other extremity of a diameter through it, and the component at right angles to the axis of the coil, due to this symmetrical component, is equal and opposite to $CB$.

If a magnet pole of strength $m$ is placed at $A$, then the force on it perpendicular to the plane of the ring is $\mathcal{F}m$, while the horizontal force due to the earth's magnetism is $\mathcal{H}m$. Hence, as in the tangent galvanometer,

$$\mathcal{F}m\cos\theta = \mathcal{H}m\sin\theta,$$

or

$$\mathcal{F} = \mathcal{H}\tan\theta.$$

Therefore
$$\frac{2\pi r^2 I}{(r^2 + d^2)^{\frac{3}{2}}} = \mathcal{H} \tan \theta.$$

or
$$K \frac{r^2}{(r^2 + d^2)^{\frac{3}{2}}} = \tan \theta.$$

$K$ is a constant if $I$ and $\mathcal{H}$ are constant. This form of the equation is convenient for use in the experimental proof of the relation between $d$ and $\theta$.

### Example.

Place the circular coil (Fig. 56) in the magnetic meridian and

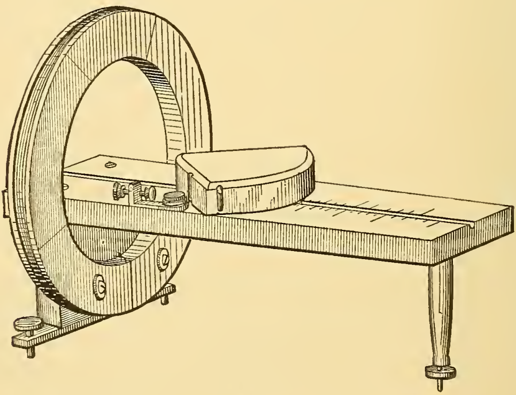


Fig. 56.

set the compass box at the centre of the coil. Pass a current through the coil from a constant source, and of such strength as to give a deflection of about 45°. By means of a commutator take deflections in both directions. Repeat the observations with the compass box at different distances from the centre of the coil. Measure the mean radius of the coil with calipers, if it is not known. Finally, compare tangents of the mean deflections with the values derived from the preceding equation and plot.[1]

---

[1] Stewart & Gee's *Prac. Phys.*, Part II., p. 321.

The following are the details of an experiment:
$r$ 12.45 cms.

| (1) Distances $x$ in cms. | (2) Mean Deflection. | (3) Tangent of Deflection. | (4) $\frac{r^2}{(r^2+x^2)^{\frac{3}{2}}}$ | (5) (3) ÷ (4) or $K$. | (6) $K\frac{r^2}{(r^2+x^2)^{\frac{3}{2}}}$ |
|---|---|---|---|---|---|
| 0 | 43° 21′ | 0.9440 | 0.08032 | 11.753 | 0.9394 |
| 1 | 43.3 | 0.9341 | 0.07955 | 11.742 | 0.9304 |
| 2 | 42.12 | 0.9067 | 0.07731 | 11.729 | 0.9042 |
| 3 | 40.51 | 0.8647 | 0.07380 | 11.717 | 0.8631 |
| 4 | 39.9 | 0.8141 | 0.06932 | 11.745 | 0.8107 |
| 5 | 37.6 | 0.7563 | 0.06418 | 11.783 | 0.7506 |
| 6 | 34.33 | 0.6886 | 0.05872 | 11.726 | 0.6858 |
| 7 | 32.3 | 0.6261 | 0.05320 | 11.769 | 0.6222 |
| 8 | 29.12 | 0.5589 | 0.04783 | 11.685 | 0.5593 |
| 9 | 26.30 | 0.4986 | 0.04275 | 11.662 | 0.5000 |
| 10 | 23.34 | 0.4431 | 0.03807 | 11.642 | 0.4452 |
| 11 | 21.27 | 0.3929 | 0.03381 | 11.623 | 0.3953 |
| 12 | 19.15 | 0.3492 | 0.02998 | 11.648 | 0.3506 |
| 13 | 17.12 | 0.3096 | 0.02658 | 11.647 | 0.3108 |
| 14 | 15.18 | 0.2736 | 0.02357 | 11.606 | 0.2757 |
| 15 | 13.42 | 0.2438 | 0.02093 | 11.650 | 0.2447 |
| 16 | 12.15 | 0.2171 | 0.01860 | 11.671 | 0.2176 |
| 17 | 10.57 | 0.1935 | 0.01655 | 11.691 | 0.1935 |
| 18 | 9.48 | 0.1729 | 0.01479 | 11.682 | 0.1729 |
| 19 | 8.48 | 0.1548 | 0.01322 | 11.706 | 0.1547 |
| 20 | 7.54 | 0.1388 | 0.01185 | 11.705 | 0.1386 |

The mean value of $K$ is 11.695. From this value and from column (4) column (6) is calculated. The curve (Fig. 57) represents the observed values of the tangents, distances $x$ being plotted as abscissas. The curve of the theoretical values of the tangents in column (6) falls so

near this one that it cannot be plotted separately. The greatest difference between the observed and computed values of the tangents is only three-fourths per cent; most of these differences are only a small fraction of one per cent.

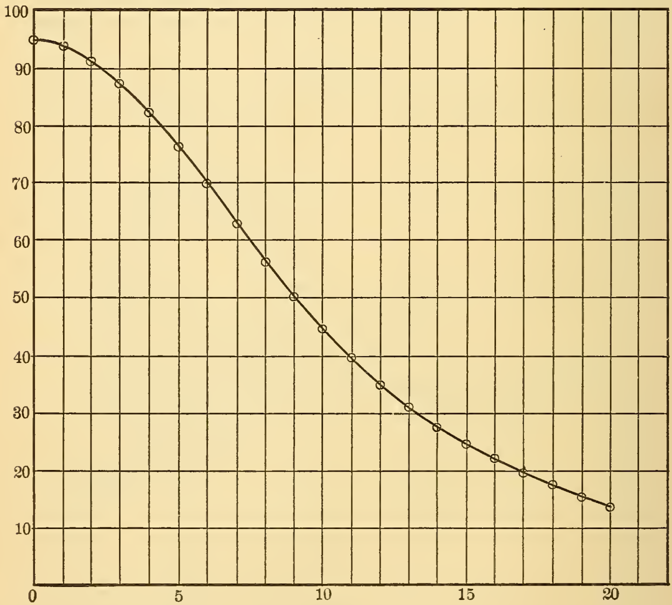



Fig. 57.

**66. The Cosine Galvanometer.** — The cosine galvanometer is made so that the coil may rotate about its horizontal diameter. Fig. 58 is a vertical section through the coil. The axis of rotation, which lies in the magnetic meridian, is perpendicular to the paper through $O$. The plane of the coil has been rotated over through the

angle $\phi$. Then the whole force on the magnet pole due to the current through a single turn of conductor is in the direction $OC$, perpendicular to the plane of the coil. The effective component moving the needle is $OD$, and

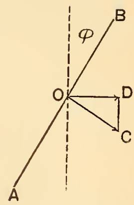


Fig. 58.

$$OD = \frac{2\pi m I}{r} \cos \phi.$$

Placing the moments of the two forces acting on the needle equal to each other, as in the case of the tangent galvanometer, we have

$$\mathcal{H} m l \sin \theta = \frac{2\pi m I}{r} l \cos \theta \cos \phi,$$

or

$$I = \mathcal{H} \frac{r}{2\pi} \frac{\tan \theta}{\cos \phi}.$$

For a given deflection of the needle the current is inversely proportional to the cosine of the angle which the plane of the coil makes with the vertical. By this means the range of the galvanometer is greatly increased.

**67. The Siemens Electrodynamometer.** — An electrodynamometer consists of two coils with their magnetic axes at right angles, one of them fixed and the other movable about a vertical axis through its plane. The motion of the movable coil is produced by the electrodynamic action between the convolutions of the two coils. The current flows through the two in series.

Let $AB$ (Fig. 59) be a single convolution of the fixed coil and $CD$ the suspended movable coil. The movable coil consists of only one turn, or at least a very limited number, according to the current which the instrument is designed to measure. A large current means a heavy conductor and a single turn, since it would be impracticable to support several turns. The instruments for smaller currents may have several turns in the movable coil. It will be seen that the movable conductor is subjected to a system of forces all tending to turn it in the same direction. It is suspended by means of silk threads or on a point resting in a jewel; and a carefully wound helix is connected rigidly with it and with the torsion head $T$ above. Fig. 60 shows the complete instrument.

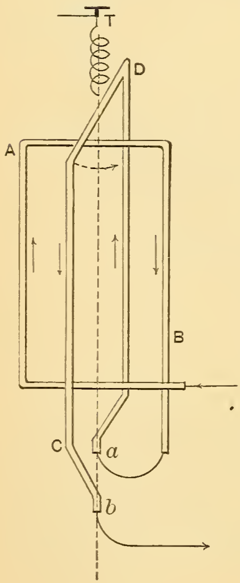


Fig. 59.

When the coil turns by passing current through it, the turning of the torsion head brings it back again to the zero or initial position. Thus the couple due to the electrodynamic action is offset by the couple of torsion of the helix connected with the torsion head. This torsion couple is therefore employed to measure the current. Now, the couple of torsion is proportional to the angle of torsion by Hooke's law, the forces of restitution which are called into action by any distortion within elastic limits being proportional to the distortion itself. But the electrodynamic action is proportional to the square of the current, since

the two coils are in series. Hence the square of the current is proportional to the twist of the counteracting helix.

We may accordingly write

$$I^2 = A^2 D,$$

or

$$I = A\sqrt{D},$$

as the equation connecting the current with the twist of the torsion helix. $A$ is a constant depending upon the windings, the torsion of the suspending spring, etc. This is the common equation of a parabola. Hence if currents and twist be plotted as coördinates, the resulting curve will be parabolic.

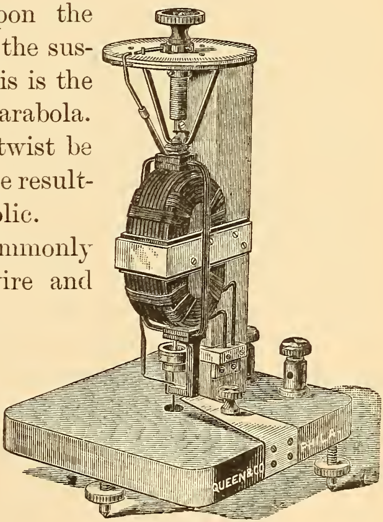


Fig. 60.

Two fixed coils are commonly employed, one of fine wire and the other of coarse wire. One end of each is connected to a binding-post on the base of the instrument. The other terminals are connected to the upper mercury cup at $a$ (Fig. 59), into which dips one end of the movable coil, the other end dipping into another mercury cup at $b$, from which a conductor leads to a third binding-post. Hence, whether the current enters by the one fixed coil or the other, it passes out through the suspended coil and the third post. Since the direction of the deflection depends upon the

manner in which the coils are connected, and not upon the direction of the current, the electrodynamometer is applicable to the measurement of alternating currents. Its period of swing must, however, be long in comparison with the period of alternation of the current. It then becomes an integrating device, and integrates the values of the squares of the current for successive equal time-intervals. The result is, therefore, the square root of the mean square of the current.

**68. The Equation of the Electrodynamometer as affected by the Earth's Field.** — When only small currents are employed with a sensitive electrodynamometer, the effect of the earth's directive force on the suspended coil, considered as a magnetic shell, must be taken into account. This force is proportional to the first power of the current, while the deflecting force due to the mutual action of the coils is proportional to the square of the current. If, therefore, the instrument is set up with the plane of the suspended coil and the axis of the fixed coils in the magnetic meridian, the fixed coils being of such dimensions as to produce a sensibly uniform magnetic field in the region of the suspended coil, we shall have for the equation of equilibrium

$$aI^2 \cos\theta + bI \cos\theta = c\theta,$$

or

$$aI^2 + bI = c\frac{\theta}{\cos\theta}, \quad . \quad . \quad . \quad . \quad (1)$$

in which $a$ is a constant depending on the windings and dimensions, $b$ one depending on the number of turns and the area of the suspended coil, as well as on the earth's horizontal field $\mathcal{H}$, $c$ the couple of torsion for a

unit angle, and $\theta$ the deflection. The current is here supposed to be in the direction in which the earth's magnetic force and the electrodynamic action between the coils act together.

If the current be reversed the dynamic action between the coils turns the suspended coil the same way round, but the direction of the couple due to the earth's field is reversed. Therefore, the deflecting couple is due to the difference of the two forces, and for the same deflection as before the current must be greater. Let it be $n$ times as great. Then we may write

$$an^2I^2 - bnI = c\frac{\theta}{\cos\theta}, \quad . \quad . \quad . \quad (2)$$

$$anI^2 + bnI = nc\frac{\theta}{\cos\theta}. \quad . \quad . \quad . \quad (3)$$

Multiplying equation (1) by $n$, we have equation (3); adding and dividing by $(n+1)$, we have

$$anI^2 = c\frac{\theta}{\cos\theta}.$$

It follows, therefore, that if the earth's influence were eliminated, the same deflection $\theta$ would be given by a current equal to $I\sqrt{n}$, numerically a mean proportion between the two oppositely directed currents required to produce the same deflection.

For small angular displacements equation (1) may be written with sufficient approximation,

$$aI^2 + bI = cd, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $d$ is the deflection in millimetres observed by the usual telescope and scale method, and $c$ is dependent on the distance of the scale from the electrodynamometer.

Equation (4) is the equation of a parabola referred to axes parallel to those of the equal parabola whose equation is

$$I^2 = \frac{c}{a} d.$$

The following equation was derived from a sensitive instrument in our laboratory:

$$I^2 - 0.842 I = 0.0298 d. \quad . \quad . \quad . \quad (5)$$

If the current through the suspended coil alone is reversed, we obtain

$$I^2 + 0.842 I = -0.0298 d. \quad . \quad . \quad . \quad (6)$$

If the observations are plotted with deflections as abscissas and currents as ordinates, the full line parabola passing through the origin is obtained (Fig. 61).

For alternating currents the term containing the first power of $I$ in equation (5) vanishes, and we have

$$I = \sqrt{0.0298 d} = 0.1726 \sqrt{d}. \quad . \quad . \quad (7)$$

This equation represents the same parabola as that of equation (5), but shifted, as shown in the dotted curve in the figure, so as to have its vertex at the origin. It is the equation for alternating currents in which the earth's field plays no part. For direct currents the instrument should be set up with the plane of the movable coil at right angles to the magnetic meridian.

**69. The Wattmeter.**—The electrodynamometer may be made to measure the power expended in any part of a circuit. The integrated product of the current and the corresponding pressure at the terminals of the circuit is the mean power expended in it. If the whole current

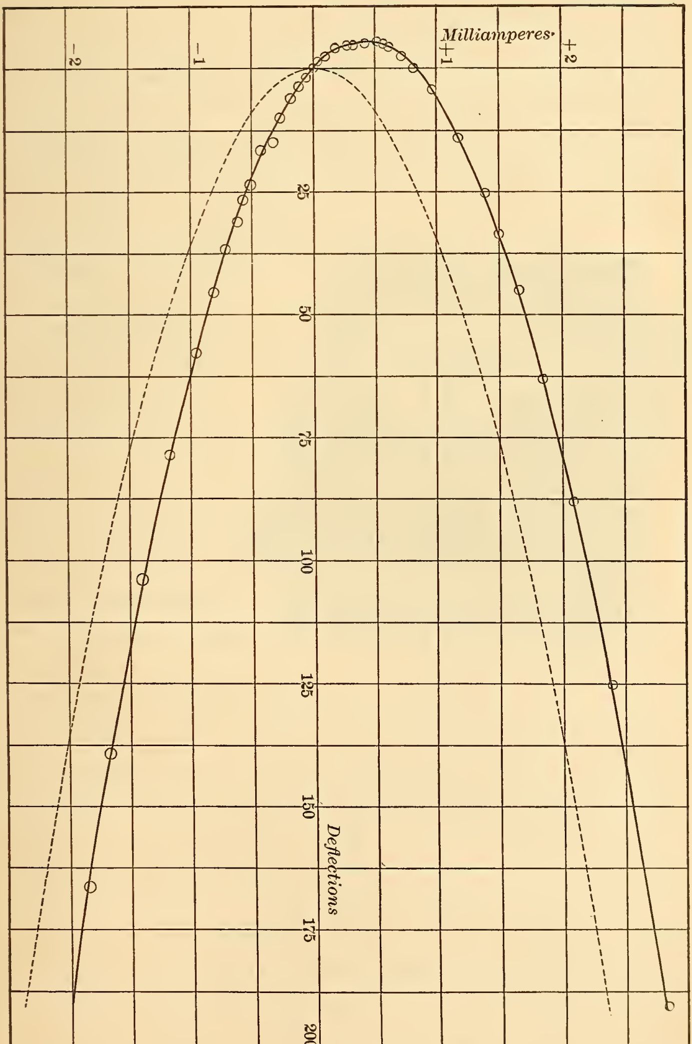



Fig. 61.

is carried through the fixed coil of the electrodynamometer, and the movable coil is connected as a shunt to the resistance on which the power to be measured is expended, so as to serve as a pressure coil, with the necessary resistance in series with it, the instrument then becomes a wattmeter, and may be calibrated to read in watts.

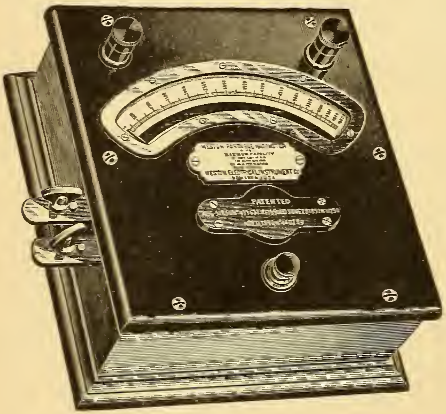


Fig. 62.

Fig. 62 is the Weston wattmeter, which is graduated to read directly in watts. Fig. 62*a* is a diagram of the internal connections. The translating device, such as a lamp, is connected across the mains from *C* to *D*. *A* and *B* are the terminals of the series or field coil, and *ab* those of the pressure coil. It will be seen that the pressure circuit through the movable coil is carried round the field coil also. This is for the purpose of compensating for the current through the pressure circuit, since this current also traverses the series coil. The connections are so made that the currents through this compensating winding and the field coil flow in opposite directions. The reading is thus diminished to such an extent as to compensate for the energy required to operate the instrument.

The independent binding-post *I* is employed in con-

nection with *b* when the instrument is used with two independent circuits, or when it is calibrated by means of two separate currents. The compensating winding is then cut out and an equivalent resistance is substituted.

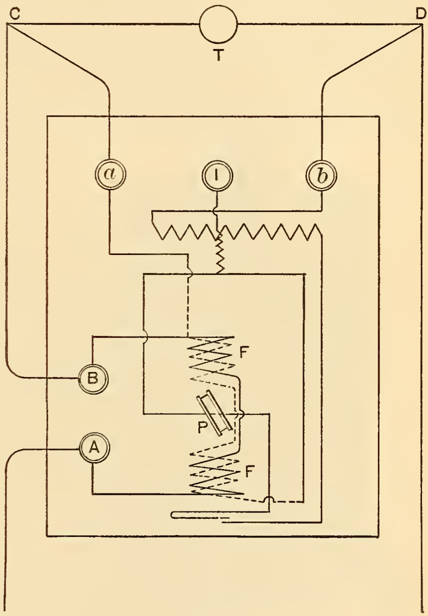


Fig. 62a.

**70. The d'Arsonval Galvanometer.** — The d'Arsonval galvanometer may be considered as an electrodynamometer in which the fixed coil is replaced by a permanent magnet; or it may be looked upon as a galvanometer in which the magnet is fixed and the coil is movable, instead of the converse arrangement of the tangent galvanometer. Since the action and reaction are equal between a coil and a magnet, it is immaterial from a magnetic point of view whether the one is made movable or the other.

The great advantage of the d'Arsonval type of galvanometer is that it has a strong magnetic field only slightly affected by the earth's magnetism, or by iron or other magnetic matter in its vicinity. It is also extremely "dead beat" under certain conditions. Further-

more, by properly shaping the pole pieces of the permanent magnet, the deflections may be made strictly proportional to the current. The Weston instruments for direct currents are a modification of the galvanometer of d'Arsonval, and both operate on the same principle as Lord Kelvin's Siphon Recorder for submarine telegraphy, which preceded both of them.

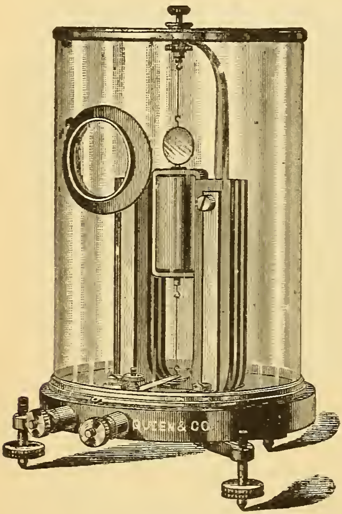


Fig. 63.

In the earlier instruments of this design the coil had a large area, and a soft iron core was inserted to strengthen the field. This arrangement is still retained in the Weston instruments. But Ayrton has pointed out [1] that galvanometers of the d'Arsonval type should not have a soft iron core, and that the coil should be long and thin.

Fig. 63 is a d'Arsonval galvanometer of ordinary pattern. The current is led in through the spring and attached wire at the bottom, thence through the coil, and out by the suspending wire and the supporting standard. The field-magnet is a compound one supported vertically. Within the coil is a soft iron core supported from the rear. The coil turns in the narrow intense field between the poles of the magnet and the iron core. When the external resistance is not large, the induced currents on

[1] "Galvanometers," *Phil. Mag.*, July, 1890, p. 58.

closing the current, with the coil in motion, quickly bring it to rest.

The coil in the Weston instrument (Fig. 64) is controlled by two helical springs which also serve as conductors to connect the coil with the external circuit. A

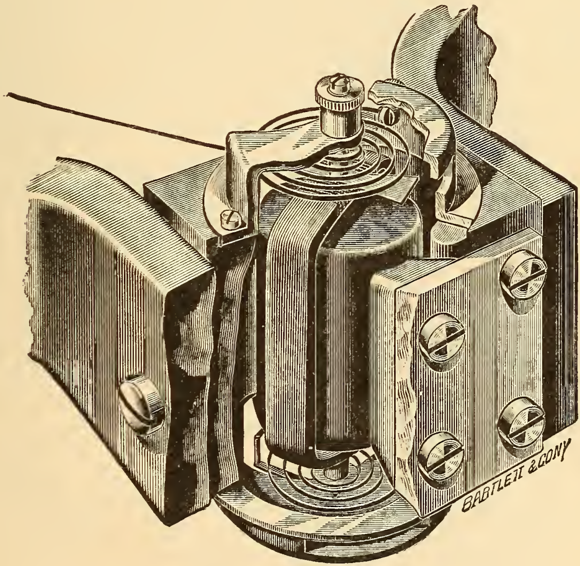



Fig. 64.

portion of one pole is shown cut away in the figure. The pivots rest in jewels, and a long aluminium pointer is attached to the coil and traverses a scale of equal parts not shown. In the voltmeter a large resistance is put in series with the movable coil. In the ammeter for large currents the movable coil is connected as a shunt to the main conductor in the instrument.

The Ayrton-Mather pattern of this galvanometer (Fig. 65) has a single ring-magnet with a narrow division at one point. In the opening is placed the tube containing the long narrow coil without any iron core. This coil is suspended by a thin wire, and has a fine helix at the bottom for a conductor. Its plane must be parallel to the lines of force in the narrow gap in which it hangs. If quick damping is desired, the coil is enclosed in a thin silver or aluminium tube.

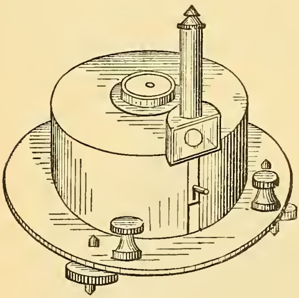


Fig. 65.

**71. The Best Shape for the Section of a Coil.** — The best shape for the section of the coil of a d'Arsonval galvanometer perpendicular to the axis about which it turns has been determined by Mather.[1]

His paper deals with coils suspended in a uniform field, but similar reasoning applies to instruments in which the field is not uniform.

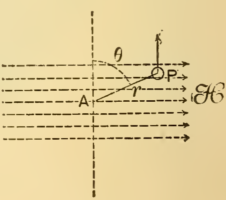



Fig. 66.

Let the field be of strength $\mathcal{H}$, and let $P$ (Fig. 66) be an element of the section of the coil turning about an axis through $A$ perpendicular to the plane of the element, and $I$ the current density per unit area. Then the deflecting moment exerted on unit length, measured perpendicular to the paper, and of cross-section $a$, is

$$\mathcal{H} I a r \sin \theta.$$

[1] *Phil. Mag.*, Vol. 29, p. 434, May, 1890.

The moment of inertia of the element about $A$ will be

$$adr^2,$$

where $d$ is the density, or mass per unit cube.

In ordinary instruments it is inconvenient to have the period of oscillation long, but for a constant period the controlling moment at unit angle must be proportional to the moment of inertia; hence the problem is to find the shape of the section such that the total deflecting moment for a given moment of inertia shall be a maximum.

If the magnetic moment of a spiral be made greater by increasing its radius, the moment of inertia will be increased in a greater ratio, and thus the period of free vibration of the coil will be increased. But this period is limited by practical considerations. We have, therefore, to consider the form, so that for a given moment of inertia there may be a maximum magnetic moment; or, what amounts to the same thing, for a given magnetic moment the coil may have a minimum moment of inertia.

The ratio of the magnetic or deflecting moment to the moment of inertia of the element considered is

$$\frac{\mathcal{H}Iar\sin\theta}{ard^2} = \mathcal{H}I\frac{\sin\theta}{rd}.$$

Since $\mathcal{H}$, $I$, and $d$ may be considered constants, the problem is to find the conditions making $\frac{\sin\theta}{r}$ a maximum for every element of the coil.

Consider the curve the polar equation to which is

$$r = c\sin\theta.$$

For a given value of $\pm c$ the equation represents two

circles tangent to $BC$ at the point $A$ (Fig. 67). The diameter of the circles is $c$. A family of such circles

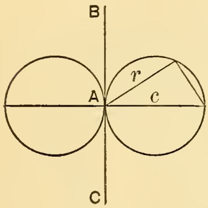


Fig. 67.

may be drawn with $A$ as the common point of tangency. If now we conceive a wire transferred from the surface of the circle to a point without, then the value of $c$ for such outer point is greater, and consequently $\frac{\sin\theta}{r}$ is less than for a point on the circumference. If it is transferred to a point inside the circle, the value of $\frac{\sin\theta}{r}$ is greater. If, therefore, the cross-section of the coil be any circle, $r = c\sin\theta$, a diminution of the value of the expression $\frac{\sin\theta}{r}$ would be produced by transferring any portion of the wire within the circle to any unoccupied space outside; that is, the ratio of the magnetic moment to the moment of inertia would be diminished.

Also, since the horizontal portions of the coil, lying parallel with the field, contribute to the moment of inertia and not to the deflecting moment, the deflecting moment will be increased by making the coil long and narrow. The cross-section of the long narrow coil must, moreover, be two tangential circles, their point of tangency being as nearly as possible on the axis of rotation of the coil.[1] The problem in hand "resolves itself into finding the shape and position of an area having a given moment of inertia about a point in its plane such that the moment of the area about a coplanar line through

[1] Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., Part II., p. 380.

the point is a maximum. Taking the point as a pole, these conditions are

$$\iint r^3\,dr\,d\theta \text{ is a constant,}$$

while $$\iint r^2 \sin\theta\,dr\,d\theta \text{ is a maximum.''}$$

**72. The Kelvin Balances.** — In the balances of Lord Kelvin the electrodynamic action between the fixed and movable coils is counterbalanced by adjustable weights or sliders instead of the torsion of a helical spring.

The coils are ring-shaped and horizontal. The two movable rings $E$ and $F$ (Fig. 68) are attached to the ends

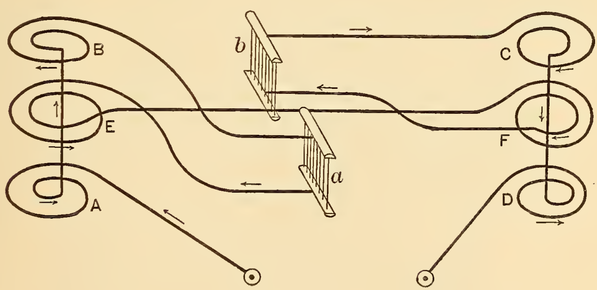



Fig. 68.

of a horizontal balance beam which is supported by two trunnions $a$ and $b$, each hung by an elastic ligament of fine wires, through which the current passes into and out of the circuit of the movable rings. These rings are placed midway between two pairs of fixed rings, $AB$ and $CD$, which are connected as shown in the diagram, so that the movable ring on either side is attracted by one of the fixed rings and repelled by the other. When a current passes through the six coils in series, the beam tends to rise at $F$ and sink at $E$.

The balancing is performed by means of a weight, which slides on a horizontal graduated arm attached to the balance beam (Fig. 69). A trough is fixed to the right-hand end of the beam, and in it is placed a weight which counterpoises the sliding weight, shown near the centre of the beam, when it is at the zero of the scale and no current is passing through the balance. By this arrangement the range of movement of the slider is the entire length of the beam. These weights can be changed so as to vary the range of the balance. Provision is made for the fine adjustment of the zero by means of a small metal flag, as in some chemical balances. A vertical scale and a horizontal pointer at each end of the balance arm determine the sighted zero position. When a current passes, the beam is brought back to the horizontal position by moving the sliding weight toward the right by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in the two directions by two silk cords passing through holes to the outside of the glass case. The balance is shown in the figure with the glass case removed. Since the force is proportional to the product of the current in the fixed and movable coils, the current is proportional to the square root of the turning moment. Hence the four pairs of weights (slider and counterpoise) supplied with each instrument are adjusted in the ratios of 1 : 4 : 16 : 64, so that for the same division of either scale the second weight indicates twice the current of the first, the third twice that of the second, and the fourth twice that of the third. Of the two scales the upper fixed one, called the inspectional scale, gives the current approximately in decimal parts of an ampere; but for more accurate reading the movable scale of equal

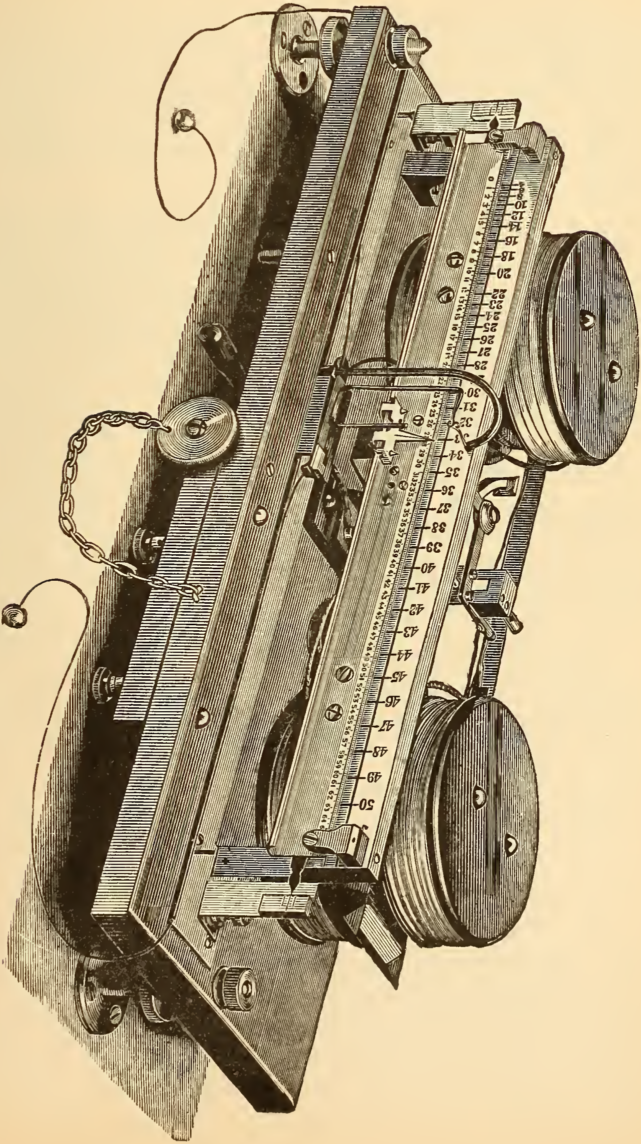


Fig. 69.

parts must be read, and the current calculated by the aid of the table of doubled square roots (Appendix, Table VI.). Thus, for example, if the balancing point is 475 on the scale of equal parts, the corresponding reading for the inspectional scale obtained from the table is 43.59.

There are several types of instruments made. The following table shows the value per division of the inspectional scale corresponding to each of the four pairs of weights for the centi-ampere, the deci-ampere, the deka-ampere, and the hekto-ampere balances:

| | I. Centi-amperes per division. | II. Deci-amperes per division. | III. Amperes per division. | IV. Amperes per division. |
|---|---|---|---|---|
| 1st pair of weights . . . | 0.25 | 0.25 | 0.25 | 1.5 |
| 2d " " " . . . | 0.5 | 0.5 | 0.5 | 3.0 |
| 3d " " " . . . | 1.0 | 1.0 | 1.0 | 6.0 |
| 4th " " " . . . | 2.0 | 2.0 | 2.0 | 12.0 |

The useful range of each instrument is from 1 to 100 of the smallest current for which its sensibility suffices. Thus the centi-ampere balance, illustrated in Fig. 69, has a range with the four weights from 1 to 100 centi-amperes, or from $\frac{1}{100}$ to 1 ampere.

Each balance is designed to carry 75 per cent of its maximum current continuously, and its maximum current long enough for standard comparisons.

The centi-ampere balance, with a thermometer to test the temperature of its coils, and in the more recent instruments with platinoid resistances up to 1,600 ohms, serves to measure potential differences of from 10 to 400 volts. The first resistance of the series includes that of the balance, which is about 50 ohms.

*Constants of Centi-ampere Balance as a Voltmeter.*

| Weight used. | Resistance in circuit. | Volts per division of fixed scale. |
|---|---|---|
| 1st pair | 400 | 1.0 |
| " " | 800 | 2.0 |
| " " | 1,200 | 3.0 |
| " " | 1,600 | 4.0 |

If the second pair of weights is used, the constants will be double those in the last column.

For the highest accuracy corrections must be made for the temperature of the balance and of the auxiliary platinoid resistance. The correction for the copper resistance of the former is about 0.4 per cent per degree centigrade, and for the latter about 0.024 per cent.

When the lowest potentials are measured the smallest platinoid resistance must be in the circuit; and one or more of the others must be included in series with it, when the potential is so high as to give a larger current than can be measured by the lightest weight on the beam.

**73. The Thomson Astatic Reflecting Galvanometer.** — For the highest sensibility the requirements of a good galvanometer are:

(*a*) An astatic magnetic system of small moment of inertia.

(*b*) A variable magnetic control.

(*c*) Four coils of nearly equal resistance.

(*d*) High insulation and large resistance.

Such an instrument is shown complete in Fig. 70. The coils are supported on grooved pillars for the purpose of increasing their insulation from the base. The binding posts on the top are the terminals of vertical brass rods which screw into special lugs on the coil frames. They

are disconnected from the case when in use. The open-

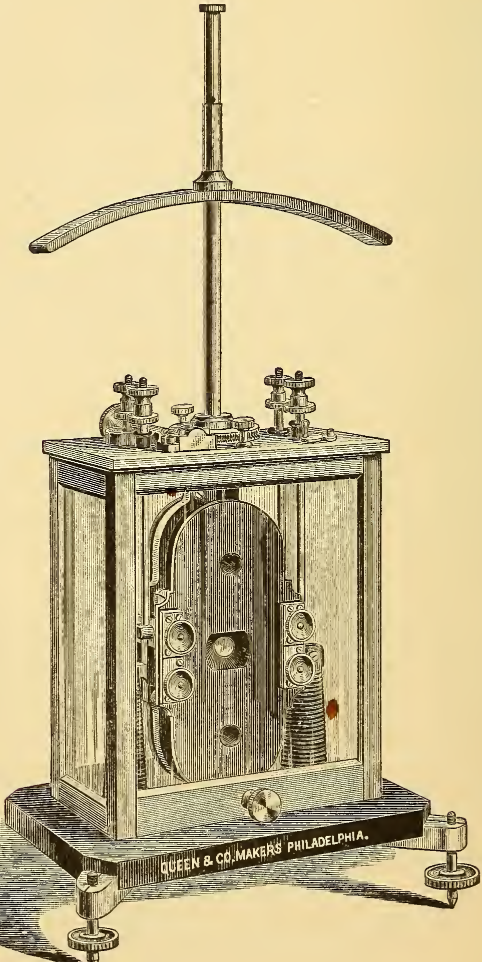



Fig. 70.

ing between the rods and the case can be closed by

rubber washers when the instrument is not in use. The control magnet on the vertical supporting rod is similar to the one on the tripod galvanometer of Fig. 8. The suspension is by means of a quartz fibre which is greatly superior to silk in strength, stability, uniformity, and smallness of torsion coefficient.

Fig. 71 is a galvanometer of similar construction. It shows the two coils on one side swung open, exposing the astatic magnetic system.

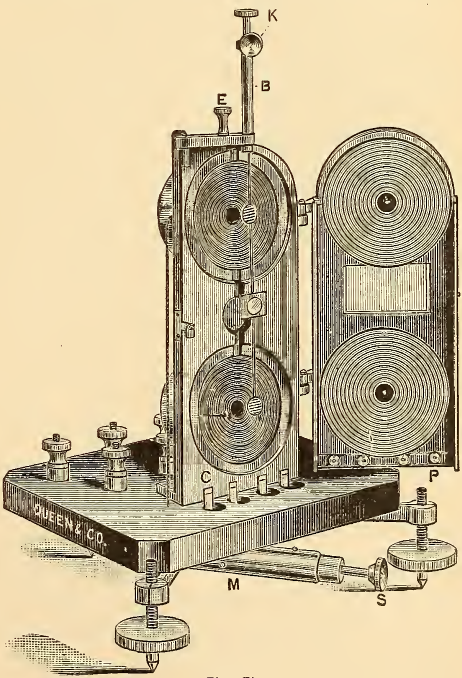


Fig. 71.

The magnetic system consists of two sets of minute magnets made of bits of fine watch-spring. Four or five of these are attached near the top of a thin aluminium wire with their north-seeking poles turned toward the north; the same number are similarly attached at the bottom, but with the north-seeking poles turned toward the south. The first set is placed at the centre of the upper pair of coils, and the other set at the centre of the lower pair. Between them

a small round mirror is hung in a very light aluminium cradle. This is either plane or concave, according as it is desired to read the deflections with a telescope and scale or with a lamp and scale.

If an incandescent lamp be available, by enclosing it in an appropriate case or hood, it may be used with a translucent scale, and may give sufficient light to read the deflections in a well-lighted room.

The movable system weighs only a fraction of a gramme. The arm carrying the suspending fibre swings out so that the system is entirely free and can be readily examined or conveniently mounted. The contact between the coils is automatic, and is made by means of platinized springs when the hinged face is closed. The use of flexible conductors is thus avoided.

The control magnet *M*, of Fig. 71, is novel and convenient. It not only turns around a vertical axis, but its effective magnetic moment can be varied by turning the milled head *S*. It consists of a permanent cylindrical magnet with threads cut on each end. On these threads turn two long nuts of soft iron which act as a magnetic shunt. They approach or recede from each other according as the magnet is turned by the milled head in the one direction or the other, since one thread is right hand and the other left. In this way the sensibility can be regulated with great exactness. The field produced by the control magnet at the needles is changed by the magnetic shunt instead of by changing the distance of the magnet from the suspended system.

It is customary to give to the upper set of magnets a slightly greater magnetic moment than that of the lower set. The entire system then places itself in the magnetic meridian, but with a very feeble directive force.

The mirror is commonly attached so as to look toward the west when the galvanometer is adjusted. The aluminium disks at the needles are intended to produce air damping, and to aid in bringing the movable system more rapidly to rest after deflection.

To adjust the galvanometer, proceed as follows:

Place it on some fixed support, such as a pier with a stone top, or on a shelf attached to a brick wall. Turn the instrument till the plane of the coils is as nearly as may be in the magnetic meridian. Next level by means of levelling screws till the movable system hangs entirely free within the coils. In lifting the system by the suspension pin, it should be raised very slowly and carefully till the needles are in the centres of the coils. They should then be entirely free, and the suspending fibre should be without torsion. The scale should then be placed at the proper distance from the galvanometer in the magnetic meridian, and horizontal. Next turn the control magnet till the plane of the mirror is in the magnetic meridian as nearly as possible. One can judge of this by looking into the mirror and getting an image of one's eye. Then move backward and observe if the line of sight is perpendicular to the face of the instrument. If not, adjust by turning the control magnet. Then make the height of the telescope and scale such that on looking directly along the tube of the telescope an image of the scale can be seen in the mirror. Focus the telescope and finally adjust the image by slightly changing the height of the scale, and by the altitude and azimuth screws on the telescope stand. It is better to have the scale numbered from one end to the other, to avoid the use of positive and negative quantities. A deflection is then taken by subtracting the reading of

rest from the reading in the deflected position, or conversely.

The north-seeking pole of the control magnet should be turned toward the north for greatest sensibility. If it is turned the other way it increases the strength of field at the needles, and so lessens the sensibility or the deflection for a given current.

**74. Calibration of any Galvanometer by Comparison with a Tangent Galvanometer.**[1] — Connect a tangent galvanometer $T$, the galvanometer to be calibrated $G$, a battery $B$, and a suitable resistance $R$, in series (Fig. 72). Note the deflections of both $T$ and $G$; vary the current by changing $R$, and again read the deflections. The resistances should be varied or so adjusted that the deflections of $G$ may be as nearly as possible equidistant. Then if the constant of the tangent galvanometer has been determined previously, the currents in amperes corresponding to the various deflections of $G$ are known. Construct a plain elastic curve, with currents as abscissas and deflections of $G$ as ordinates. This will be the calibration curve of $G$, from which may be read off the currents corresponding to other deflections.

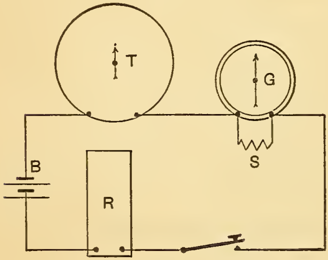


Fig. 72.

If the constant of $T$ has not been determined, the calibration of $G$ will be only relative and not absolute;

[1] Ayrton's *Practical Electricity*, p. 58.

that is, the deflections serve merely to compare currents, but not to measure them in amperes.

It may happen that $G$ is more sensitive than $T$. In that case a suitable deflection of $T$ produces too great a one in $G$. The difficulty may be avoided by putting a shunt or by-path around $G$, indicated at $S$. The calibration will then be relative, unless the ratio of the resistances of $G$ and $S$ is known.

**Example.**

| $G$ Deflections (1). | Tangent Galvanometer. | | Currents (2). | (2) ÷ (1). |
|---|---|---|---|---|
| | Deflections. | Tangents. | | |
| 5° | 2.2° | 0.038 | 0.00192 | 0.000384 |
| 10 | 4.4 | 0.077 | 0.00389 | 0.000389 |
| 15 | 6.95 | 0.122 | 0.00616 | 0.000410 |
| 20 | 9.8 | 0.173 | 0.00874 | 0.000437 |
| 25 | 12.95 | 0.230 | 0.01161 | 0.000464 |
| 30 | 16. | 0.287 | 0.01451 | 0.000483 |
| 35 | 19.3 | 0.350 | 0.01772 | 0.000506 |
| 40 | 22.5 | 0.414 | 0.02096 | 0.000524 |

The curve (Fig. 73) expressing the relation between deflections and currents is plotted as described above.

**75. Relative Calibration of a Galvanometer by Ohm's Law.** — Connect a suitable constant potential battery to a slide-wire bridge $PQ$ (Fig. 74), with sufficient resistance at $R'$ to adjust the current through the bridge wire to a proper value. A key should be inserted in this circuit so as to keep the current flowing only so long as it is needed. Join the galvanometer to be calibrated and a resistance box to one end of the bridge wire at $P$, and the other end of this circuit to a suitable contact-maker on the wire.

The experiment consists in placing the contact-maker $A$ at successive equal divisions on the scale and observ-

ing the deflections of the galvanometer. A series of observations should first be made with the battery current flowing in one direction, and then another similar series with the current reversed. The mean of the readings should be taken for each division on the bridge scale.

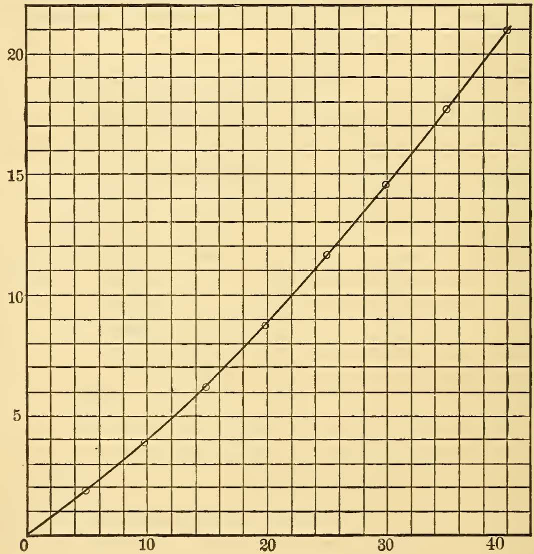



Fig. 73.

The differences of potential along the wire are, by Ohm's law, proportional to the resistances passed over, or to the length of wire between the two points of the divided circuit. But the resistance in the circuit of the galvanometer remaining unchanged, the currents through it will be proportional to the P.D. between its terminals — that is, to the lengths of the bridge wire included between the points of derivation *A* and *P*.

It is assumed that the E.M.F. of the battery remains constant, and that the resistance in circuit with it remains fixed. A storage battery is, therefore, to be preferred to a primary polarizable cell, and the student should carefully guard against heating the conductor by keeping

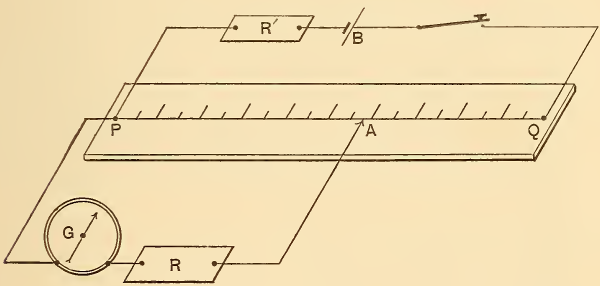



Fig. 74.

the circuit closed longer than is absolutely necessary. Since we have a divided circuit between $A$ and $P$, an appreciable error will be introduced unless the resistance in circuit with the galvanometer is high in comparison with that of the bridge wire.

**Example.**

*Calibration of a d'Arsonval Galvanometer.*

| Readings on bridge wire. | Mean deflections in mm. | Common difference. | Deflection per cm. |
|---|---|---|---|
| 10 cm. | 48.0 | 48.0 | 4.80 |
| 20 | 96.5 | 48.5 | 4.82 |
| 30 | 144.0 | 47.5 | 4.80 |
| 40 | 191.5 | 47.5 | 4.79 |
| 50 | 238.5 | 47.0 | 4.77 |
| 60 | 285.0 | 46.5 | 4.75 |
| 70 | 331.0 | 46.0 | 4.73 |
| 80 | 378.0 | 47.0 | 4.72 |
| 90 | 424.0 | 46.0 | 4.71 |

These observations are plotted with deflections of the galvanometer as ordinates and distances on the wire as abscissas (Fig. 75). The calibration curve is nearly straight, showing that the deflections are nearly proportional to the currents.

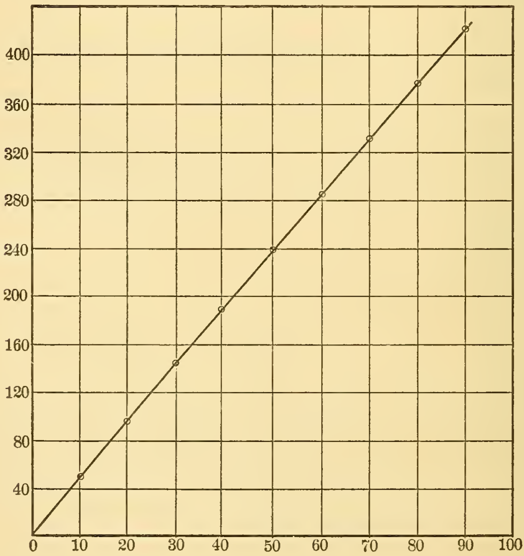



Fig. 75.

**76. Calibration of a Galvanometer by Known Resistances.** — The necessary apparatus consists of a battery of very low internal resistance, preferably a storage cell, and resistances reliably adjusted. The resistance of the galvanometer must also be known if it is enough to be appreciable in comparison with the remaining resistance in circuit. Connect the battery, the galvanometer, and the adjustable resistance in series. Adjust the resistance for successive readings of the gal-

vanometer and record galvanometer readings and total resistances in circuit. Then by Ohm's law the successive currents are inversely proportional to the corresponding resistances; and if the E.M.F. of the battery is known, the calibration will be in amperes. The internal resistance of the battery is supposed to be negligible in comparison with the remaining resistance in circuit. The following data illustrate the method. The resistance of the instrument and connecting wires was found to be 1.6 ohms. This must be added to the resistances taken from the resistance box.

**Example.**

| (*a*) Readings of Instrument. | (*b*) Total Resistance in Circuit. | (*c*) Reciprocals of Resistance. $\frac{1}{b}$ |
|---|---|---|
| 20 | 860 + 1.6 | .001160 |
| 30 | 563 " | .001771 |
| 40 | 420 " | .002372 |
| 50 | 334 " | .002979 |
| 60 | 280 " | .003551 |
| 70 | 238 " | .004174 |
| 80 | 209 " | .004748 |
| 90.2 | 185 " | .005359 |
| 100 | 167 " | .005931 |
| 110.3 | 151 " | .006553 |
| 119.8 | 139 " | .007112 |
| 130 | 128 " | .007716 |
| 140 | 118.6 " | .008319 |
| 151 | 110 " | .008960 |
| 161 | 103 " | .009560 |
| 170.8 | 97 " | .010142 |
| 180 | 92 " | .010684 |
| 190 | 87 " | .011286 |
| 199 | 83 " | .011820 |

Columns ($a$) and ($c$) have been plotted as coördinates (Fig. 76), and the result is very accurately a straight line passing through the origin. The instrument of the table was a Weston milli-voltmeter, reading from 2 to 20 milli-volts, and the scale readings are directly proportional to the currents and therefore to the volts measured.

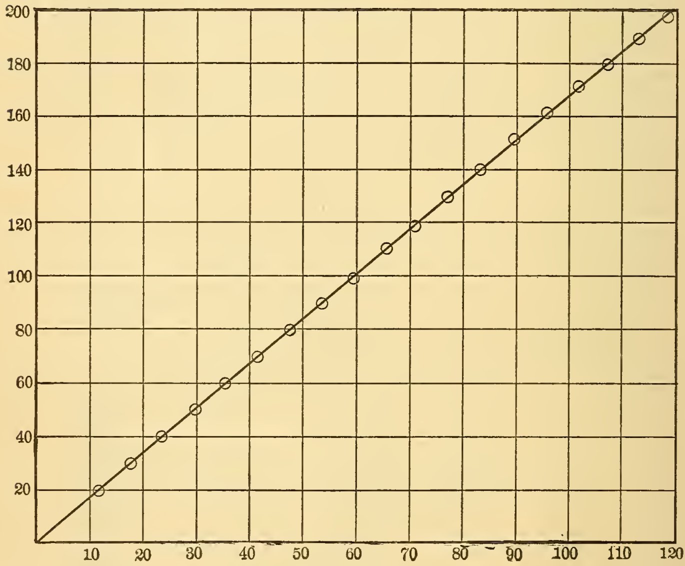



Fig. 76.

**77. Measurement of Current by Electrolysis.** — When an electric current passes through a chemical compound in the liquid state, the compound is decomposed. The process is called *electrolysis*, and the component parts into which the substance is divided are called *ions*. These collect at the *electrodes*, or the conductors by which the current enters and leaves the electrolyte.

The electrode by which the current enters is called the *anode;* and the one by which it leaves the electrolyte is the *cathode.*

Faraday demonstrated that the quantity of an ion deposited is proportional to the quantity of electricity which has passed. Hence the quantity deposited in unit time is proportional to the current strength.

He further showed that the same quantity of electricity deposits weights of different ions proportional to their chemical equivalents; that is, proportional to the relative quantities which chemically replace one another. Thus the quantity which will release one gramme of hydrogen will deposit 32.5 grammes of zinc, 31.66 of copper, 108 of silver, and so on. These quantities are the atomic weights of univalent substances and the half atomic weight of bivalent ones. It follows that if the weight of one of the substances deposited by one coulomb can be found by experiment, the known atomic weights of the chemical elements will give the electrochemical equivalents of the others, or the weights of the several elements which are released or deposited by one coulomb of electricity.

When the electrochemical equivalent of some convenient element has been ascertained, then the weight of it deposited in an observed interval of time serves as a measure of the quantity of electricity which has passed. If further the current has been maintained at a constant value, then this value may be determined by dividing the whole quantity of electricity by the time in seconds, or by dividing the weight of the ion by the product of the electrochemical equivalent and the time. The electrolytic process furnishes the practical method of determining the international ampere (Art. 19).

If $w$ is the weight of the ion deposited, $z$ its electrochemical equivalent, and $t$ the time of deposit, then the current will be

$$I=\frac{w}{zt}.$$

**78. The Silver Voltameter.** — For currents as large as one ampere the cathode on which the silver is deposited should take the form of a platinum bowl not less than 10 cms. in diameter and from 4 to 5 cms. in depth.

The anode should be a plate of pure silver some 30 sq. cms. in area and 2 or 3 millimetres in thickness. This is supported horizontally in the liquid near the top of the solution by platinum wires passing through holes in the plate. To prevent the disintegrated silver or particles of silver oxide or carbon falling from the anode into the platinum bowl, the anode should be wrapped around with pure filter paper and secured at the back with sealing wax.

The liquid should consist of a neutral solution of pure nitrate of silver, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

The method of making the measurement is as follows:

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution and connected to the rest of the circuit by being placed on a clean copper support, to which a binding-screw is attached. The copper support must be insulated.

The anode is then immersed in the solution, so as to be well covered by it, and supported in that position; the connections to the rest of the circuit are then made. Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during the interval.

The solution is now removed from the bowl and the deposit is washed with distilled water and left to soak for at least six hours. It is rinsed successively with distilled water and absolute alcohol, and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator the bowl is weighed again. The gain in weight gives the silver deposited.

To find the current in amperes, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed and by 0.001118.

The result will be the time average of the current, if during the interval the current has varied.

In determining by this method the constant of an instrument the current should be kept as nearly constant as possible, and the readings of the instrument taken at frequent observed intervals of time. These observations should give a curve from which the reading corresponding to the mean current (time average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading.

Instead of dividing by the time of deposit in seconds and by 0.001118, it is usually easier to divide by the time in hours (fractions) and by 4.025.

Instead of the costly platinum bowl as cathode, a convenient substitute, which is superior in some respects, is a flat silver plate, mounted between two anode plates of pure silver, as shown in Fig. 77. The plates are mounted on a hard-rubber strip *A* by means of stiff spring clips. By loosening the screw *B*, the plates can all be removed together from the solution. The plates can be raised or lowered by means of a rack and pinion. This is a convenient method of effecting a fine adjustment of the resistance of the circuit in making and maintaining an electrical balance. The anode plates do not need to be covered with filter paper, since any dislodged particles will fall to the bottom of the jar. Great care is necessary in washing, drying, and weighing the gain plate. It may be handled and weighed by means of a hook of stiff brass wire for suspension. This is a better plan than to run the risk of detaching particles of silver by laying the plate down, except in the bottom of a glass tray in washing. This form of voltameter provides better insulation than those in which the

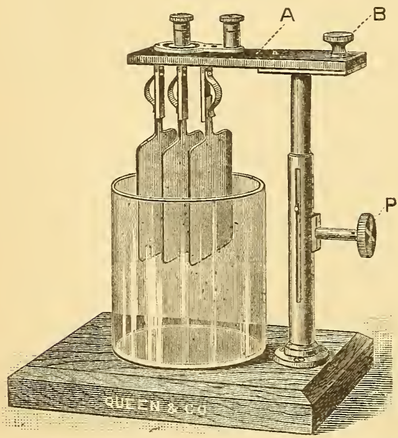


Fig. 77.

bowl rests on a base on which the nitrate of silver solution is almost certain to be spilled by lack of extreme care. In this form neither the base nor the standard forms any part of the conducting circuit.

**79. The Copper Voltameter.** — When large currents are measured by electrolysis the copper voltameter is employed instead of the corresponding one of silver, because the size of the plates required would make the latter too expensive. The copper voltameter scarcely equals the silver voltameter in accuracy, partly because of oxidation and partly because the electrochemical equivalent of copper is much smaller than that of silver, so that for a given current the quantity of copper deposited is less than that of silver, and it cannot be weighed with so small a percentage of error. On the other hand, the copper has the advantage of simplicity in manipulation. Silver is always deposited in a crystalline form, and requires careful washing and handling to avoid losses. It is difficult to make it adhere firmly to the gain plate or platinum bowl unless the surface is not less than 200 nor more than 400 sq. cms. per ampere. The deposited copper is much more firmly adherent, and 50 sq. cms. per ampere will give good results. Thus for large currents, the copper plates need not be more than one-fifth as large as the silver.

The solution is made by dissolving copper sulphate crystals in distilled water and adding one per cent of sulphuric acid. It may have a density varying from 1.1 to 1.2 without any difference in the nature of the deposit. A density of about 1.15 to 1.18 is to be preferred.

The solution should not be used too often, since the

acid is exhausted by action on the plates; and unless the solution is distinctly acid the results will be very irregular.

The loss plates should never have an area of less than 40 sq. cms. per ampere. If they are smaller than this, the resistance of the cell becomes variable and the current cannot be kept constant.

The gain plates, or cathode, should never be less than 20 sq. cms. per ampere. An area of from 50 to 100 sq. cms. per ampere is best. The smaller the area the less firmly adherent is the crystalline copper deposit. When the deposit is continued for a long time the larger area should be used.[1] At the current density of one-fiftieth of an ampere per sq. cm. there is a slight tendency for the deposit to thicken at the edges of the plates and become rough, but this tendency becomes less marked as the current density diminishes. A uniform and solid deposit is very desirable, and this is interfered with if the plates roughen at the edges.

The plates may be prepared by rounding and smoothing the edges and corners, and then polishing thoroughly with glass paper and washing in a rapid stream of water. They may then be rubbed with a clean cloth. On removing from the electrolytic cell, wash at once thoroughly in water containing a few drops of sulphuric acid, finally in distilled water, and dry on a clean blotting-pad. The plate may then be held before a fire and carefully warmed. It must not be weighed till it has cooled.

For large currents a rectangular glass or earthenware vessel may be used to contain the solution, and the plates

---

[1] A. W. Meikle, *The Electrolysis of Copper Sulphate*, Physical Soc. of Glasgow University.

may be of the shape shown in Fig. 78. They are held in spring clips on one side, the anode and cathode plates alternating, one set connected by the clips on one side and the other set on the other. Each plate may then be lifted out and cleaned separately. The following table is given by Mr. Meikle, connecting the area of the plate, the temperature, and the electrochemical equivalent:

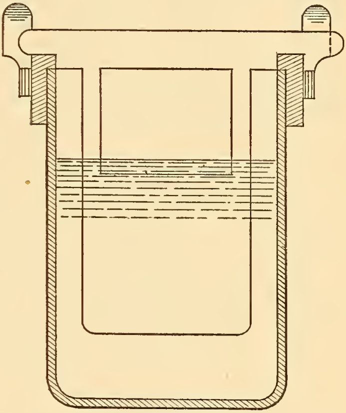


Fig. 78.

| Sq. cms. of cathode per ampere. | 12° C. | 23° C. | 28° C. |
|---|---|---|---|
| 50 | .0003288 | .0003286 | .0003286 |
| 100 | .0003288 | .0003283 | .0003281 |
| 150 | .0003287 | .0003280 | .0003278 |
| 200 | .0003285 | .0003277 | .0003274 |
| 250 | .0003283 | .0003275 | .0003268 |
| 300 | .0003282 | .0003272 | .0003262 |

The process of obtaining the current from the weight of copper deposited in an observed time is the same as in the case of silver.

The following solution for a copper voltameter is said to give good results:[1]

| | | |
|---|---|---|
| Copper sulphate . . . . . . . . . | 15 | gms. |
| Sulphuric acid . . . . . . . . . . | 5 | " |
| Alcohol . . . . . . . . . . . . . | 5 | " |
| Water . . . . . . . . . . . . . . | 100 | " |

[1] *Electrician* (London), May 19, 1893.

This can be used with a current density from 0.06 to 1.5 amperes per square decimetre.

**80. To find the Constant or Reduction Factor of any Current Meter by Electrolysis.** — If the currents to be measured by the instrument in question do not much exceed one ampere, the silver voltameter is to be preferred; but for currents in excess of one ampere the copper voltameter may be used.

When applied to a tangent galvanometer the operation consists in finding the reduction factor $A$, which multiplied by the tangent of the angle of deflection gives the current in amperes. With an electrodynamometer the process has for its object the determination of the constant in the equation

$$I = A\sqrt{D},$$

in which $D$ is the torsion in divisions of the scale and $A$ is the constant to be determined. When applied to a direct-reading ammeter it can find only the error of the scale corresponding to the number of amperes flowing through the voltameter. The apparatus may be set up as follows:

$B$ is a storage battery of a sufficient number of cells to furnish the requisite current through the parallel resistances $R$ and $R'$ and the voltameter $V$ (Fig. 79). When the E.M.F. of the battery and the approximate current which is to be measured by the voltameter are known the resistances $R$ and $R'$ can be determined beforehand. $R'$ is put in parallel with $R$ for the purpose of keeping the current constant through the voltameter and galvanometer. It may be either a carbon rheostat of the proper construction, or any other resistance

adjustable by insensible or at least very small gradations. Any small change in the current can thus be very readily compensated by adjusting the resistance $R'$.

A convenient form for currents not exceeding three or four amperes may be made by winding a flexible cable, such as heavy picture-wire, on an insulating tube supported by an iron rod through it and around insulating pins at the bottom (Fig. 80). The conductor is

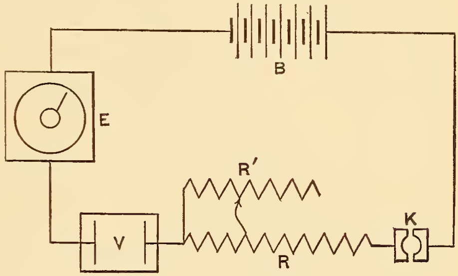



Fig. 79.

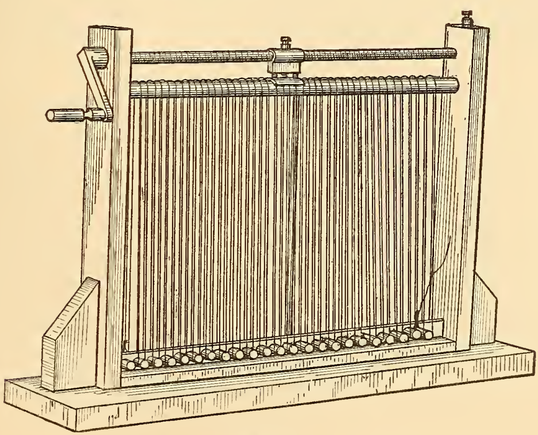


Fig. 80.

thus wound non-inductively. If it were wound round and round on the frame or on a cylinder, it would pro-

duce a magnetic field within it. The long brass screw at the top is traversed by a contact-maker. Instead of a nut this contains a screw pin, so that the contact-maker may slide readily from one end of the screw to the other by merely unscrewing the pin. When the pin is screwed in, the contact-maker may be moved slowly along the wires, so as to vary the portion in circuit, by turning the handle.

If the constant of the electrodynamometer is to be determined, the instrument should be set up with the plane of its movable coil at right angles to the magnetic meridian, or with its magnetic axis in the earth's magnetic meridian, and variable currents should be avoided.

As a check, it is desirable to employ two electrolytic cells in series. One-half the weight of the electrolyte or metal deposited in the two is then taken for use in the formula with either the silver or the copper voltameter.

**Example I.**

*To find the Reduction Factor of a Tangent Galvanometer.*

The galvanometer was set up in series with a silver voltameter, two Daniell cells, and a commutator for reversing the current through the galvanometer. The coil used was marked 29.893 ohms. The current deposited silver for thirty minutes, and the deflections were read every minute, except when the current was reversed, when one reading was omitted. The observations were as follows:

| Time. | Deflections. | | Time. | Deflections. | |
|---|---|---|---|---|---|
| | Left. | Right. | | Left. | Right. |
| 11.09 | | | 25 | 43.2 | |
| 10 | 41.3 | | 26 | | |
| 11 | 41.3 | | 27 | | 44.0 |
| 12 | | | 28 | | 44.0 |
| 13 | | 42 | 29 | | 44.4 |
| 14 | | 42 | 30 | | 44.5 |
| 15 | | 42.5 | 31 | | 44.6 |
| 16 | | 42.5 | 32 | | 44.6 |
| 17 | | 42.6 | 33 | | |
| 18 | | 42.7 | 34 | 44.2 | |
| 19 | | | 35 | 44.4 | |
| 20 | 42.5 | | 36 | 44.5 | |
| 21 | 42.6 | | 37 | 44.6 | |
| 22 | 42.7 | | 38 | 44.6 | |
| 23 | 43.0 | | 39 | 44.7 | |
| 24 | 43.1 | | | | |
| Mean | | | | 43.34 | 43.37 |

| | | |
|---|---|---|
| Mean deflection | 43.36 | |
| Tangent of mean deflection | 0.94435 | |
| Weight of cathode before deposit | 30.3726 | gms. |
| Weight of cathode after deposit | 30.4685 | " |
| Gain | 0.0959 | " |

Average current equals $\dfrac{0.0959}{4.025 \times \frac{1}{2}} = 0.04765 = A \tan \theta.$

Therefore $$A = \frac{0.04765}{0.94435} = 0.05046.$$

**Example II.**

*To find the Constant of Siemens Electrodynamometer, No. 97 Q.*

Two copper voltameters were connected in series with the electrodynamometer, 14 cells of storage battery, and a resistance which served to regulate the current.

The table gives the observations at one-minute intervals :

| Readings. | $\sqrt{\text{Readings}}$. | Readings. | $\sqrt{\text{Readings}}$. |
|---|---|---|---|
| 81 | 9 | 79.8 | 8.933 |
| 81 | 9 | 79.5 | 8.916 |
| 80.5 | 8.972 | 79.5 | 8.916 |
| 80 | 8.944 | 79.5 | 8.916 |
| 80 | 8.944 | 81 | 9 |
| 80 | 8.944 | 81 | 9 |
| 80 | 8.944 | 81.5 | 9.028 |
| 80 | 8.944 | 81.5 | 9.028 |
| 80 | 8.944 | 81.5 | 9.028 |
| 80 | 8.944. | 81.9 | 9.050 |
| 78.8 | 8.877 | 81.9 | 9.050 |
| 79.9 | 8.939 | 81.9 | 9.050 |
| 79.8 | 8.933 | 82.1 | 9.061 |
| 79.9 | 8.939 | 82.2 | 9.066 |
| 79.8 | 8.933 | 82 | 9.055 |
| Mean . . . . . . . . . . . . . . . . . . . . . . | | | 8.977 |

| | I. | II. |
|---|---|---|
| Weight of cathode plate before deposit . . | 103.6476 | 83.4925 |
| " " " " after " . . | 104.6026 | 84.4475 |
| Gain . . . . . . . . . . . . . . . | 0.955 | 0.955 |

$$I = A\sqrt{D} = \frac{0.955}{\frac{1}{2} \times 1.1838} = 1.6134 \text{ amperes.}$$

Therefore $$A = \frac{1.6134}{8.977} = 0.1797.$$

**81. Arrangement for Strong or Weak Currents.**[1] — When a very strong or a very weak current is used, the apparatus illustrated in Fig. 81 may be employed. In the former case the current which it is desired to measure is larger than the capacity of the electrolytic cell; in the latter case it is smaller than it is necessary to use for the purpose of obtaining an accurate result by electrolysis. The figure shows the arrangement for the first case of heavy currents, in which the current through the instrument for measuring current is nine times as great as through the two electrolytic cells in series.

[1] Gray's *Absolute Measurements in Electricity and Magnetism*, Vol. II., Part II., p. 428.

A set of parallel straight wires of German silver, platinoid, or manganin are soldered to thick terminal bars of copper, $b$, $b_1$, $b_2$, as shown, so that they can be connected in two groups in parallel. The wires in position must have accurately the same resistance. A sensitive reflecting galvanometer $g$ of high resistance connects $b_1$ and $b_2$. The resistances $R$ and $R'$ must be so adjusted that no

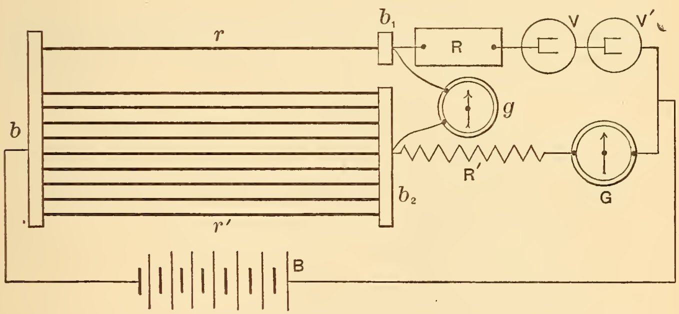



Fig. 81.

current flows through $g$; or, in other words, so that $b_1$ and $b_2$ are at the same potential. The current through $G$ will then be nine times the current measured by the electrolytic cells $V$ and $V'$, or in the ratio of the conductances of the two groups of wires $r$ and $r'$.

$G$ is the galvanometer or other current measurer to be calibrated.

**82. Measurement of Current by Means of a Standard Cell.** — A standard Clark cell will be described later (Art. 85). For the present, it is only necessary to say that a Carhart-Clark cell gives a constant E.M.F. of 1.440 volts at 15° C. (Latimer-Clark cell, 1.434 v.) Such a cell may be employed in connec-

tion with standard resistances to measure currents in amperes.

The method consists in balancing the E.M.F. of a standard cell against the fall of potential over the whole or a part of a known resistance through which the current to be measured flows.

Let $r$ (Fig. 82) be the known resistance placed in the main circuit in which flows the current to be measured.

This resistance should consist of a metallic conductor capable of carrying the current without undue heating.

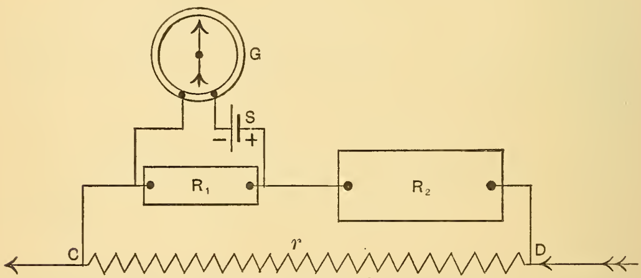



Fig. 82.

If it is so mounted that it can be immersed in kerosene or oil the temperature can be kept nearly constant; and, what is quite as important, it can be measured accurately.

Two resistance boxes of high resistance are then placed in a derived circuit as a shunt to the resistance $r$. From the terminals of one, as $R_1$, another derived circuit is set up containing a standard cell $S$ and a sensitive galvanometer $G$. This circuit should also contain a key. The poles of the cell must be turned so that the P.D. over $R_1$ shall be opposed to the E.M.F. of the cell. The balance is then made by adjusting $R_1$ or $R_2$ till no

current flows through the galvanometer on closing the key in its circuit. We have then

$$R_1 : R_1 + R_2 : : 1.44 : E,$$

where $E$ is the P.D. between $C$ and $D$.

Then $$E = 1.44 \frac{R_1 + R_2}{R_1}.$$

If the temperature of the standard cell is not 15° C. a correction must in general be made.

Finally,

$$I = \frac{1.44}{r} \cdot \frac{R_1 + R_2}{R_1}.$$

It is evident that the resistance $r$ must be such that the P.D. between its terminals shall be equal to or greater than the E.M.F. of the standard cell.

**Example.**

*To determine the Constant of a Thomson "Graded Galvanometer" (ammeter) without its Field-Magnet.*

*Formula:* $$I = \frac{A \times D}{\text{Base number}},$$

where $A$ is the constant to be determined, $D$ the deflection, and by "base number" is meant the number indicating the position of the sliding magnetometer box on the base of the instrument.

*Data:* $R_1 = 2110$; $R_2 = 1254$; and $r = 10$ ohms at 24° C.

E.M.F. of standard cell at 20.5° C. = 1.437 volts.

Therefore, $$I = \frac{1.437}{10} \cdot \frac{2110 + 1254}{2110} = 0.229 \text{ ampere}.$$

$$D = 38.5 \text{ divisions}.$$

$$\text{Base number} = 32.$$

Hence from the above formula,

$$A = \frac{0.229 \times 32}{38.5} = 0.190.$$

This constant is the value of the magnetic field at the needle when no current is flowing.

**83. Second Method by Means of a Standard Cell.** — This method, the connections for which are shown in the diagram (Fig. 83), admits of using a resistance $r$ of such dimensions that the difference of potential between its terminals may be greater or less than the

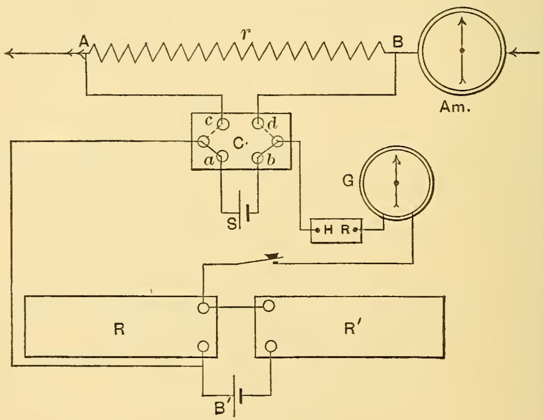



Fig. 83.

E.M.F. of the standard cell or cells employed. The resistance must be capable of carrying the current during the time required to effect the balance without appreciable heating; or, better, it may be immersed in oil, with a stirrer, so that its temperature may be known.

Set up two 10,000 ohm resistance boxes in series with a battery $B'$ of higher E.M.F. than the standard cell or the P.D. between $A$ and $B$. From the terminals of $R$ form a shunt circuit containing a sensitive high resistance galvanometer and a standard cell. It is better also

to include a high resistance HR in this circuit. The poles of the standard cell must be turned in such direction that the P.D. between the terminals of $R$ opposes the E.M.F. of the cell. Then, *keeping a total of* 10,000 *ohms in the two boxes $R$ and $R'$*, vary the part contained in each box till, on closing the key, the galvanometer $G$ shows no deflection. The P.D. between the terminals of $R$ then equals the E.M.F. of the standard cell. The high resistance HR may be so arranged, if necessary, that it can be short-circuited when a balance is nearly effected, so as to increase the sensibility. Then with the circuit closed through $AB$, transfer the terminals of the derived circuit from $ab$ to $cd$ by means of the commutator $C$ and balance again. The fall of potential over the resistance now in $R$ will be equal to that over $AB$. But the two P.D.'s are proportional to the two resistances in $R$ required to balance. Call these $R_1$ and $R_2$.

Then

$$R_1 : R_2 :: 1.44 : x,$$

and

$$x = 1.44 \frac{R_2}{R_1},$$

where $x$ is the P.D. between $A$ and $B$. Then as before

$$I = \frac{x}{r} = \frac{1.44}{r} \cdot \frac{R_2}{R_1}.$$

The E.M.F. of the standard cell must always be corrected for temperature. So also should the resistance $r$.

This method is much more flexible than the first one, since it is not necessary to balance the E.M.F. of the cell directly against a part of the P.D. between the terminals of the resistance in the circuit in which the current to be measured is flowing. Hence with the same

resistance $r$ a balance may be effected with a considerable range of current. This method may therefore be used to calibrate an ammeter $Am$.

**Example.**

*To test the Accuracy of a Weston Milli-ammeter.*

The ammeter was connected in series with $r$, a storage battery, and a resistance to control the current.

Reading of milli-ammeter 0.828.

$r = 1.637$ ohms at 25° C.

$R_1 = 6885$ ohms.

$R_2 = 6502.5$ ohms.

E.M.F. of standard cell, 1.437 volts at 20° C.

Hence $$I = \frac{1.437 \times 6502.5}{1.637 \times 6885} = 0.829 \text{ ampere.}$$

**84. Standard Resistances for the Preceding Methods.** — When large currents are measured by the preceding methods, special standard resistances adapted to carry the desired currents should be employed. Such standards have been designed at the Physikalisch-Technische Reichsanstalt, in Berlin.[1] They have a resistance of 0.01, 0.001, and 0.0001 ohm respectively, and are made of manganin in sheet form or cast. Special terminals, from the exact points between which the resistance is measured, are brought out to separate binding-posts for the measurement of the potential difference by comparison with a Clark cell. Any small E.M.F. of contact between the manganin and the copper terminals and leading-in conductors is thus left out of the comparison. The smallest resistance is adapted to carry a few thousand amperes. These standards are mounted in nickel-plated

[1] *Elektrotechnische Zeitschrift*, 1895.

cases (Fig. 84) which can be filled with oil. The large case for heavy currents is fitted with a cooling coil

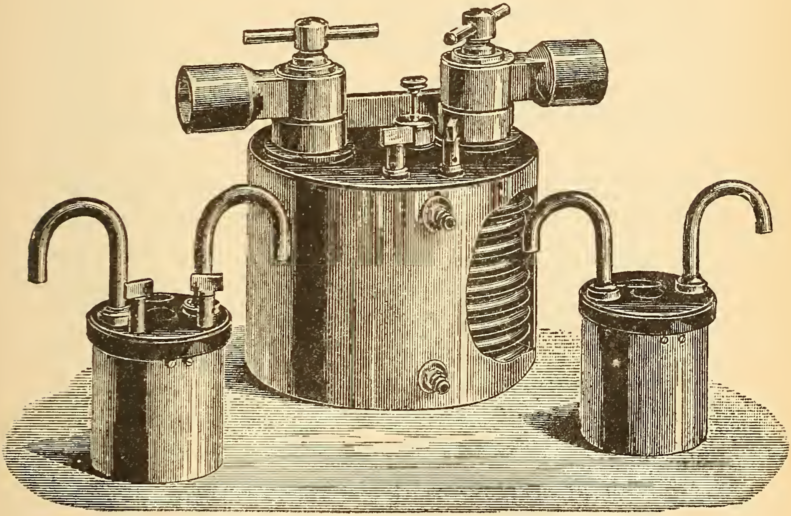


Fig. 84.

through which water may be made to flow. It contains also a diminutive turbine-stirrer which can be driven by any small motor.

# CHAPTER IV.

## MEASUREMENT OF ELECTROMOTIVE FORCE.

**85. The Clark Standard Cell.** — In accordance with the decision of the Chamber of Delegates of the Chicago International Congress of Electricians (Appendix B), the Clark cell has become the legal standard of E.M.F. (Art. 19). The cell consists of zinc, or an amalgam of zinc with mercury, and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with both sulphates in excess.

The preparation of the materials entering into the cell and the setting up of the standard will be described with some detail.

### *A. Preparation of the Materials.*

1. *The Mercury.* — All mercury used in the cell should first be chemically purified in the usual manner, and subsequently distilled in a vacuum.

2. *The Zinc.* — Pure redistilled zinc-rods can be used without further treatment. For the preparation of the zinc amalgam add one part by weight of zinc to nine parts of mercury, and heat both in a porcelain dish until by gentle stirring at about 100° C. the zinc completely disappears in the mercury.

3. *The Mercurous Sulphate.* — If the mercurous sulphate, purchased as pure, is not colored yellow with a basic salt, mix with it a small quantity of pure mercury,

and wash the whole thoroughly with two parts by weight of cold distilled water to one part of the salt, by agitation or by stirring with a glass rod. Drain off the water and repeat the process at least twice, or until a very faint yellow tint appears. After the last washing drain off as much of the water as possible, but do not dry by heating. It is better to wash only so much of the salt as may be needed for immediate use.

4. *The Zinc Sulphate Solution.*—Prepare a neutral saturated solution of chemically pure zinc sulphate, free from iron, by mixing in a flask distilled water with nearly twice its weight of pure zinc sulphate crystals, and adding pure zinc oxide in the proportion of about 2 % by weight of the zinc sulphate crystals, to neutralize any free acid. The crystals should be dissolved by the aid of gentle heat, but the temperature of the solution must not be raised above 30° C. After warming for about two hours with frequent agitation, set the solution away over night. Then add mercurous sulphate, prepared as described in 3, in the proportion of about 12 % by weight of the zinc sulphate crystals, to neutralize any free zinc oxide remaining; the solution should again be warmed, and should be filtered, while still warm, into a glass-stoppered bottle. Crystals should form as it cools.

5. *The Mercurous Sulphate and Zinc Sulphate Paste.*—To three parts by weight of the washed mercurous sulphate add one part of pure mercury. If the sulphate is dry it may be rubbed together with a mixture of the zinc sulphate crystals and concentrated solution of zinc sulphate, so as to make a stiff paste, which shows throughout crystals of zinc sulphate and minute globules of mercury. If, on the contrary, the mercurous sulphate

is moist, the paste should be made by adding the zinc sulphate crystals only, taking great care that they are present in excess and do not disappear after the paste has stood for some time. The mercury globules must also be plainly visible. The zinc sulphate crystals may with advantage be crushed fine before admixture with the mercury salt.

The above process insures the formation of a saturated solution of the zinc and mercurous sulphates in water.

*B. To set up the Cell.*

The glass vessel containing the cell, represented in Fig. 85, consists of two limbs closed at the bottom and joined above to a common neck fitted with a ground-glass stopper. The diameter of the limbs should be at least 2 cms., and their length 3 cms. The neck should be not less than 1.5 cms. in diameter, and 2 cms. long. In the bottom of each limb a platinum wire of about 0.4 mm. diameter is sealed through the glass.

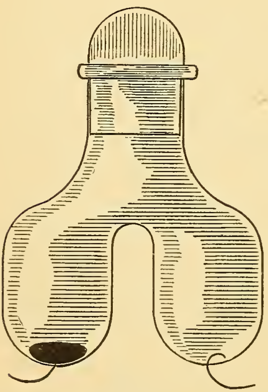


Fig. 85.

To set up the cell, place in one limb pure mercury, and in the other hot fluid amalgam containing 90 parts mercury and 10 parts zinc. The platinum wires in the bottom must be completely covered by the mercury and the amalgam respectively. On the mercury place a layer 1 cm. thick of the zinc and mercurous sulphate paste described in 5. Both this paste and the zinc amalgam must then be covered with a layer of the

neutral zinc sulphate crystals 1 cm. thick; and the whole vessel must then be filled with the saturated zinc sulphate solution, so that the stopper, when inserted, shall just touch it, leaving, however, a small bubble to guard against breakage when the temperature rises.

To prepare for placing the hot zinc amalgam in one limb of the glass vessel, after thoroughly cleaning and drying the latter set it in a hot-water bath. Then pass through the neck of the vessel and down to the bottom a thin glass tube to serve for the reception of the amalgam. This tube should be as large as the glass vessel will admit. It serves to protect the upper part of the cell from being soiled with the amalgam.

To fill in the amalgam, a clean dropping-tube about 10 cms. long and drawn out to a fine point has its fine end brought under the surface of the amalgam heated in a porcelain dish, and by pressing the rubber bulb some of the amalgam is drawn up into the tube. The point is then quickly cleaned of dross with filter paper, and is passed through the wider tube to the bottom and emptied by pressing the bulb. The point of the tube must be so fine that the amalgam will come out only on squeezing the bulb. This process is repeated till the limb contains the desired quantity of the amalgam. The vessel is then removed from the water bath; and, after cooling, the amalgam must be fast to the glass, and must show a clean surface with metallic lustre.

For insertion of the mercury a dropping-tube with a long stem will be found convenient. The paste may be poured in through a wide tube reaching nearly down to the mercury and having a funnel-shaped top. If it does not move down freely it may be pushed down with a small glass rod. The paste and the amalgam are then

both covered with the zinc sulphate crystals before the concentrated zinc sulphate solution is poured in. This should be added through a small funnel, so as to leave the neck of the vessel clean and dry.

Before finally inserting the glass stopper it should be brushed round its upper edge with a strong alcoholic solution of shellac, and should then be firmly pressed in place.

For convenience and security in handling, the cell thus set up may be mounted in a metal case which can be placed in a petroleum or paraffin oil bath. Its top may be provided with two insulated binding-posts to be connected with the two electrodes by the platinum wires, and the bottom should be perforated to allow the petroleum or oil to enter freely.

In order to ascertain the temperature of the cell, the metal case should enclose a thermometer which can be read from without. The thermometer may be fused into the glass stopper, or it may be entirely separate with its bulb immersed in the petroleum or oil bath within the case. The latter method is to be preferred.

In using the cell sudden variations of temperature should, as far as possible, be avoided, since the changes in electromotive force lag behind those of temperature.

The E.M.F. of this cell is 1.434 volts at 15° C.

For a small range of temperature above or below 15° C. the following formula may be employed to reduce to 15°:

$$E_t = 1.434\,[1 - 0.00080\,(t - 15)]\,.$$

Dr. Kahle gives for the formula connecting the E.M.F. at $t°$ with that at 15° the following:

$$E_t = E - 116 \times 10^{-5}\,(t - 15) - 1 \times 10^{-5}\,(t - 15)^2\,.$$

This holds between 10° and 30° C. The E.M.F. of this cell decreases by about 0.00115 volt per degree C.

**86. The Carhart-Clark Standard Cell.** — As a standard for practical commercial purposes a cell is needed which has the advantages of portability and a lower temperature coefficient than the normal Clark cell. These advantages have been secured in the following manner:

A piece of No. 28 platinum wire is heated red hot in a blow-pipe flame, and is then sealed into the bottom of a small tube about 5 cms. long and 1.5 cms. in diameter. In contact with this is pure redistilled mercury. A layer about 1 cm. thick of pure neutral mercurous sulphate mixed with neutral zinc sulphate saturated at 0° C. is placed on the mercury. The paste is then covered with purified asbestos; on this rests the broad foot of the zinc, cast as shown in Fig. 86. To the top of the zinc is soldered a thin copper wire. For the purpose of holding the seal a cork disc surrounds the top of the zinc. This must be thoroughly boiled in distilled water to remove the tannin, and after drying may be saturated with pure paraffin. The zinc sulphate solution surrounding the zinc must be poured in through a small funnel before the zinc is inserted. Finally the cell is sealed by pouring in hot a cement composed of gutta-

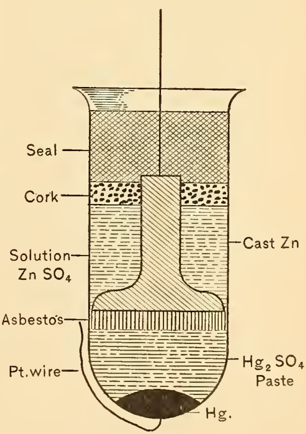



Fig. 86.

percha and Burgundy pitch, with enough balsam of fir added to make the compound flow when hot. After this has cooled, it is of advantage to add a mixture of finely powdered glass and sodium silicate.

The temperature coefficient is reduced to one-half that of the Clark cell by the use of a zinc sulphate solution saturated at a temperature lower than any at which the cell is to be used. A convenient temperature for this solution is 0° C. In the normal Clark cell a rise of temperature causes more zinc sulphate to go into solution. The consequent increase of density lowers the E.M.F. of the cell, and this effect is added to the real temperature coefficient which is due to the superposition of the two thermo-electromotive forces between the metal and the solution on the two sides of the cell.[1] Moreover the slowness with which the solution reaches the density corresponding with a new temperature causes the E.M.F. of the Clark cell to lag behind the temperature change. Both of these difficulties are avoided by the employment of a solution saturated at zero degrees.

The equation connecting the E.M.F. and temperature of the Carhart-Clark cell is

$$E_t = 1.440 \left\{ 1 - 0.000387\,(t-15) + 0.0000005(t-15)^2 \right\}.$$

Near 15° C. a formula sufficiently accurate for practical purposes is

$$E_t = 1.440 \left\{ 1 - 0.0004\,(t-15) \right\}.$$

The temperature coefficient of this cell is thus one-half that of the normal Clark standard.

[1] Carhart's *Primary Batteries*, p. 136; *Amer. Jour. of Science*, Vol. XLVI., p. 60.

**87. A One-Volt Calomel Cell.** — The calomel cell, consisting of mercury in contact with mercurous chloride and zinc in zinc chloride solution, was invented by von Helmholtz in 1882.[1] One of the present writers has investigated it with a view to adjust to exactly one volt.[2]

In 1879 D. H. Fitch patented a cell in which mercurous chloride was used as the depolarizer, but in other respects it differed from the Helmholtz form.

The E.M.F. of a chloride cell with zinc immersed in its chloride increases with decrease in density of the zinc chloride solution. Within limits, therefore, the E.M.F. of the calomel cell can be varied by varying the density of the zinc chloride solution. An increase of about 4.6 per cent in the density of the solution produces a decrease of 1 per cent in the E.M.F. The density required to give one volt is 1.391 measured at 15° C.

This cell is made in precisely the same form as the preceding. Such a cell is perfectly portable; and cells in our possession over a year old show no appreciable change in E.M.F. compared with normal Clark cells.

The temperature coefficient is small and is positive. The following equation connects the E.M.F with temperature for changes of a few degrees in the neighborhood of 15° C., or between 10° and 30° C.:

$$E = 1 + 0.000094\ (t - 15).$$

A near approach to the coefficient is 0.01 per cent per degree. A neglected variation of 10 degrees can cause an error of only 0.1 per cent.

Since the modified Clark cell described in the last

---

[1] *Sitzber. der Akad. der Wiss.*, p. 26, Berlin, 1882.

[2] *Amer. Jour. of Science*, Vol. XLVI., p. 60.

article has a negative coefficient and the calomel cell a small positive one, it becomes possible to combine the two varieties in such a way that the combined set shall have a zero coefficient. Let $x$ equal the number of calomel cells required to offset one Carhart-Clark.

Then $$0.000094\,x = 1.44 \times 0.00039,$$

or $$x = 6 \text{ nearly.}$$

**88. The Weston Standard Cell.** — Mr. Edward Weston has invented a standard cell consisting of mercury in contact with mercurous sulphate and cadmium amalgam immersed in a saturated solution of cadmium sulphate. The H form of the cell, similar to Fig. 85, has been selected as the best. A platinum wire is sealed into the bottom of each limb. In one limb is the pure mercury, and resting on it the mercurous sulphate paste mixed with the cadmium sulphate solution. In the other limb is the cadmium amalgam. The vessel is finally filled so as to connect the two limbs with the cadmium sulphate solution, and is sealed in the usual manner. The only difference in the structure between this cell and the Clark is that cadmium and cadmium sulphate are used in place of zinc and zinc sulphate. The scheme of the cell is as follows:

$$-\left|\, Cd - CdSO_4 - Hg_2SO_4 - Hg \,\right| + .$$

Weston's patent gives the E.M.F. of the cell as 1.019, and the temperature coefficient 0.01 per cent per degree centigrade.

This cell has also been investigated by Jager and Wachsmuth[1] at the Berlin Reichsanstalt. An amalgam

[1] *Zeit. für Instrumentenkunde*, November, 1894.

of 1 part of cadmium to 6 parts of mercury was covered with a layer of cadmium sulphate crystals. The mercurous sulphate was rubbed together with cadmium sulphate crystals, metallic mercury, and concentrated cadmium sulphate solution, so as to form a stiff paste. This was placed on the mercury of the positive pole. The remainder of the H element was filled with concentrated cadmium sulphate solution, the negative pole containing the cadmium amalgam.

Between 0° and 26° the temperature coefficient is expressed by the following formula:

$$E_t = E_0 - 1.25 \times 10^{-5}\, t - 0.065 \times 10^{-5}\, t^2.$$

Near 20° the change of E.M.F. per degree C. is only about 0.00004 volt. The following table shows the comparative temperature coefficients of the Clark and the Weston cell in $\frac{1}{1000}$ per cent:

| $t$ | Temperature Coefficient. | |
|---|---|---|
| | Clark. | Weston. |
| 0° | — 70.9 | — 1.3 |
| 10° | — 77.9 | — 2.5 |
| 20° | — 84.9 | — 3.7 |
| 30° | — 91.9 | — 5.0 |

Near 20° the E.M.F. of the cadmium element changes only about $\frac{1}{23}$ as much as the Clark element for the same temperature variation. When two per cent of zinc was added to the cadmium the increase of E.M.F. was only about 0.0004 volt. The cadmium sulphate of commerce contains only small traces of foreign substances, and these produce no appreciable effect on the E.M.F. It

is very essential, however, that the cadmium sulphate solution should be thoroughly neutral. Any trace of acid raises the E.M.F. To neutralize any acid cadmium hydroxide is used, and the filtered solution is treated with mercurous sulphate for the reduction of any basic salt that may have been formed. When the salt is thus treated different cells agree to within 0.0001 volt.

The solubility of cadmium sulphate changes only slightly with temperature. This is one reason for the smallness of the temperature coefficient, and in consequence the cell quickly reaches an electrical equilibrium after a variation of the temperature.

The constancy of the Weston cell can only be determined after long trial. Observations extending over four months showed that the element remained constant within 0.0001 volt. Compared with the Clark element its E.M.F. was found to be 1.022 volts.

**89. Comparison of E.M.F.'s by a Galvanometer in Shunt.** — Let there be two or more cells the E.M.F.'s of which are to be compared. Connect one of them in series with a resistance of from 10,000 to 15,000 ohms and another small resistance $R$ (Fig. 87). It is not necessary to know the value of either of these resistances, but one of them should be large enough to prevent appreciable polarization of the cells during the time required to take a reading with the circuit closed. A d'Arsonval galvanometer, or some other aperiodic form, is connected in a circuit joined as a shunt to the small resistance $R$.

Close the key $K$ and observe the deflection $d_1$. This should not exceed about 200 scale parts, with the scale one and a half metres from the mirror. It is best to

take a series of observations for $d_1$ and to make use of the mean. Next replace $B$ with another cell and repeat observations for $d_2$. Then

$$E_1 : E_2 :: d_1 : d_2.$$

This method neglects any difference in the internal resistance of the cells. If this resistance is small no appreciable error will result. But if the battery itself, or one of the cells compared, should have a high internal

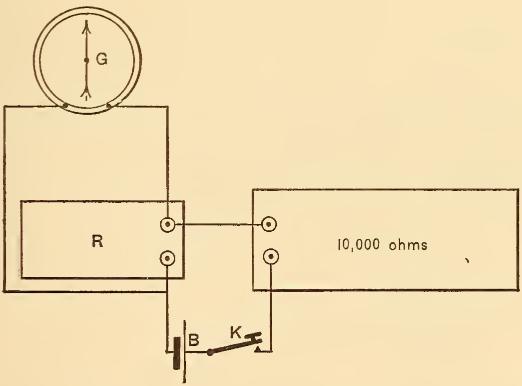



Fig. 87.

resistance the method cannot be used. A comparison of a Daniell cell, for example, with a standard Clark, having an internal resistance of 2000 ohms or more, would give a result which would make the E.M.F.'s of the two cells apparently more nearly equal than they really are. But so long as the internal resistance of the cells compared is negligible in comparison with the other resistance in circuit, then no change in the circuit is made in substituting one cell for another except a change in the E.M.F.; and if the currents are proportional to

deflections, the E.M.F.'s, being proportional to the currents, are also proportional to the corresponding deflections.

**Example.**

$R = 20$ ohms; $R' = 15{,}000$ ohms.

| Cell. | Deflection. | E.M.F. |
|---|---|---|
| Daniell, | 64 | 1.1 volts. |
| "Diamond" Carbon, | 67 | 1.15 " |
| Gassner Dry Cell, | 75 | 1.29 " |
| Ajax Dry Cell, | 63 | 1.08 " |

The Daniell cell was freshly set up, but the others were old cells.

**90. The Condenser Method of comparing E.M.F.'s.** — Let $G$ be a sensitive galvanometer with a small damping coefficient. Connect with the condenser $C$ and the battery $B$ by means of a charge and discharge key $K$ (Fig. 88). The condenser will need to have a capacity of from 0.05 to 0.3 of a microfarad. Observe the first swing several times when the condenser is discharged through the galvanometer and take the mean for $d_1$. The complete period of swing of the galvanometer, for convenience in reading, should be from 5 to 10 seconds. Next repeat the observations with a second battery and let the mean of the deflections be $d_2$. Then if $E_1$ and $E_2$ are the E.M.F.'s of the two cells,

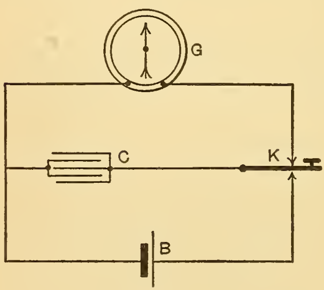


Fig. 88.

$$E_1 : E_2 :: d_1 : d_2.$$

To save time in waiting for the galvanometer needle to come to rest after each observation, a small coil may be placed near the needle, and a single cell may be connected in circuit with it. By tapping the key in this control circuit at the proper moment the needle may be quickly brought to rest.

If the ballistic form of the d'Arsonval galvanometer be used, the motion of the coil may be arrested by short-circuiting the galvanometer by means of an extra key for the purpose.

In this method the first swing of the needle from rest is nearly proportional to the quantity of electricity discharged through the galvanometer; and, since the capacity of the condenser remains unchanged, the quantities are proportional to the E.M.F.'s charging the condenser. If instead of a change in electromotive force another condenser of different capacity be used, the deflections $d_1$ and $d_2$ will be proportional to the capacities of the two condensers.

**Example I.**

| Cell. | Deflection. |
|---|---|
| Clark, | 120 mm. |
| "Diamond" carbon, | 114.5 mm. |

Therefore 120 : 114.5 : : 1.434 : $x$ (= 1.368 volts).

**Example II.**

| Cell. | Deflection. | E.M.F. |
|---|---|---|
| Clark, | 265 | 1.428 (at 20° C.) |
| Daniell, | 205 | 1.105 |

**91. Lord Rayleigh's Potentiometer Method.** — The preceding methods are deflection methods and do not admit of great accuracy. If the deflection is 200 scale parts, and if it can be read to only a single division, then no greater accuracy than one-half per cent can be

secured. Zero methods are much to be preferred, and the following one leaves nothing to be desired, where the E.M.F.'s to be compared are only a few volts. Let $R$ and $R'$ (Fig. 89) be two well-adjusted resistance boxes of 10,000 ohms each. Connect them in series with a cell having a higher E.M.F. than either of the E.M.F.'s to be compared. A total resistance of 10,000 ohms must be kept in circuit. A shunt circuit is taken from the terminals of one box $R$, and in this is placed a sensitive galvanometer, a key, one of the cells to be

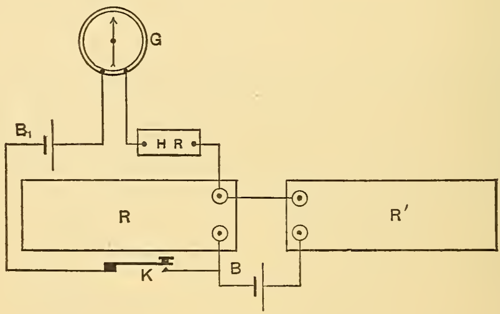



Fig. 89.

compared, and usually a high resistance to protect the cell from polarization, if a standard, as well as to avoid too large a deflection of the galvanometer. The cell $B_1$ should be so connected that its E.M.F. may be balanced against the P.D. between the terminals of $R$. Obtain a balance, so that the galvanometer shows no deflection on closing the key $K$, by transferring resistance from one box to the other, *being careful to keep the sum of the two* 10,000 *ohms*. When a balance has been secured to the nearest ohm, the E.M.F. of the cell $B_1$ equals the fall of potential over the resistance in $R$.

Repeat the operation with a second cell or other source of E.M.F. Then if $R_1$ and $R_2$ are the resistances in $R$ in the two cases to balance, we have

$$E_1 : E_2 :: R_1 : R_2.$$

The resistance in the circuit is kept so large that no appreciable polarization takes place while the comparisons are being made. Then the P.D. between the terminals of $R$ is strictly proportional to that portion of the 10,000 ohms contained in the box $R$. If the galvanometer is sensitive to a change of a single ohm from $R$ to $R'$, or the reverse, then the E.M.F. of the battery in the main circuit should be only slightly higher than that of the highest E.M.F. to be compared. Larger numbers will then be obtained to represent the E.M.F.'s, and hence greater accuracy in the result.

If one of the cells compared is a standard with known E.M.F., the method gives the E.M.F. of each of the cells compared. Two cells to be compared may be connected in opposition to each other. In this way the difference of E.M.F. between them may be compared with the E.M.F. of either.

**Examples.**

| Cell. | Temp. C. | Res. to balance. |
|---|---|---|
| No. 30 Clark, | 15° | 9475 |
| No. 3 Calomel, | 15° | 6607 |

Hence $$9475 : 6607 :: 1.434 : x,$$

or $$x = 0.9999 \text{ volt}.$$

| Cell. | Temp. C. | Res. to balance. |
|---|---|---|
| No. 30 Clark, | 17.7° | 9151 |
| No. 7 Calomel, | 19° | 6395 |
| No. 9 " | " | 6396 |
| No. 10 " | " | 6396 |
| No. 11 " | " | 6395 |

$$E \text{ (Clark)} = 1.434\,[1 - 0.00077\,(17.7 - 15)] = 1.431.$$

Hence $9151 : 6396 :: 1.431 : x,$

or $x = 1.0002$ volts at 19° C.

for Nos. 9 and 10.

And $9151 : 6395 :: 1.431 : x,$

or $x = 1.0000$ volt

for Nos. 7 and 11 at 19° C.

**92. Comparison of E.M.F.'s by Rapid Charge and Discharge.** — Two platinum wires dip into mercury cups $a$ and $b$ (Fig. 90). The wires are attached to the prongs of a large tuning-fork, and are insulated from them. When the prongs separate, one of the wires dips into the cup $b$ and completes the connections so as to charge the condenser $C$. As soon as the prongs approach each other, connection is broken at $b$ and the other wire enters the cup $a$, thus discharging the condenser through the galvanometer. If this operation is repeated a sufficient number of times a second, a steady deflection of the galvanometer will result. Let the deflection with a standard cell be $d_1$, and let $E_1$ equal 1.44 volts. Replace the standard with the cell to be compared, and obtain the deflection again and let it be $d_2$.

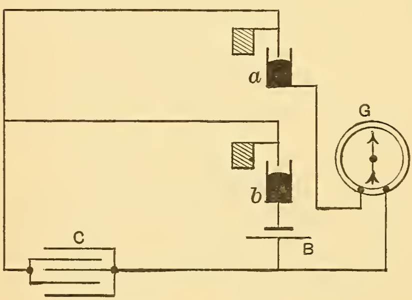


Fig. 90.

Then if $x$ be the E.M.F. of the cell,

$$d_1 : d_2 :: 1.44 : x,$$

or

$$x = 1.44 \frac{d_2}{d_1}.$$

Great care must be taken to so adjust the contacts that one platinum wire will leave the mercury surface in *b* before the other touches the mercury surface in *a*, otherwise the E.M.F. of the cell would be applied directly to the galvanometer. The accuracy of the method is dependent upon keeping constant the number of charges and discharges per second, since with a fixed capacity and E. M. F. the quantity discharged through the galvanometer in one second is proportional to the number of times the condenser is discharged.

**Example.**

| Cell. | Steady Deflection. | E.M.F. |
|---|---|---|
| Carhart-Clark, | 350 | 1.44 volts. |
| Ajax Dry, | 310 | 1.27 " |
| Bichromate, | 430 | 1.77 " |
| "Diamond" Carbon, | 295 | 1.21 " |
| Leclanché, | 380 | 1.56 " |

**93. Measurement of E.M.F. of a Standard Cell by a Kelvin Balance.** — The apparatus at the bottom of Fig. 91 is set up as in Lord Rayleigh's method of comparing E.M.F.'s. Find first with key $K$ open the number of ohms in the box $B$ required to balance the E.M.F. of the standard cell $S$ in the shunt circuit. Then close key $K$ and balance again while the current is flowing through the centi-ampere balance $TB$ and the standard coil $C$ immersed in oil. The connections are made in the figure on the assumption that the fall of potential between the terminals of the coil $C$ is less than the E.M.F. of the standard cell. Then when a balance is secured, the E.M.F. of the standard cell is balanced against the P.D. between the terminals of the coil $C$ plus the P.D. between the terminals of $B$. At the same

time that this last balance is made, the current is measured by means of the centi-ampere balance.

A high resistance should be put in circuit with the galvanometer and standard cell, but it can always be cut out when the balance is nearly complete.

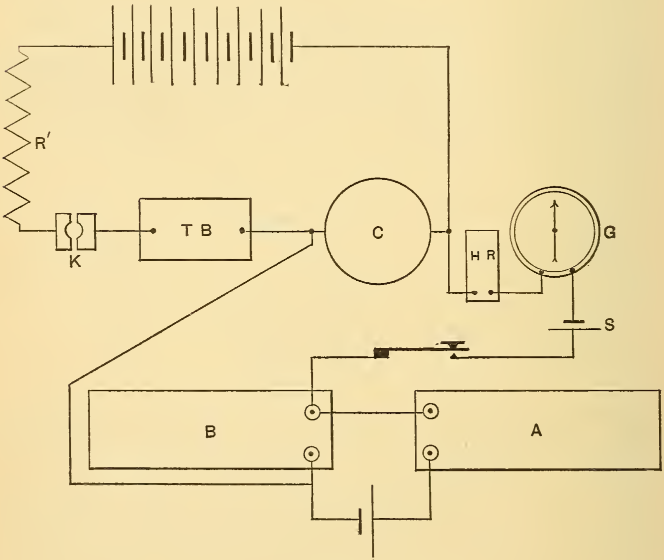



Fig. 91.

Then if $R$ is the resistance of coil $C$, $R_1$ and $R_2$ the resistances in $B$ to balance with key $K$ open and closed respectively, and $I$ the current measured by the centi-ampere balance, we have

$$E = IR \frac{R_1}{R_1 - R_2}.$$

$IR$ is the P.D. in volts between the terminals of the coil $C$. This is represented by the loss of potential over

the resistance $(R_1 - R_2)$. But the E.M.F. of the standard equals the fall of potential over the resistance $R_1$ in the auxiliary circuit of the Rayleigh method. Hence the P.D., $IR$, must be multiplied by the fraction $\frac{R_1}{R_1 - R_2}$ to obtain the E.M.F. of the standard.

The operations may be performed in a slightly different way. First, balance in the auxiliary circuit with the standard cell alone, as in the other case. Next, cut out the standard cell entirely, close key $K$ and balance again. The current through the Thomson balance must then be reversed as compared with the figure. Let $R_1$ and $R_2$ be the resistances in the auxiliary circuit to balance in the two cases. Then

$$E = IR\frac{R_1}{R_2}.$$

The accuracy of this method can be no greater than that of the centi-ampere balance, even with resistances $A$ and $B$ accurately adjusted.

The reverse reasoning gives a test of the accuracy of the balance. Given the E.M.F. of the standard cell, the equation determines the current.

**Example.**

*Standard Cell, No. 25.*

| | |
|---|---|
| Resistance in $B$ to balance with $K$ open . . . . . . | 9416 |
| Resistance in $B$ to balance with $K$ closed . . . . . . | 1802 |
| Temp. of standard cell . . . . . . . . . . . . . . | 17.2° C. |
| Temp. of coil $C$ . . . . . . . . . . . . . . . . | 17.2° C. |

Coil $C$ equalled 10 ohms at 9° C.
Temperature coefficient, 0.0002.
Hence at 17.2° the resistance of the coil was 10.0164 ohms.
Current through centi-ampere balance, 0.1162 ampere.

Hence the electromotive force of the cell was

$$0.1162 \times 10.0164 \times \frac{9416}{9416 - 1802} = 1.4393.$$

This is at 17.2° C. At 15° C.,

$$E = 1.4393\ [1 + .00039\ (17.2 - 15)] = 1.4405 \text{ volts.}$$

**94. Measurement of the E.M.F. of a Standard Cell by Means of the Silver Voltameter.** — This method of measuring E.M.F. consists in comparing the P.D. between the terminals of a known resistance with the E.M.F. to be measured. To get the P.D. we must know not only the resistance between the two points, but the current flowing. The current is measured by means of the silver voltameter, while the intermediate means of comparing the P.D. with the E.M.F. of the cell is the Rayleigh method of comparing E.M.F.'s, as in the last method.

First, there must be provided as constant an E.M.F. as possible, so that the current to be measured by the voltameters may be nearly constant. Let $B_1$ (Fig. 92) be a storage battery of a number of cells connected in series with a resistance $R'$ and the standard or accurately known resistance $R$. It is desirable to include in this circuit also a carbon resistance, or some other one capable of changing continuously, or at least by very small steps. $V_1$ and $V_2$ are the silver voltameters. By means of the commutator either a resistance $r$ or the two voltameters can be thrown into circuit. By this means the current can be adjusted to the desired value before the voltameters are put into circuit. The resistance $r$ should be made, as nearly as convenient, equal to that of the two voltameters. The advantage in using a number of storage cells and a considerable resistance $R'$ is

that any small change in the resistance of the voltameters, or any small difference between their resistance and $r$, will be nearly or quite inappreciable. The other part of the apparatus consists of the two 10,000-ohm boxes, $A$ and $B$, with one or two cells of Leclanché battery, a sensitive galvanometer $G$, a standard cell $S$, the E.M.F. of which is to be measured, and a commutator as shown, made by boring holes in a block of paraffin. By connecting $ac$ and $bd$, the E.M.F. of the standard cell is first balanced against the difference of potential between the terminals of the box $A$. At the same time the temperature of the cell is noted. Then by connecting $a$ and $b$ to $e$ and $f$, a balance can be made between the fall of potential over the resistance $R$ and over that in $A$. When the preliminary balance has been secured and the temperature of $R$ taken, the connections may be made with the voltameters and the current sent through them. The balance for the P.D. of $R$ is again obtained. If the change of a

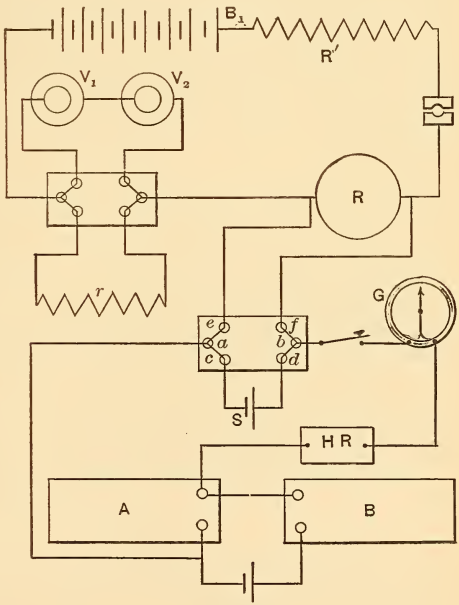



Fig. 92

single ohm in *A* reverses the deflection of the galvanometer, the exact balance may be effected by means of the carbon resistance mentioned above. The current should be allowed to flow for half an hour, and it may either be kept constant by means of the adjustable resistance, or it may be observed at equal time-intervals by means of the resistance in *A* required to balance. The balance for *S* should be tested occasionally, and the temperature of the cell should be kept constant if possible.

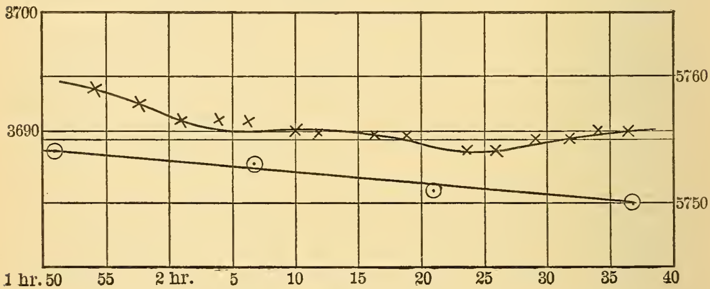



Fig. 93.

The resistance *R* should be made of manganin wire immersed in paraffin oil, and the case should be provided with a stirrer to equalize the temperature. Any small change in this resistance is practically negligible, but allowance may be made for it, since the temperature coefficient of the manganin wire is supposed to be known.

Fig. 93 shows the method of plotting the observations for a normal Clark cell and for the current. The mean value for the Clark is 5751.5 and for the current 3691.2. These values represent the mean ordinates for the two curves.

Let $R_1$ be the resistance in box $A$ required to balance the Clark cell, and $R_2$ the resistance required to balance $RI$ of the known resistance $R$.

Let $M$ be the mass of silver deposited, $t$ the time of deposit, and $z$ the electrochemical equivalent of silver in grammes per coulomb. Then

$$z = 0.001118.$$

$$M = Itz.$$

Therefore
$$\frac{E}{RI} = \frac{R_1}{R_2},$$

and
$$E = R\,\frac{R_1}{R_2} \cdot \frac{M}{zt}.$$

The value of $E$ thus found requires correction to reduce to temperature 15° C.

**Examples.**[1]

In the experiment to which the two curves of Fig. 93 relate $R_1 = 5751.5$; $R_2 = 3691.2$; $R = 0.9877$ at 17° C.

Temp. of Clark, 16.45° C.

$M = 2.8095$ gms.

$t = 2700$ seconds.

Hence
$$E = 0.9877\,\frac{5751.5}{3691.2} \cdot \frac{2.8095}{2700 \times 0.001118} = 1.4324.$$

Correction to 15° C. with coefficient 0.00077 = 0.0016.

Hence $E = 1.4324 + 0.0016 = 1.4340$ volts at 15° C.

---

Again, $R_1 = 5722.5$; $R_2 = 3904.5$.

$R = 0.9877$ at 17° C.

Temp. of Clark, 16.5° C.

$M = 2.6071$ and $t = 2357$ seconds.

[1] Glazebrook and Skinner, *Phil. Trans.*, Vol. 183 (1892) A, pp. 567–628.

Then $E = 0.9877 \dfrac{5722.5}{3904.5} \cdot \dfrac{2.6071}{2357 \times 0.001118} = 1.4322.$

Correction to 15° C. $= 0.0017$.

Whence $E = 1.4322 + 0.0017 = 1.4339$ volts.

**95. Electrostatic Voltmeters.** — The forces operating in an electrostatic voltmeter are due to the attraction and repulsion between static charges. Like the

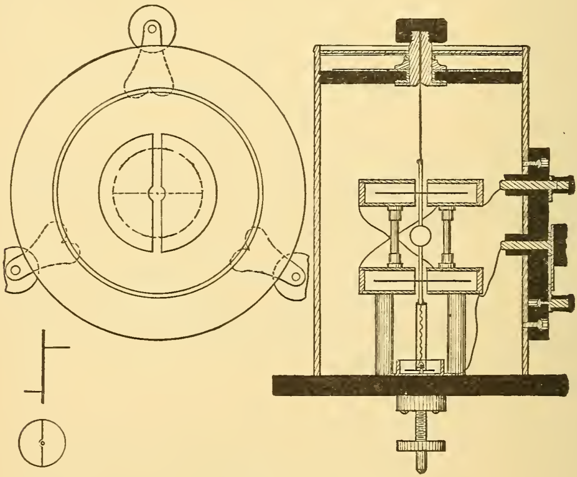


Fig. 94.

electrodynamometer, it is applicable to both direct and alternating currents. It has no self-induction and takes no appreciable current, even on an alternating current circuit, because of its small capacity (Art. 111).

The instruments illustrated in Figs. 94 and 96 may very properly be called electrostatic electrodynamometers. Each contains a mirror from which a beam of light from a lamp is reflected to a fixed scale; and in using them the spot of light is brought back to the zero

or initial position by turning the torsion head before the reading is taken. The beam of light, about a metre long, takes the place of the pointer of a Siemens dynamometer.

Referring to Fig. 94, which consists of a horizontal and a vertical section, it will be seen that the fixed portions of the electrostatic part of the instrument consist of four half-circular flat boxes, three inches in diameter and half an inch in depth inside. The lower pair is supported on ebonite pillars, and the upper one is carried on the lower by means of lead-glass rods set into appropriate sockets.

The needle consists of two half-circles of very thin aluminium mounted on a wire of the same metal, as shown in the lower left-hand corner of the figure. It is evident that when the half-circular boxes are cross-connected and one pair of these inductors is electrically connected with the needle, the forces acting on the movable system all tend to turn it in one direction.

The needle is suspended by a phosphor-bronze wire, about 0.038 mm. in diameter, from a brass torsion-head with a hard-rubber top. The suspending wire is perfectly free except at the point of support at the top of the brass head. The axis of the needle is connected below by means of a platinum-silver spiral to the cup containing paraffin oil as a damper. The damper itself is a horizontal disk supported by two wires from the axis of the needle, and having at its centre a hole through which passes the pin holding the lower end of the spiral. The needle is charged through this spiral; and, since the instrument is a zero one, the spiral does not affect its sensitiveness if the beam of light composing the pointer can be brought accurately back to zero

before the reading is taken; for the instrument is set up so that the spiral is entirely without torsion when the beam of light is at the zero of the scale. The torsion scale rests on the hard-rubber top and is divided into 400 equal divisions. The pointer is set to the zero of this scale after all other adjustments have been made. A key, shown in the charging position, is made to discharge the semi-circular inductors by turning it through 180°.

When the instrument is charged, the system swings, twisting both the supporting wire and the steadying spiral at the bottom. This spiral has more torsion than the wire. The torsion head is turned till the spot of light returns to zero, and the twist of the suspending wire is then read by the pointer on the circular scale. The spiral is without torsion when the torsion head stands at zero, but it serves to overcome the surface viscosity of the damping fluid, and to give a constant zero reading. The instrument is practically dead-beat and its performance is very satisfactory. The one represented in Fig. 94 was intended to measure up to 1,100 volts. Fig. 95 is its calibration curve. Since the instrument is used idiostatically, this curve, like that of the electrodynamometer, should be a parabola. It departs from a parabola only very slightly. The constant increases a little on the upper readings. The points on the upper part of the curve were obtained by means of a platinoid resistance of 4,000 ohms, wound non-inductively on three frames supported in a horizontal position, so that all portions of the wire remain at the same temperature. This wire is divided into four sections, and the resistance of each section is accurately known. The smallest is about $\frac{1}{11}$ of the entire amount. The whole

was connected across the mains leading to an alternating dynamo, while conductors led from the terminals of the smallest section to a Kelvin multicellular voltmeter. The performance of this particular multicellular instrument is not satisfactory, partly because of an uncertain zero. Hence the vagaries of the points on the upper part of the curve. The points nearer the origin were taken by comparison with a Weston voltmeter and with

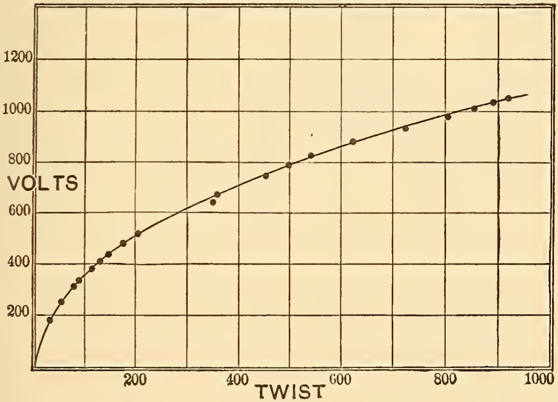



Fig. 95.

additional known resistance in circuit with it. A later calibration by means of the smaller instrument (Fig. 96) gave a better result.

Fig. 96 represents a similar instrument of smaller dimensions designed to measure from about 20 to 100 volts. Its principle is identical with that of the other, and its construction is similar. The suspending fibre is in this case quartz. Instead of semi-circular boxes for the inductors, parallel semi-circular plates are secured at fixed distances, and the entire system of inductors is

hung from the hard-rubber cross-bar which is adjustable on the supporting brass pillars carrying the top plate, scale, and torsion head. Fig. 97 is its calibration curve. The suspending fibre has since been replaced by a slightly thicker one, so that one revolution of the torsion head corresponds almost exactly to 100 volts. Vertical cylindrical quadrants and a vertical cylindrical needle were first tried,[1] but these did not prove

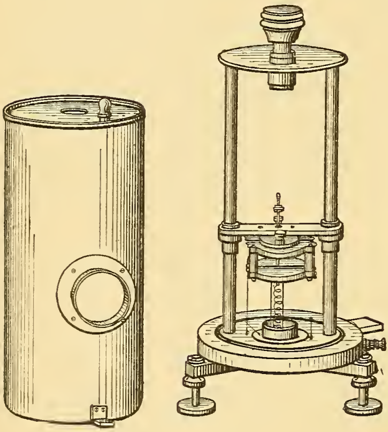


Fig. 96.

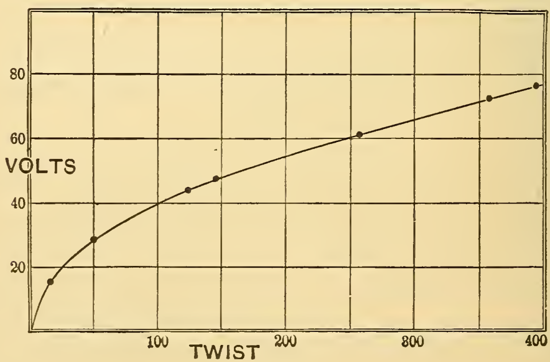



Fig. 97.

so satisfactory as the horizontal form of inductor plates and needle.

[1] *Proceedings of the International Electrical Congress,* 1893, p. 208.

**96. Calibration of a Voltmeter by Means of Standard Cells.** — The method consists in balancing the electromotive force of one or more standard cells against a fraction of the potential differences applied to the binding-posts of the voltmeter, and determining this fraction by means of well-adjusted resistance boxes. Let $R$ and $R'$ (Fig. 98) be two good resistance boxes, the first preferably as large as 100,000 ohms. The range of the second one will depend upon the range of the calibration and the number of standard cells used. $B$ is a storage battery of a sufficient number of cells to give the requisite potential difference. Vary the resistances $R$ and $R'$ till on closing $K_1$ and $K_2$ in order, the galvanometer shows a minimum deflection. Until the balance is nearly completed it is better to insert in the shunt circuit containing the galvanometer and standard cells $S$ a high resistance. If no current passes through the galvanometer the electromotive force of the standard cells is equal to the potential difference between the binding-posts of $R'$. Read now the voltmeter $V$.

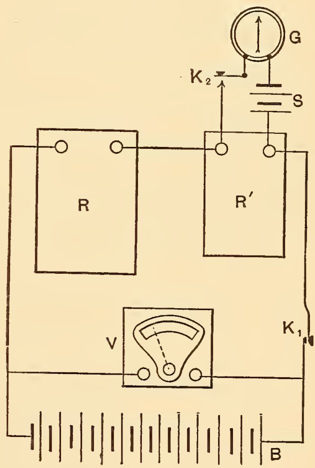


Fig. 98.

Then

$$V = 2\,E\,\frac{R + R'}{R'} \text{ (for two standard cells),}$$

where $V$ is the number of volts and $E$ the electromotive

force of the standard corrected for temperature. If the voltmeter is direct reading, the difference between $V$ and the reading will be the error at that part of the scale.

The voltage may then be changed and another balance taken, continuing the process till the entire scale has been traversed.

# CHAPTER V.

## QUANTITY AND CAPACITY.

**97. The Ballistic Galvanometer.** — The quantity of electricity discharged through a galvanometer during a transient flow may be measured by means of the first swing of the needle, provided its period of vibration is sufficiently long to permit the passage of the discharge before the needle moves through an appreciable angle. Such a galvanometer is called a ballistic galvanometer.

The general expression for a continuous current with any galvanometer is

$$I = \frac{\mathcal{H}}{G} f(\theta),$$

where $\mathcal{H}$ equals the magnetic field, $G$ is the galvanometer constant, and $\theta$ is the angular deflection.

When the deflection is small, with any galvanometer

$$I = \frac{\mathcal{H}}{G} \theta.$$

The present problem resolves itself into finding what function of the deflection must be multiplied by the constant $\frac{\mathcal{H}}{G}$ to give the quantity discharged through the galvanometer.

The maximum moment of the deflecting couple, due to a current $I$, is

$$2mlIG = \mathcal{M}IG \text{ (Art. 62)},$$

where $l$ is the half length of the needle and $\mathfrak{M}$ its magnetic moment, $2ml$. The moment of a couple producing an angular acceleration $\frac{d\omega}{dt}$ is $K\frac{d\omega}{dt}$, in which $K$ is the moment of inertia of the movable system.

Therefore

$$\mathfrak{M}IG = K\frac{d\omega}{dt}.$$

The instantaneous value of $I$ is $\frac{dQ}{dt}$, for $I = \frac{Q}{t}$ when the current is constant.

Therefore $$\mathfrak{M}G\frac{dQ}{dt} = K\frac{d\omega}{dt}.$$

If $\omega$ is zero at the instant when the circuit is closed, then integrating,

$$\mathfrak{M}GQ = K\omega. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

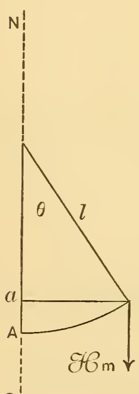



Fig. 99.

We must now obtain the expression for the energy of motion of the system at the instant when $\theta$ becomes zero and place it equal to the work done in producing a deflection. The kinetic energy of a rotating system in terms of moment of inertia and angular velocity is

$$\tfrac{1}{2}K\omega^2.$$

Now, if the total work done on the needle is represented by the kinetic energy of the system as it passes through the position of zero deflection, that is, if there is no damping of any kind, then this energy may be equated to the work done on the needle against the force of control. If the impulse on the needle moves it from the position of

equilibrium through an angle $\theta$, the work done on it in moving its poles a distance $Aa$ (Fig. 99) against the controlling force $\mathcal{H}m$ on each pole is $2\mathcal{H}m.\,Aa$.

But $Aa = l\,(1 - \cos\theta)$. Hence the work done on both poles in producing a deflection $\theta$ is

$$2\mathcal{H}ml\,(1 - \cos\theta) = \mathcal{H}\mathcal{M}\,(1 - \cos\theta).$$

Therefore $\quad \frac{1}{2}K\omega^2 = \mathcal{H}\mathcal{M}\,(1 - \cos\theta).$

But from equation (1)

$$\omega^2 = \left(\frac{\mathcal{M}GQ}{K}\right)^2.$$

Hence

$$\frac{(\mathcal{M}GQ)^2}{K} = 2\mathcal{H}\mathcal{M}\,(1 - \cos\theta) = 4\mathcal{H}\mathcal{M}\sin^2\frac{\theta}{2}.$$

Solving,

$$Q = \frac{2}{G}\sqrt{\frac{\mathcal{H}K}{\mathcal{M}}}\,\sin\frac{\theta}{2} = \frac{\mathcal{H}}{G}\sqrt{\frac{K}{\mathcal{H}\mathcal{M}}}\,.\,2\sin\frac{\theta}{2}. \qquad (2)$$

The time of a single vibration of the magnet is given by the equation

$$T = \pi\sqrt{\frac{K}{\mathcal{H}\mathcal{M}}},$$

from which $\quad \dfrac{T}{\pi} = \sqrt{\dfrac{K}{\mathcal{H}\mathcal{M}}}.$

Substituting in (2),

$$Q = \frac{\mathcal{H}}{G}\cdot\frac{T}{\pi}\cdot 2\sin\frac{\theta}{2}. \quad . \quad . \quad . \qquad (3)$$

This is the full equation for quantity without any damping coefficient.

If $\theta$ is small, $\sin\frac{1}{2}\theta$ may be taken equal to $\frac{1}{2}\theta$, and

$$Q = \frac{\mathcal{H}}{G}\cdot\frac{T}{\pi}\theta,$$

or the quantity is proportional to the first angular throw.

If $T$ is the observed time of a single oscillation for an amplitude $a$, then the time for an infinitely small arc is given by the equation

$$T = T\left(1 - \frac{1}{4}\sin^2\frac{a}{4} - \frac{5}{64}\sin^4\frac{a}{4} - \right).$$

Table III. in the Appendix contains the corrections.

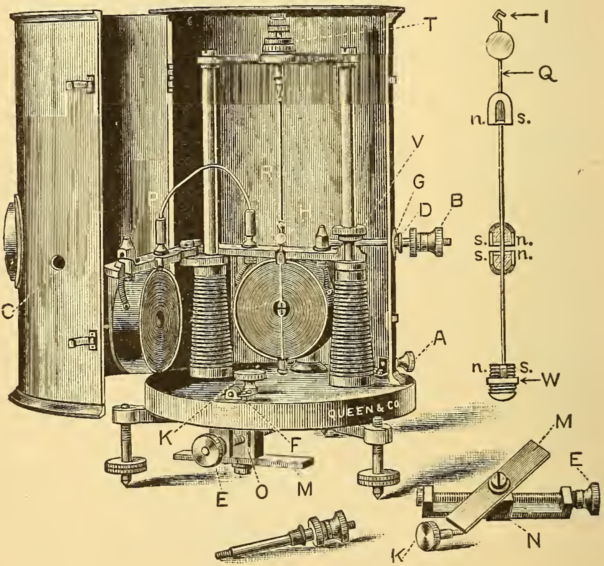



Fig. 100.

If $d$ is the deflection in scale parts and $a$ the distance between the mirror and scale, then

$$T_o = T\left(1 - \frac{1}{256}\frac{d^2}{a^2}\right).$$

For accuracy the value of $T_o$ should be used in the above formulas for $Q$.

A ballistic galvanometer complete is shown in Fig. 100.

The astatic system, consisting of four bell magnets, is at the right of the cut. $W$ is the soft iron ring nut which is employed as a magnetic shunt to adjust the sensibility of the whole astatic system. When it is turned up toward the poles $ns$ the magnetic moment of this lowest magnet is diminished

**98. Correction for Damping.** — A correction to the deflection $\theta$ may be necessary on account of the damping action on the needle due to setting the air in motion, and to the induced currents produced in the coil by the movement of the needle.

If the deflection is small, so that we may write the angle for the sine of the angle, and if the damping also is small, we may write

$$Q = \frac{\mathcal{H}}{G} \cdot \frac{T}{\pi} \left[\theta + \tfrac{1}{4}(\theta - \theta')\right],$$

where $\theta$ is the first deflection, and $\theta'$ is the following one in the same direction.[1] Here the decrement of the first half-swing is taken equal to one-fourth the total decrement of the succeeding four half-swings; or the decrement of the first quarter of a period is taken equal to one-fourth the decrement of the complete period following.

The logarithmic decrement of the motion is the Naperian logarithm of the ratio of any one amplitude to that which succeeds it after an interval of half a period. Let it be denoted by $\lambda$.

To apply it to the correction for damping, let $n_1$, $n_2$,

[1] *L'Électricité*, Mascart and Joubert, Tome 1, p. 558.

$n_3$, etc., be successive scale readings. Then the ratio of one amplitude to the next following is

$$\rho = \frac{n_1 - n_2}{n_3 - n_2},$$

and $\log_e \rho = \lambda$.

The constancy of the ratio $\rho$, or of the logarithmic decrement $\lambda$, means that the successive amplitudes decrease in a geometrical series. Let the differences between the successive scale readings, that is, the successive amplitudes, be denoted by $a_1$, $a_2$, $a_3$, etc.

Then $\quad a_2 = \frac{a_1}{\rho}; \; a_3 = \frac{a_2}{\rho} = \frac{a_1}{\rho^2}; \; a_n = \frac{a_1}{\rho^{n-1}}.$

Whence $\quad \log_e a_n = \log_e a_1 - (n-1) \log_e \rho,$

and $\quad \log_e a_m = \log_e a_1 - (m-1) \log_e \rho,$

where the first equation applies to the $n^{th}$ amplitude and the second to the $m^{th}$. Subtracting the first equation from the second,

$$\log_e a_m - \log_e a_n = (n-m) \log_e \rho = (n-m)\, \lambda.$$

Therefore

$$\lambda = \frac{1}{n-m} \log_e \left(\frac{a_m}{a_n}\right).$$

If $a_m$ is the first amplitude and $a_n$ is the $n^{th}$, then

$$\lambda = \frac{1}{n-1} \log_e \left(\frac{a_1}{a_n}\right).$$

If now $a$ represent the first amplitude not diminished by damping, $a_1$ being the observed amplitude, then $n-m$ for the two is $\frac{1}{2}$, and

$$\lambda = \frac{1}{\frac{1}{2}} (\log_e a - \log_e a_1),$$

or

$$\frac{1}{2}\lambda + \log_e a_1 = \log_e a.$$

But

$$a = e^{\log_e a} = e^{\frac{1}{2}\lambda + \log_e a_1} = e^{\frac{1}{2}\lambda} \times e^{\log_e a_1} = e^{\frac{1}{2}\lambda} \times a_1.$$

Now

$$e^x = 1 + \frac{x}{1} + \frac{x^2}{1 \cdot 2} + \quad . \quad . \quad . \quad . \quad . \quad .$$ [1]

Therefore

$$e^{\frac{1}{2}\lambda} = 1 + \frac{1}{2}\lambda + \quad . \quad . \quad . \quad .$$

and $\qquad a = a_1 \left(1 + \frac{1}{2}\lambda\right)$, omitting higher powers.

If then $\lambda$ be determined by observing the first and $n^{th}$ amplitudes and substituting in the equation

$$\lambda = \frac{1}{n-1} \log_e \left(\frac{a_1}{a_n}\right),$$

the equation for quantity becomes

$$Q = \frac{\mathscr{H}}{G} \cdot \frac{T}{\pi} \left(1 + \frac{1}{2}\lambda\right) \theta_1,$$

where $\theta_1$ is the first angular deflection,[2] and the damping is small.

**Example.**

Scale readings $+ 130, - 120, + 105, - 97, + 85$.

Hence $\qquad \lambda = \frac{1}{4} \log_{10} \frac{130}{85} \cdot \frac{1}{0.4343} = 0.1062,$

and $\qquad 1 + \frac{1}{2}\lambda = 1.0531.$

The damping correction amounts to 5.3 per cent.

**99. Standard Condensers.** — Standard condensers are made of tin foil interlarded with mica, and finally embedded in solid paraffin. The experimental deter-

---

[1] Williamson's *Differential Calculus*, p. 62.

[2] Maxwell's *Electricity and Magnetism*, Vol. II., p. 357.

mination of the capacity of such condensers is more or less affected by conductivity and by absorption. The capacity with solid dielectrics is a function of the duration of the charging. For a primary standard of capacity it is necessary to use a condenser with air as the dielectric, an instrument which Lord Kelvin calls an air-Leyden. The insulation resistance, which should be several thousand megohms, may be measured by one of the methods in Chapter III.; and if any portions of a subdivided condenser are found to have faulty insulation, they cannot be used. The paraffin used by the best foreign makers has been known to contain traces of acid which attacks the metal embedded in it, and causes the insulation to deteriorate. When the top is clean and dry a good condenser should not lose an appreciable part of its charge in an hour. The influence of absorption can be eliminated only by the application of the method of rapidly alternating charges and discharges.

A subdivided condenser is usually made in the form shown in Fig. 101, in which one side of all the sections is connected to the brass bar marked *Earth*, and the other sides to the blocks *A*, *B*, *C*, *D*, *E*, as indicated by the dotted lines. When any section is to be used it is connected by a brass plug to the bar marked *Condenser*. The other sections may at the same time be completely discharged by connecting to *Earth*. For example, the condenser has a capacity of 0.3 microfarads when *A*, *B*, and *C* are connected to *Condenser*, *D* and *E* being to *Earth*. It is evident that great care must be exercised in putting in the plugs, for the battery applied may be short-circuited if plugs are inserted at both ends of any block.

The accuracy of a standard condenser may be tested by comparing the different sections with one another when a second condenser is not available. Thus charge *A* by connecting to *Condenser*, all the other blocks being joined to *Earth*. Then remove all plugs and divide *A's* charge with *B* by connecting both blocks to *Condenser*. *A* and *B* should then have equal charges if their capacities are equal. This can be determined by discharging first one and then the other through a ballistic gal-

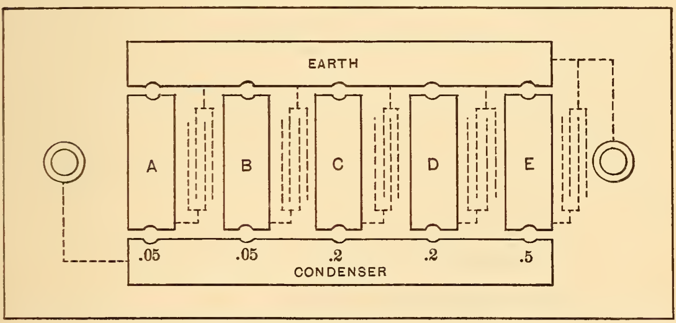



Fig. 101.

vanometer and observing the throw. Use sufficient E.M.F. to get a satisfactory deflection. Next compare *C* and *D* in the same manner. Then charge *A*, *B*, and *C* simultaneously, divide *C's* charge with *D*, and ascertain whether the charge of *A* and *B* together is equal to that of *C* and *D* separately. Finally, charge *A*, *B*, *C*, and *D* together, and divide their charge with *E*. The discharge of *E* should then give the same throw of the galvanometer as that of the other four together. For this method the tops of the plugs should be well insulated.

Any one of the sections may be made the basis of a comparison for the remainder. In every case the charges compared by the ballistic galvanometer will be very nearly equal. Hence, the deflections may be taken proportional to the charges without error; and since the charges are proportional to the capacities,

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{d_1}{d_2}.$$

Hence $$C_2 = C_1 \frac{d_2}{d_1} = C_1 a.$$

Let $a$ be the ratio between $A$ and $B$.

" $b$ " " " " $A + B$ and $\frac{1}{2} C$.

" $c$ " " " " $C$ and $D$.

" $d$ " " " " $A + B + C + D$ and $E$.

Then
$$A = 0.05.$$
$$B = 0.05 a.$$
$$C = 0.05 (1 + a) 2b.$$
$$D = 0.05 (1 + a) 2bc.$$
$$E = 0.05 (1 + a) (1 + 2b + 2bc) d.$$

**100. Comparison of Capacities by the Method of Divided Charge.** — The method of calibrating a standard condenser described in the last article may be applied to the comparison of any capacity $C_2$ with that of the standard $C_1$.

The standard is first charged by a potential difference which need not be known, but which must remain of fixed value. The charge $Q_1$ is then measured by discharging through a ballistic galvanometer. The standard is again charged to the same potential difference, and therefore with the same quantity $Q_1$, and is then

connected for a few seconds in parallel with the cable or condenser whose capacity is to be measured. The charge $Q_1$ divides in proportion to the capacities of the two connected condensers. The charge remaining in the standard is then measured by the ballistic galvanometer. Call it $Q$. Then the charge in the condenser of unknown capacity $C_2$ is $Q_1 - Q$, and

$$\frac{C_2}{C_1} = \frac{Q_1 - Q}{Q}.$$

Whence

$$C_2 = C_1 \frac{Q_1 - Q}{Q}.$$

For the highest accuracy $C_1$ should be equal to $C_2$. This may be demonstrated by the general principle of Art. 36. In this case we wish to find the partial derivative of $C_2$ with respect to $Q$.

$$F = -f \frac{dC_2}{dQ} = f\, C_1 \frac{Q_1}{Q^2} = f\, C_2 \frac{Q_1}{Q(Q_1 - Q)}.$$

The minus sign is used with the derivative, because $C_2$ decreases as $Q$ increases. The relative error is

$$\frac{F}{C_2} = f \frac{Q_1}{Q(Q_1 - Q)}.$$

For a minimum the denominator $Q(Q_1 - Q)$ must be a maximum, since $Q$ is the variable and $Q_1$ the constant. But $Q + (Q_1 - Q) = Q_1$, a constant; and when the sum of two factors is a constant their product is a maximum when they are equal to each other, or when $Q = Q_1 - Q$. Hence for a relatively minimum error,

$$Q = Q_1 - Q = \frac{Q_1}{2}.$$

This means that the charges must divide equally, or

$C_1$ must equal $C_2$. After a preliminary trial, therefore, adjust the subdivided standard condenser so that the capacity used shall be as nearly as possible equal to the capacity to be measured.

The chief objection to this method lies in the fact that the two charges compared bear the ratio of about two to one. Hence, the observed deflections of the ballistic galvanometer must be corrected to render them proportional to $\sin \frac{1}{2}\theta$ instead of $\tan 2\theta$ (Table I.).

**101. Comparison of Capacities by the Bridge Method.** — Let the two condensers be placed in two of the arms of a Wheatstone's bridge, and two resistances in the other two (Fig. 102), the galvanometer joining the branches on either side connecting a capacity to a resistance. Adjust the resistances $R_1$ and $R_2$ till the galvanometer shows no deflection on charging and discharging by means of the key $K$. When there is a balance the potentials of the points $A$ and $B$ remain equal to each other during charge and discharge. Hence the two condensers, being charged with the same difference of potentials, will contain quantities proportional to their capacities, or

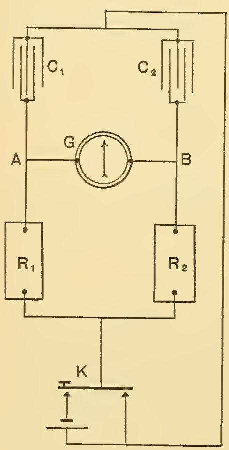


Fig. 102.

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2}.$$

But the quantities flowing into the condensers in the

same time are inversely proportional to the resistances $R_1$ and $R_2$. Hence

$$\frac{Q_1}{Q_2} = \frac{R_2}{R_1} = \frac{C_1}{C_2},$$

or

$$C_2 = C_1 \frac{R_1}{R_2}.$$

The resistances $R_1$ and $R_2$ must be non-inductive and without capacity. It is desirable for accuracy that the two capacities should be nearly equal to each other, and that the resistances should be moderately large.

The charge and discharge of long cables or of cables coiled in tanks is much retarded by absorption and electromagnetic induction. Hence when the time constants of the two condensers compared are very different the bridge method may give a result largely in error, particularly for rapid charge and discharge. To avoid this error the key $K$ should be worked slowly.

**Example.**

Comparison of a subdivided condenser with one marked $\frac{1}{3}$ microfarad, but found by an absolute determination to have a capacity of 0.3345 mf.

| Subdivisions. | $R_1$ | $R_2$ | $C_2$ |
|---|---|---|---|
| 0.05 | 1046 | 7000 | 0.0500 |
| 0.05 | 1042 | 7000 | 0.0498 |
| 0.2 | 4140 | 7000 | 0.1978 |
| 0.2 | 4151 | 7000 | 0.1983 |

**102. Comparison of Capacities by Gott's Method.** — This is also a bridge method, but differs from the last one in exchanging the places of the galvanometer and battery. The arrangement is shown in Fig. 103.

Two resistances $R_1$ and $R_2$ are selected inversely pro-

portional to the supposed values of $C_1$ and $C_2$. The key $K_1$ is then closed and clamped. After a few seconds key $K_2$ is closed, and if any deflection of the galvanometer occurs, the condensers are discharged by opening $K_1$ and closing $K_2$. After readjusting $R_1$ or $R_2$ the operation is repeated and continued till on closing $K_2$ with the battery still in circuit no deflection is produced.

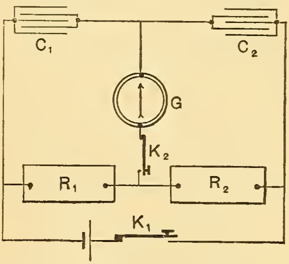


Fig. 103.

Since the two condensers are connected in cascade they must contain the same quantity and $C_1 V_1 = C_2 V_2$, where $V_1$ is the fall of potential over $R_1$, and $V_2$ that over $R_2$.

Hence

$$\frac{C_1}{C_2} = \frac{V_2}{V_1} = \frac{R_2}{R_1}.$$

The battery remains in circuit except during the discharge of the condensers. For highest accuracy the resistances should be quite large and the capacities equal.

The galvanometer key should be well insulated, as well as the conductors leading to the condensers. It is not necessary to insulate the battery.

**103. Correction for Absorption.** — The last method furnishes a means of measuring the absorption of one of the condensers compared. Assume $C_1$ as the one which absorbs a charge. Obtain a balance exactly as with the Gott method. The inverse ratio of the resistances will not be then the ratio of the true capacities. For, since

the same quantity $Q$ has entered each condenser, while a portion $q$ has been absorbed, the potential difference between the two sides of $C_1$ is due to a charge $Q-q$, while the potential difference of $C_2$ is due to the charge $Q$. Then

$$V_1 = \frac{Q-q}{C_1}, \text{ and } V_2 = \frac{Q}{C_2},$$

where $V_1$ and $V_2$ are the differences of potential between the terminals of $R_1$ and $R_2$ respectively when a balance has been obtained. From the two preceding equations

$$C_1 V_1 + q = C_2 V_2.$$

Therefore, $$\frac{C_1}{C_2} = \frac{V_2}{V_1} - \frac{q}{C_2 V_1} = \frac{R_2}{R_1} - \frac{q}{C_2 E_1}\left(1 + \frac{R_2}{R_1}\right),$$

where $E_1$ is the electromotive force of the battery.

To find $q$, with the key $K_1$ closed adjust $R_1$ and $R_2$ so that the galvanometer shows a small deflection due to the discharge of a fraction of the charge of $C_2$ on closing the key $K_2$. This is effected by diminishing $R_2$ slightly relative to $R_1$.

Then open $K_2$, break the circuit at $K_1$, and after a few seconds close $K_2$ and observe the deflection. The galvanometer needle should now swing in the opposite direction to that observed before opening the battery circuit. If necessary readjust the resistances till the two opposite deflections are equal to each other. The quantity discharged through the galvanometer in either direction is then equal to $q$.

To find now the value of $q$, charge a condenser of known capacity with a known E.M.F. and discharge through the ballistic galvanometer. Let the deflection.

corrected for damping, be $d_2$, and let the deflection due to $q$ be $d_1$. Then

$$q = \frac{d_1}{d_2} CE,$$

where $C$ is the known capacity and $E$ the known E.M.F.

**104. Comparison of Capacities by Thomson's Method of Mixtures.** — This method takes its name from the process of mixing the charges of opposite sign of the two condensers compared in order to determine whether those charges are equal. $C$ (Fig. 104) is a Pohl's commutator, which must be well insulated. When it is turned so as to connect the terminals of the battery with the inner coatings of the two condensers, $C_1$ and $C_2$, they are charged with the potential differences existing between the terminals of the two resistances $R_1$ and $R_2$ respectively. When the commutator is turned

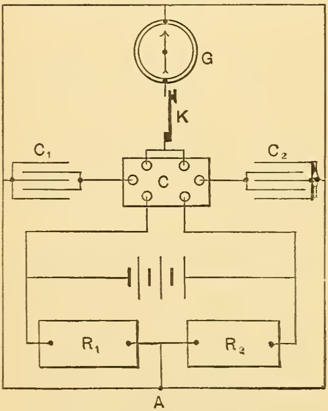


Fig. 104.

the other way, the two charges of opposite sign mix. To ascertain whether they are equal and completely neutralize each other, the key $K$ is then closed and any residue remaining in either condenser is discharged through the galvanometer $G$. The resistances $R_1$ and $R_2$ should be large and the capacities about equal. The electromotive force should be as large as the resistance

boxes will safely permit, especially for the final adjustment, since only the residue of the two charges remains to affect the galvanometer.

The point $A$ is sometimes grounded. This is essential when the capacity of a cable is to be measured. The core of the cable is then connected to the commutator and the earth is the outer coat. High insulation of the rest of the apparatus is essential.

**Example.**

To compare a special mica condenser $C_2$ with a Marshall condenser $C_1$ of 0.3345 microfarad capacity.

| $R_2$ | $R_1$ | $C_1$ | $C_2$ |
|---|---|---|---|
| 590 | 340 | 0.3345 | 0.1928 |
| 1400 | 807 | 0.3345 | 0.1928 |

**105. Discharge of a Condenser through a High Resistance.** — When a non-absorbing condenser leaks through a high resistance $R$, the fall of potential is expressed by the equation

$$V = V_o e^{-\frac{t}{RC}} \text{ (Art. 51)},$$

in which $V_o$ is the initial potential or charging electromotive force, and $V$ is the potential after the condenser has been leaking $t$ seconds through a resistance $R$. If potentials are plotted as ordinates and the times of leaking as abscissas the curve will be exponential in form.

Since the quantity held by a condenser of capacity $C$ is proportional to its potential, we may also write

$$Q = Q_o e^{-\frac{t}{RC}}.$$

We also have

$$R = \frac{t}{C} \cdot \frac{1}{\log_e \frac{Q_o}{Q}} = \frac{t}{C} \cdot \frac{1}{\log_{10} \frac{Q_o}{Q} \times 2.303}$$

as the resistance through which the condenser leaks, expressed in terms of common logarithms and the deflections of the ballistic galvanometer employed to measure the charges.

The actual curves obtained by experiment will differ from the theoretical exponential ones because of the complication introduced by absorption. So also the resistance computed from observations made at different time-intervals of leakage will not be constant, but will increase with the time.

The apparatus may be set up as in Fig. 105, in which *K* is a charge and discharge key. When the lever *b* is brought in contact with *a* the condenser is charged by the battery *B*. If the lever *b* is thrown over to *c* the whole charge is at once passed through the galvanometer *G*. This gives the deflection $d_o$. Then charge again and place the lever midway between *a* and *c* for five minutes or more, the time depending upon the insulation resistance of the condenser. If that is too high to permit of frequent observations, a resistance of about 25 or 30 megohms, if available, may connect the two sides of the condenser. At the end of the observed time of leaking, the lever *b* is again made to touch *c*, and the deflection corresponding to the charge remaining in the

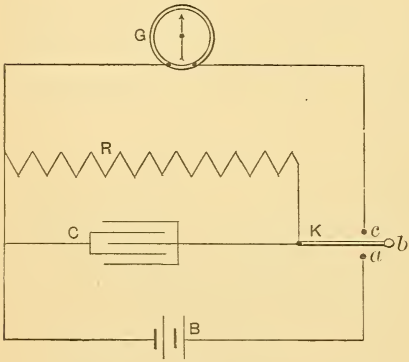


Fig. 105.

condenser is observed. Charge again and proceed in the same way, increasing each time the period of leaking till a sufficient number of observations have been secured.

It is obvious that all parts of the circuits, including the galvanometer and the battery, must be highly insulated. The deflections, or the corresponding quantities, may then be plotted as ordinates and the periods of leaking as abscissas.

**106. Residual Discharges.** — For the purpose of studying the residual charge it is advisable to experiment with a cable of sufficient capacity and with an insulation which constitutes a dielectric of large absorbing power when the cable is immersed in water. A cable of high insulation resistance should be selected.

Proceed as follows: Charge the cable with an electromotive force of 50 to 100 volts for several hours. It will often continue to absorb a charge for twenty-four hours. Discharge it through a low resistance by closing the key for a very short interval. This is best accomplished by using the pendulum apparatus (Art. 58) and setting three keys so that the first one opens the charging circuit, the second discharges through the low resistance, and the third insulates the cable. Let it stand insulated for five seconds and then discharge through a ballistic galvanometer. Next charge again to the full by applying the same electromotive force as at first for a period about twice as long as the cable has been left insulated. This is done by resetting the keys on the automatic pendulum device in the proper order.

Then again discharge through the low resistance, and increase the time of standing insulated to ten seconds.

passing the residual charge as before through the galvanometer. Recharge for about double the time the residual charge occupied in coming out, and repeat the observations with increasing intervals of insulation. Finally, plot the deflections (or quantities) and the corresponding periods of insulation.

**Example.**

A coil of insulated wire, which had been in a tank of water for 15 days, was charged by a storage battery of 73 volts electromotive force for several hours. The length under water was 997 feet, its capacity 0.075 microfarads, and its insulation resistance 400,000 megohms.

The keys on the charge and discharge apparatus were so set that the cable was discharged through a low external resistance for about $\frac{1}{5}$ second. The insulation periods ranged from one second to two minutes. The following are the data of the experiment:

| Intervals in seconds. | Mean Deflections. | Quantities in microcoulombs. | Rise in volts during the intervals. |
|---|---|---|---|
| 1 | 19.4 | 0.357 | 4.76 |
| 2 | 22.8 | 0.420 | 5.60 |
| 3 | 25.4 | 0.467 | 6.23 |
| 5 | 28.3 | 0.521 | 6.95 |
| 10 | 34.9 | 0.632 | 8.43 |
| 15 | 37.9 | 0.697 | 9.29 |
| 20 | 40.2 | 0.740 | 9.87 |
| 25 | 43.1 | 0.793 | 10.57 |
| 30 | 45.5 | 0.837 | 11.16 |
| 40 | 48.0 | 0.883 | 11 77 |
| 50 | 50.5 | 0.929 | 12.39 |
| 60 | 51.9 | 0.955 | 12.73 |
| 90 | 57.1 | 1.051 | 14.01 |
| 120 | 62.6 | 1.152 | 15.36 |

It was necessary to shunt the galvanometer with the $\frac{1}{10}$ shunt, because without it after the fifteen-second period the deflection became too large. Its constant was then 0.0184 microcoulomb per mm. deflection. The observations are plotted in Fig. 106.

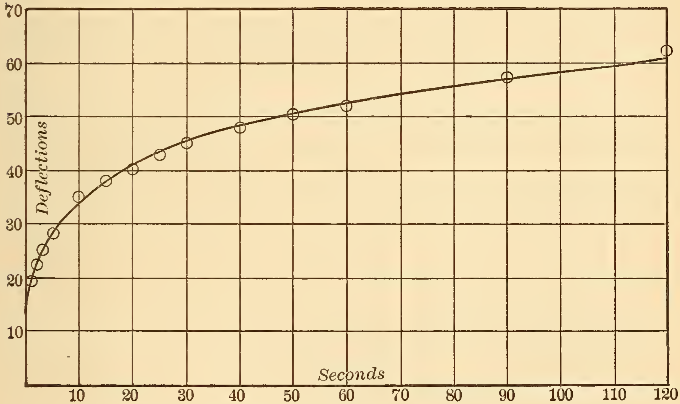



Fig. 106.

**107. To measure the Absolute Capacity of a Condenser — First Method.**[1] — When a quantity of electricity $Q$ is discharged through a ballistic galvanometer,

$$Q = \frac{\mathcal{H}}{G} \cdot \frac{T}{\pi}(1 + \frac{1}{2}\lambda)\,\theta.$$

$\theta$ is the first angular throw.

Let $A$ represent the constant $\frac{\mathcal{H}}{G}$, and for $\theta$ put $\frac{d}{2a}$, in which $d$ is the deflection and $a$ the distance of the scale from the mirror, both in millimetres. Then

$$Q = A\,\frac{T(1 + \frac{1}{2}\lambda)}{2\pi a}\,d. \quad . \quad . \quad . \quad (1)$$

[1] Stewart and Gee's *Practical Physics*, Part II., p. 407.

If a condenser of capacity $C$ be charged with an E.M.F., $E$, then

$$Q = EC. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

$$C = A\,\frac{T\,(1+\tfrac{1}{2}\lambda)\,d}{2\pi a E}. \quad . \quad . \quad . \quad . \quad (3)$$

If now we use the same battery to produce a steady deflection $d_1$ through a resistance $R$, including that of the battery and the galvanometer, then

$$\frac{E}{R} = A\,\frac{d_1}{2a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

for small deflections.

Therefore, $$\frac{A}{E} = \frac{2a}{Rd_1}.$$

Substitute in (3) and

$$C = \frac{T\,(1+\tfrac{1}{2}\lambda)}{\pi R}\cdot\frac{d}{d_1}. \quad . \quad . \quad . \quad . \quad (5)$$

In practice first determine $d$ by charging the condenser with an electromotive force $E$, as in Fig. 88, discharging through the ballistic galvanometer, and notice the deflection or first swing $d$.

Next, find the time of a single vibration, correcting for reduction to an infinitely small arc.

Third, determine $R$ and $d_1$. $R$ must be a high resistance, and probably the $\frac{1}{1000}$ shunt will need to be used with the galvanometer. Increase $R$ until the deflection is within the proper limits. Then if $R_1$ is the external resistance, $b$ that of the battery, and $\dfrac{gs}{g+s}$

that of the galvanometer and shunt in parallel, the total resistance in circuit will be

$$R = R_1 + b + \frac{gs}{g+s}.$$

But since the shunt is used, the equivalent resistance for the current measured is

$$R = \left(R_1 + b + \frac{gs}{g+s}\right)\frac{g+s}{s}.$$

Finally, substitute in equation (5). If $R$ is in ohms $C$ will be in farads.

**108. Absolute Capacity of a Condenser — Second Method.** — This method rests upon the production of a steady deflection of the galvanometer by a succession of rapid discharges through it from the condenser. If the rate of discharge is a large number of times the frequency of oscillation of the galvanometer needle, the effect of these discharges in producing a deflection is the same as that due to a current numerically equal to the quantity of the discharges a second.

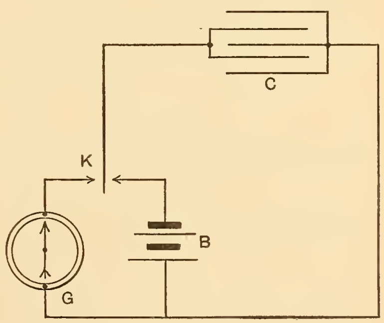


Fig. 107.

The apparatus may be set up as in Fig. 107. $K$ is an automatic device for charging the condenser and discharging it through the galvanometer at an unvarying rate. The tuning-fork with the attachment described in Art. 92 may be employed.

If $n$ be the number of discharges per second, $C$ the capacity of the condenser, and $E$ the charging electromotive force, then for one discharge $q = EC$, and for $n$ discharges $nq = nEC$. This quantity is equal to the current $I$, which will produce the same deflection.

If $d_1$ is the deflection in scale parts, corrected by Table II. for proportionality to tan $\theta$, then

$$md_1 = nEC \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $m$ is a constant equal to the current corresponding to a deflection of one scale part.

Next connect in series the same battery, the galvanometer, and a high resistance $R_1$, the galvanometer being shunted with a resistance $s$. Then if $d_2$ is the deflection, corrected as before,

$$md_2 = \left\{ \frac{E}{R_1 + \frac{sg}{s+g}} \right\} \frac{s}{s+g} \quad . \quad . \quad . \quad (2)$$

Divide (1) by (2) and

$$\frac{d_1}{d_2} = nRC\,\frac{s+g}{s}.$$

$R$ is the total resistance of the circuit, neglecting the internal resistance of the battery.

Therefore, $$C = \frac{d_1}{d_2} \cdot \frac{1}{nR} \cdot \frac{s}{s+g}.$$

**109. Absolute Capacity of a Condenser — Third Method.** — The condenser whose capacity is to be measured is placed in one of the branches of a Wheatstone's bridge (Fig. 108). One side of the condenser is alternately connected to $S$ for charging and to $R$ for dis-

charging $n$ times a second by means of a vibrating plate $P$, or a tuning-fork (Art. 92). The condenser is thus charged and discharged $n$ times a second. During the charging of the condenser a part of the charge passes through the galvanometer in the opposite direction to the steady current flowing when the condenser is fully charged and while it is discharging.

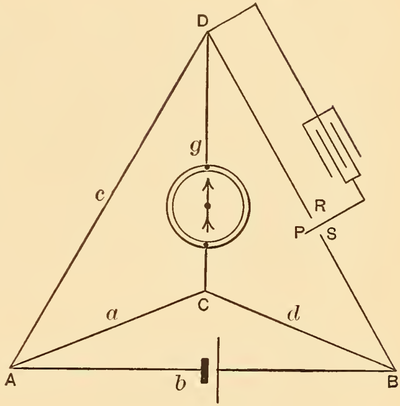


Fig. 108.

The resistances are varied until a balance is obtained as in the use of the Wheatstone's bridge for the measurement of resistance. Then if the resistances of the several branches are represented by the small letters in the figure,

$$nC = \frac{\left\{(a+c+g)(a+b+d)-a^2\right\}a}{\left\{(a+b+d)(a+c)-a(a+d)\right\}\left\{(a+d)(a+c+g)-a(a+c)\right\}}.^{1}$$

In practice it has been found unnecessary to use the complete formula. Where $a$ and $b$ are small in comparison with $c$, $g$, and $d$, we may write

$$nC = \frac{a}{cd\left\{1+\dfrac{a}{d\left(1+\dfrac{c}{g}\right)}\right\}}.$$

[1] J. J. Thomson, in *Phil. Trans.*, 1883, Part III., p. 707; R. T. Glazebrook, in *Phil. Mag.*, 1884, Vol. 18, p. 98.

This approximate formula may be demonstrated as follows:

The quantity required to charge the condenser equals the product of its capacity and the maximum value of the potential difference between $D$ and $B$ which is reached when the condenser is fully charged. Assuming that the time required to charge the condenser is a very small fraction of the period of the fork, we may suppose a steady current flowing through the galvanometer for $\frac{1}{n}$ of a second, followed by a momentary rush through it in the opposite direction of that part of the charge which goes through the branch $g$. The galvanometer needle will appear to stand still in its zero position if the total quantity passing through the galvanometer is algebraically zero. The period of the galvanometer must be large in comparison with $\frac{1}{n}$.

The value of the steady current through $d$ is

$$\frac{E}{b + \frac{a(c+g)}{a+c+g} + d}$$

if $E$ is the E.M.F. of the battery. Put $R$ for the resistance

$$b + \frac{a(c+g)}{a+c+g} + d.$$

Then the steady current through $g$ is

$$\frac{E}{R} \cdot \frac{a}{a+c+g}.$$

These currents cause a fall of potential between $D$ and $B$ of

$$\frac{E}{R}\left(d + \frac{ag}{a+c+g}\right).$$

Hence the total quantity required to charge the condenser to this potential difference $n$ times a second is

$$nC\frac{E}{R}\left(d+\frac{ag}{a+c+g}\right).$$

Neglecting self-induction, the portion of this charge passing through the galvanometer is $\frac{c}{a+c+g}$ times the whole, and this discharge is balanced by the steady current through the galvanometer in the opposite direction for the rest of the period.

Therefore,

$$nC\frac{E}{R}\left(d+\frac{ag}{a+c+g}\right)\left(\frac{c}{a+c+g}\right)=\frac{E}{R}\cdot\frac{a}{a+c+g}.$$

Hence,

$$nC=\frac{a}{cd\left\{1+\dfrac{a}{d\left(1+\dfrac{a+c}{g}\right)}\right\}}=\frac{a}{cd\left\{1+\dfrac{a}{d\left(1+\dfrac{c}{g}\right)}\right\}}$$

if $a$ is negligible in comparison with $c$.

**Example.**

Measurement of the absolute capacity of a Marshall one-third microfarad condenser:

| $n$ | $g$ | $a$ | $c$ | $d$ | $C$ |
|---|---|---|---|---|---|
| 32 | 13,720 | 5 | 1000 | 467 | 0.331 |
| 32 | 13,720 | 1 | 1000 | 95 | 0.329 |
| 32 | 13,720 | 3 | 1000 | 281 | 0.330 |
| 32 | 13,720 | 2 | 1000 | 187 | 0.330 |
| | | | | Mean, | 0.330 mf $\frac{\text{ohm}}{\text{B.A. unit}}$. |

The resistances were in B.A. units. The dimensional formula of a capacity is $L^{-1}T^2$, while that of a resistance is $LT^{-1}$. Hence, the unit of time remaining the same, any change in the unit of resistance is directly as a length, while the change in the unit of capacity is inversely as a length. Therefore, the resulting change in the numeric of a capacity, measured in terms of a resistance, will be directly as a length, or directly as the unit of resistance. The international ohm is 1.01358 B.A. units. Hence, 0.330 microfarad measured in B.A. units equals

$$0.330 \times 1.01358 = 0.3345 \text{ mf.}$$

The charge and discharge was effected by means of a large Koenig fork, and its rate was measured by means of a device based on electrolytic action. Its rate both immediately before and immediately after the balance was found to be just 32.

# CHAPTER VI.

## SELF-INDUCTION AND MUTUAL INDUCTION.

**110. Preliminary Relations.** — The electromotive force of self-induction in any circuit or part of a circuit is the product of its inductance $L$ and the rate of change of the current. If the resistance is strictly non-inductive, then $L$ is zero and there is no self-induced electromotive force. If the circuit or coil contains no magnetic material and has no iron within or about it, then $L$ is a constant, and the electromotive force of self-induction is

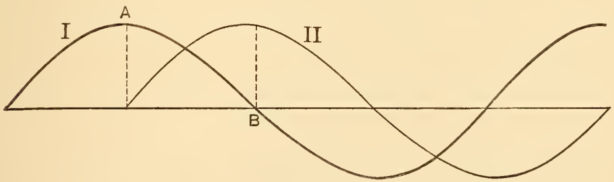



Fig. 109.

proportional to the rate of change of the current. The phase of this electromotive force is then a quarter of a period behind that of the current, when the latter is simple harmonic.

Let an alternating current, following the simple harmonic law, be represented by the heavy sine curve $I$ of Fig. 109. Then the induced electromotive force due to its variations may be represented by the thin line $II$. This is also a sine curve, since the differential coeffi-

cient of a sine function is itself a sine function. When the current has reached its maximum value at $A$, the electromotive force has its zero value, because at that instant the change-rate of the current is zero; but when the current passes through its zero value at $B$, its change-rate is a maximum and the induced electromotive force has its greatest value. The electromotive force therefore reaches its maximum value one-quarter of a period later than the current, and the two are said to be in quadrature. The effective electromotive force producing the current in accordance with Ohm's law must correspond in phase with the current itself. We may therefore represent the maximum effective and induced electromotive forces by the two adjacent sides of a right triangle[1] (Fig. 110), where $ab$ is the effective electromotive force, and $bc$ the inductive electromotive force; the hypotenuse $ac$ is therefore the maximum impressed electromotive force applied to the circuit. Since the current is in the same phase as $ab$, it must lag behind the impressed electromotive force by an angle $\phi$. This angle becomes zero when $L$ is zero. Self-induction therefore explains the lag of the current behind the impressed electromotive force.

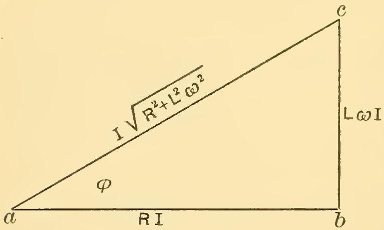


Fig. 110.

The instantaneous value of an alternating current following the simple sine law is

$$i = I \sin \theta = I \sin 2\pi nt,$$

[1] Carhart's *University Physics*, Part I., p. 36.

where $I$ is the maximum value of the current, and $n$ is the number of full periods per second. Hence $2\pi n$ is the angular velocity $\omega$.

Therefore, $$i = I \sin \omega t.$$

Then, $$L\frac{di}{dt} = L\omega I \cos \omega t.$$

The maximum value of this induced electromotive force is $L\omega I$. Therefore in the triangle of electromotive forces, if the base $ab$ is the maximum effective electromotive force, producing a current $I$ through a resistance $R$, by Ohm's law it is equal to $RI$. Also $bc$, the maximum inductive electromotive force, is $L\omega I$. Consequently the hypotenuse $ac$ equals $I\sqrt{R^2 + L^2\omega^2}$, or

$$E = I\sqrt{R^2 + L^2\omega^2}.$$

Therefore, $$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}}.$$

The expression $(R^2 + L^2\omega^2)^{\frac{1}{2}}$ is called the *impedance*.

Also, $$\tan \phi = \frac{L\omega}{R}.$$

In these equations $I$ and $E$ may represent either the maximum values of the current and electromotive force, or the "square root of the mean square" values. The latter are those measured by all the practical current and pressure instruments which are operated by forces varying as the square of the current and electric pressure respectively. Such are the electrodynamometer and the electrostatic voltmeter.

**111. To solve for the Current when the Circuit contains both Self-Induction and Capacity.** — If the electromotive force applied follows the simple law of

sines, its value for any instant is $e = E \sin \omega t$. This applied electromotive force equals the vector sum of the effective electromotive force producing a current, the electromotive force of self-induction, and that due to the charge of a condenser in series with the resistance.

Then, $$E \sin \omega t = Ri + L\frac{di}{dt} + \frac{\int idt}{C}.$$

The last term is the electromotive force introduced by capacity. From the definition of capacity the potential $V = \frac{Q}{C}$.

But $$Q = \int idt. \quad \text{Hence } V = \frac{\int idt}{C}.$$

It is entirely valid to assume a general solution of the above equation and then find the constants. Since the applied electromotive force is a sine function of the time, it may be assumed that the current also will be a sine function if the circuit contains no iron. The general equation for the current may then be written

$$i = k \sin (\omega t - \phi).$$

The angle $\phi$ is introduced to express the lag of the current behind the applied electromotive force. Then

$$L\frac{di}{dt} = Lk\omega \cos (\omega t - \phi).$$

$$\frac{\int idt}{C} = -\frac{k}{C\omega} \cos (\omega t - \phi).$$[1]

[1] Strictly speaking, this equation should be written

$$\frac{\int idt}{C} = -\frac{k}{C\omega} \cos (\omega t - \phi) + A,$$

in which $A$ is a constant of integration. It will however be easily seen that the value of $A$ is zero, as the maximum and minimum values of

$$\frac{\int idt}{C} = +\frac{k}{C\omega} + A \text{ and} -\frac{k}{C\omega} + A$$

must be numerically equal, which is true only when $A$ is zero.

Substituting in the equation of electromotive forces,

$$E \sin \omega t = Rk \sin (\omega t - \phi) + \left(Lk\omega - \frac{k}{C\omega}\right) \cos (\omega t - \phi).$$

Since this equation is generally true, it is true when the angle $(\omega t - \phi)$ equals zero and when it equals $\frac{\pi}{2}$.

In the first case $E \sin \phi = Lk\omega - \frac{k}{C\omega}$. . . . . $(a)$

In the second case $E \cos \phi = Rk$. . . . . $(b)$

Squaring $(a)$ and $(b)$ and adding,

$$E^2 = R^2k^2 + k^2\left(L\omega - \frac{1}{C\omega}\right)^2,$$

and

$$k = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}.$$

Therefore,

$$i = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \sin (\omega t - \phi).$$

To find the angle of lag, divide $(a)$ by $(b)$ and

$$\tan \phi = \frac{L\omega - \frac{1}{C\omega}}{R}.$$

While self-induction causes the current to lag behind the impressed electromotive force, capacity tends to give to it a lead ahead of the electromotive force. The one will neutralize the other when

$$L\omega = \frac{1}{C\omega}.$$

The fraction $\frac{1}{C\omega}$ is the impedance due to capacity alone. It may be expressed numerically in ohms.

If the circuit contains self-induction but not capacity, then the third term in the equation of electromotive forces drops out and

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}},$$

where $I$ and $E$ are either maximum values or the square roots of the mean squares, as measured by an electro-dynamometer.

If the circuit contains capacity but no self-induction, then

$$I = \frac{E}{\sqrt{R_2 + \frac{1}{C^2\omega^2}}}.$$

Further, if the resistance of the circuit is zero,

$$I = C\omega E.$$

This last equation furnishes an independent method of measuring the capacity of a condenser.

**112. Measurement of the Capacity of an Electrostatic Voltmeter.**[1] — The voltmeter is first employed to measure the potential difference $e$ between the alternating mains. A non-inductive graphite resistance of several megohms is then joined in series with the voltmeter. It will now indicate a smaller potential difference $e_2$. This potential difference is one-quarter of a period behind the charging current, while the potential difference $e_1$ between the terminals of the graphite resistance agrees in phase

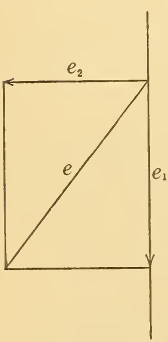


Fig. 111.

[1] Dr. Sahulka, in the *Proceedings of the Chicago International Electrical Congress*, p. 379.

with the current, since this resistance $r$ is non-inductive and without capacity.

Hence (Fig. 111) $e = \sqrt{e_1^2 + e_2^2}$.

Therefore $e_1$ may be computed and $I$ equals $\frac{e_1}{r}$. Then since $I$ also equals $2\pi n e_2 C$,

$$C = \frac{e_1}{e_2} \cdot \frac{1}{2\pi n r}.$$

**Example.**

The alternating current had 2500 full periods per minute.

Hence $\omega = 2\pi n = 262.$

The table gives the results with a Kelvin multicellular voltmeter. The values of $r$ are in megohms, the potential difference in volts, the current in millionths of an ampere, and the capacity in millionths of a microfarad.

| $r$ | $e$ | $e_1$ | $e_2$ | $i$ | $C$ |
|---|---|---|---|---|---|
| 11.05 | 207.2 | 69.2 | 195.3 | 6.26 | 122 |
| 20.78 | 207.6 | 108.3 | 177.1 | 5.21 | 112 |
| 33.16 | 207.6 | 138.6 | 154.6 | 4.18 | 103 |
| 41.90 | 207.9 | 153.4 | 140.3 | 3.66 | 99.6 |
| 52.40 | 208.0 | 166.7 | 124.4 | 3.18 | 97.6 |

The capacity was greater for the higher values of $e_2$ than for the lower ones, because the movable system is deflected so as to increase the capacity of the instrument as an air condenser for the higher readings.

**113. Measurement of Capacity by Alternating Currents.**—Employing small letters for the square root of mean square values,

$$C = \frac{i}{e_2 \omega},$$

where $e_2$ is the potential difference between the two sides of the condenser. If $i$ is expressed in amperes and $e_2$ in

volts, $C$ will be in farads. Let the condenser be put in series with a graphite resistance, about numerically equal to the impedance of the condenser $\frac{1}{C\omega}$ expressed in ohms. By means of an electrostatic voltmeter measure the potential difference between the terminals of the graphite resistance and between those of the condenser. Call the former $e_1$ and the latter $e_2$. Then $e_1$ agrees in phase with the current, while $e_2$ differs from it in phase somewhat less than 90° if the condenser has a solid dielectric. Measure also $e$, the potential difference between the mains. Then since $i$ equals $\frac{e_1}{r}$,

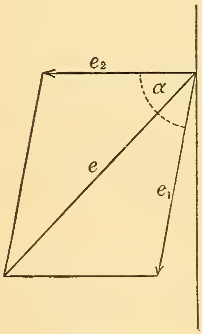



Fig. 112.

$$C = \frac{e_1}{e_2} \cdot \frac{1}{2\pi n r}.$$

The angle of lag $a$ may be calculated from Fig. 112.

$$e^2 = e_1^2 + e_2^2 + 2e_1e_2 \cos a.$$

Whence $$\cos a = \frac{e^2 - e_1^2 - e_2^2}{2e_1e_2}.$$

The energy in watts absorbed by the condenser is

$$w = e_2 i \cos a.$$

In an air condenser, where $a$ is 90°, the energy absorbed by the condenser during the charging is equal to that restored to the circuit in the discharge, or the positive work done equals the negative. In condensers with solid dielectrics energy is absorbed in excess of that given out and the condenser heats.

**Example.**

To measure the capacity of a nominal $\frac{1}{10}$ microfarad made by Elliott Bros. the smaller electrostatic voltmeter of Art. 95 was employed. The alternator had 10 poles and made 1,632 revolutions per minute.

| $r$ | $e$ | $e_1$ | $e_2$ | $C$ | $i$ | $a$ | $\omega$ |
|---|---|---|---|---|---|---|---|
| 16700 | 105.7 | 83.5 | 62.25 | 0.093 | 0.005 | 88° 13′ | 0.0097 |

The capacity of a condenser with a solid dielectric is smaller when measured with alternating currents than with direct ones.

**114. Impedance Method of measuring the Coefficient of Self-Induction.**[1] — The value of the coefficient of self-induction of a coil of known resistance $R$ may be found by passing through it an alternating current and measuring the potential difference between its terminals by means of an electrostatic voltmeter. At the same time the current through the coil must be measured by an appropriate ammeter. Then

$$I = \frac{E}{\sqrt{R^2 + L^2\omega^2}},$$

where $E$ is the measured potential difference, $I$ the current, $R$ the ohmic resistance of the coil, and $L$ the inductance in henrys.

The term $L\omega$ is now called the *reactance*. The resistance must be measured independently, and $\omega$ is obtained from the speed of the dynamo and the number of poles. Thus a small bipolar machine, making 3000 revolutions a minute, gives for $n$ a value of 50, and for $\omega$ or $2\pi n$,

[1] Nichol's *Laboratory Manual of Physics*, Vol. II., p. 109.

314.2. The value of $L$ may then be found by substituting the values of $E$, $I$, $R$, and $\omega$ in the equation for the current.

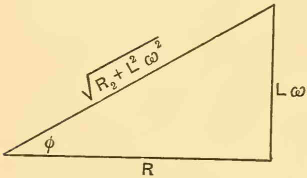



Fig. 113.

Draw a right triangle (Fig. 113) with the three sides equal to resistance, reactance, and impedance respectively, and measure the angle of lag $\phi$. Compute the time constant of the coil $\frac{L}{R}$. If the resistance of the coil is large, the result may be vitiated by its static capacity.[1]

The value of $L$ found by this method depends upon an ammeter and a voltmeter reading. It may be made to depend upon voltmeter readings alone.

**115. Three-Voltmeter Method of measuring Inductance.**[2] — A non-inductive resistance $R_1$ (Fig. 114) is placed in series with the coil of resistance $R_2$ whose inductance $L_2$ is to be measured. An alternating cur-

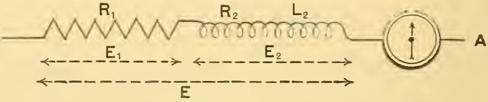



Fig. 114.

rent is then sent through the circuit, and it may be measured by the ammeter $A$ as a check. Three voltmeter readings as nearly simultaneous as possible are taken — $E$ the total potential difference between the

---

[1] *Electrical World*, July 13, 1895.

[2] Nichol's *Laboratory Manual*, Vol. II., p. 113.

terminals of the whole resistance, $E_1$ between those of $R_1$, and $E_2$ between those of $R_2$.

Then draw a triangle $OBA$ (Fig. 115) with the three sides equal to the three voltmeter readings, or the readings reduced to volts. Produce $OB$ to $C$, making $OCA$ a right triangle. Then $AC$ is equal to $L_2\omega I$. It may be taken directly from the figure, and $L_2$ may then be found from the known values of the frequency $n$ and the current. $BC$ is the electromotive force producing the current $I$ through $R_2$, and $CA$ the electromotive force of self-induction. It is evident that $I$ equals $\frac{E_1}{R_1}$, since $R_1$ is non-inductive. Besides the three electromotive forces, we must therefore measure either $I$ or $R_1$.

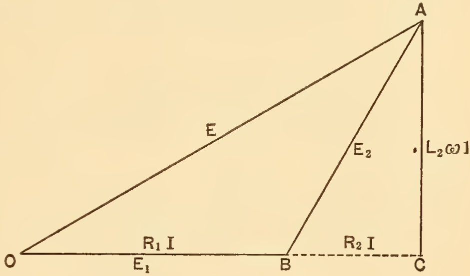


Fig. 115.

If the coil surrounds an iron core, the inductance should be measured for different values of the current. It will be found to decrease as the core becomes saturated. The currents may then be plotted as abscissas and the inductances as ordinates.

**116. Comparison of the Capacity of a Condenser with the Self-Inductance of a Coil.**[1] — The four resistances in the arms of the Wheatstone's bridge (Fig. 116) are $Q$, $P$, $R$, $S$. When the battery circuit is closed, the

[1] Maxwell's *Electricity and Magnetism*, Vol. II., p. 387.

potential difference at the terminals of $R$ causes a current through it and at the same time charges the condenser $C$. The potential difference rises as the condenser receives its charge, and therefore the current through $R$ requires a definite time-interval to rise to its final value.

The current through the coil $Q$ will increase from zero to its maximum value in a precisely similar way on account of the counter E.M.F. of self-induction. Both the condenser and the coil have a time constant,

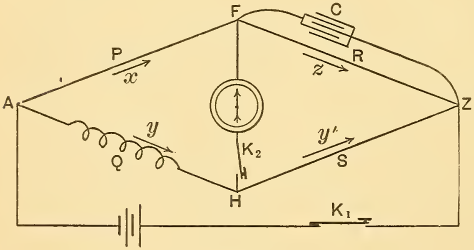


Fig. 116.

and the effect of the condenser in delaying the current in one branch may be made to offset that of the coil in the other, so that the rise of potential at $F$ may be the same as at $H$. In that case no current will pass through the galvanometer. We have to determine the conditions under which the potential at $F$ remains equal at every instant to that at $H$.

Let $x$ and $z$ be the quantities which have passed through $P$ and $R$ respectively at the end of the interval $t$ after closing the circuit. Then $x-z$ will be the charge of the condenser at the same instant.

The potential difference between the two sides of the condenser is by Ohm's law $R\frac{dz}{dt}$, since $\frac{dz}{dt}$ is the value of the current. Therefore

$$x-z=RC\frac{dz}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let $y$ be the quantity traversing $Q$ in the same time $t$. Then the potential difference between $A$ and $H$ is equal to that between $A$ and $F$ when there is a balance and no current flows through the galvanometer; or

$$Q\frac{dy}{dt}+L\frac{d^2y}{dt^2}=P\frac{dx}{dt}. \quad . \quad . \quad . \quad (2)$$

The first member consists of the effective E.M.F. producing a current and E.M.F. of self-induction. The sum of the two is the potential difference between $A$ and $H$.

Since there is no current through the galvanometer the quantity passing along $HZ$ must be the same as that along $AH$, or $y'=y$. Therefore

$$S\frac{dy}{dt}=R\frac{dz}{dt}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

since the potential difference between $F$ and $Z$ is the same as that between $H$ and $Z$, when no current flows through the galvanometer.

From (1) $$\frac{dx}{dt}-\frac{dz}{dt}=RC\frac{d^2z}{dt^2},$$

the rate at which the condenser is charged.

Substitute in (2) and

$$Q\frac{dy}{dt}+L\frac{d^2y}{dt^2}=P\left(RC\frac{d^2z}{dt^2}+\frac{dz}{dt}\right).$$

From (3) $\frac{dy}{dt}=\frac{R}{S}\frac{dz}{dt}$. Substituting in the last equation and $Q\frac{R}{S}\frac{dz}{dt}+L\frac{R}{S}\frac{d^2z}{dt^2}=P\left(RC\frac{d^2z}{dt^2}+\frac{dz}{dt}\right)$.

Multiply by $S$ and integrate and

$$QRz+LR\frac{dz}{dt}=PRSC\frac{dz}{dt}+PSz,$$

or $$QR\left(1+\frac{L}{Q}\frac{d}{dt}\right)z = PS\left(1+RC\frac{d}{dt}\right)z. \quad . \quad . \quad (4)$$

This is the equation of condition that no current shall pass through the galvanometer.

The condition for a steady current with a Wheatstone's bridge is

$$QR = PS. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence the condition that no current shall traverse the galvanometer when the battery circuit is opened and closed is

$$\frac{L}{Q} = RC. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$\frac{L}{Q}$ and $RC$ are called the "time constants" of the coil and the condenser respectively. If by varying $P$ and $R$ the bridge can be adjusted so that no current traverses the galvanometer on opening and closing the battery circuit, as well as when it is kept closed, then the two "time constants" are equal and

$$L = QRC.$$

To show that a time constant $\frac{L}{R}$ is a time, since a resistance has the dimensions of a velocity, and a capacity is the square of a time divided by a length, we have from the equation $\frac{L'}{Q} = RC$ (calling the coefficient of self-induction $L'$ to distinguish it from a length $L$)

$$L' \div \frac{L}{T} = \frac{L}{T} \cdot \frac{T^2}{L} = T.$$

Also $$L' = \left(\frac{L}{T}\right)^2 \cdot \frac{T^2}{L} = L,$$

or self-induction is a length. The unit of induction is the henry and equals $10^9$ cms. It varies directly as the ohm.

If $C$ is in microfarads the value of $L$ from the equation above will be a million times too large and must be multiplied by $10^{-6}$ to reduce to henrys.

**117. Anderson's Modification of Maxwell's Method.**[1] — In the preceding method of Maxwell a double adjustment must be made in order to effect a balance. First, one of the branches $P$ has to be adjusted for a balance

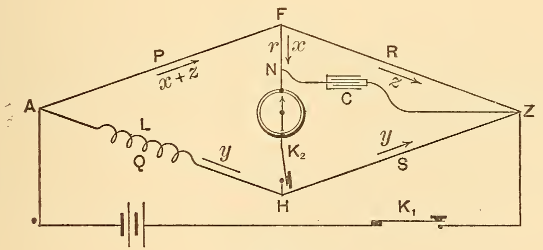



Fig. 117.

with a steady current. Then, in order to obtain a balance when the galvanometer circuit is closed first, the resistance $R$ will have to be adjusted. This necessitates a fresh adjustment of $P$, and so on. Anderson's modification of Maxwell's method is designed to facilitate the adjustments.

Suppose a balance has been obtained for steady currents by closing $K_1$ before $K_2$ (Fig. 117). This balance will not be disturbed by introducing the resistance $r$ between $F$ and $N$. Adjust $r$ therefore till the galvanom-

[1] *Phil. Mag.*, Vol. XXXI., 1891, p. 329.

eter shows no deflection when $K_2$ is closed before $K_1$. The potentials at $H$ and $N$ then remain equal to each other. Let $x$ be the quantity which has flowed into the condenser at the time $t$, and $z$ the quantity which has passed through $FZ$. Then $x + z$ has passed through $AF$. Then if $C$ is the capacity of the condenser, since the fall of potential from $F$ to $Z$ is the same by the two paths, we have

$$R\frac{dz}{dt} = \frac{x}{C} + r\frac{dx}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also since $N$ and $H$ must be of the same potential,

$$\frac{x}{C} = S\frac{dy}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Further, the change of potential from $A$ through $F$ to $N$ is the same as from $A$ to $H$. Hence

$$r\frac{dx}{dt} + P\left(\frac{dx}{dt} + \frac{dz}{dt}\right) = Q\frac{dy}{dt} + L\frac{d^2y}{dt^2}. \quad . \quad (3)$$

Substituting from (1) and (2),

$$(r + P)\frac{dx}{dt} + \frac{P}{R}\left(\frac{x}{C} + r\frac{dx}{dt}\right) = \frac{Q}{S}\cdot\frac{x}{C} + \frac{L}{SC}\frac{dx}{dt}.$$

This equation expresses both conditions necessary for a balance with variable currents. For steady currents

$$\frac{P}{R} = \frac{Q}{S}.$$

Hence the other condition is found by equating the coefficients of $\frac{dx}{dt}$; or

$$r + P + \frac{Pr}{R} = \frac{L}{SC}.$$

This condition gives the formala

$$L = C\{r(Q+S) + PS\}.$$

If $r$ is zero, $L = CPS = CQR$, which is Maxwell's formula.

To apply the above equation for $L$, first obtain a balance in the ordinary way, and then adjust $r$ and, if possible, $C$ till there is no deflection of the galvanometer needle on working $K_1$ with $K_2$ closed.

For sensitiveness of the final adjustment it is desirable to make $R$ and $S$ large, and $r$ small. Since $Q$ is usually small, $P$ will also be small.

**Example.**

*Calibration of the Standard of Inductance.*

1. For a balance with steady currents,

$P = 13.27$ ohms. $R = 125.2$ ohms.
$Q = 10.6$ " $S = 100$ "

When corrected for temperature, $Q + S = 111.1$ ohms; $P \times S = 1337$.

2. For a balance with variable currents,

$$C = 0.335 \text{ mf.}$$

| $r$ in ohms. | Nominal value of inductance. | Calculated value of inductance. |
|---|---|---|
| 124 | 0.005 | 0.0051 |
| 253 | 0.010 | 0.0099 |
| 390 | 0.015 | 0.0150 |
| 525 | 0.020 | 0.0200 |
| 660 | 0.025 | 0.0250 |
| 798 | 0.030 | 0.0301 |
| 925 | 0.035 | 0.0349 |

**118. Russell's Modification of Maxwell's Method.**[1] — Connect the coil exactly as in Maxwell's method and balance for steady currents. Then if the galvanometer

[1] *London Electrician*, May 4, 1894.

key be closed first, there will be a throw of the needle when the battery key is closed; and if the battery key be opened first the throw of the needle will be the other way. Now connect the condenser, which should be a subdivided one, as a shunt to the branch $R$. The effect will be to reduce the throws of the needle. Use different values of the condenser capacity, one giving a throw in one direction on opening or closing the battery circuit, and the other a throw in the other direction. Then by interpolation find the capacity which would reduce the deflection to zero. This capacity, substituted in the equation $L = QRC$, gives the desired inductance $L$.

**Example.**

*To measure the Self-Inductance of Two Coils.*

The bridge consisted of special non-inductive resistances.

$$R = 131.7 \text{ ohms.} \qquad S = 131.2 \text{ ohms.}$$

$$Q = 25.88 \text{ ohms} + \text{resistance of the coil.}$$

The coils consisted of 450 turns in three layers each, the smaller having a mean diameter of 3.3 cms., the larger, 4.0 cms. The larger coil could be slipped over the smaller one.

1. The smaller coil.

$$Q = 30.05; \; R = 131.7.$$

With $C = 0.45$ mf., the deflection was $+ 15$ scale parts.
" $C = 0.5$ " " " " $- 25$ " "
To balance, $C = 0.47$ mf.
Therefore,

$$L = 0.00000047 \times 30.05 \times 131.7 = 0.00186 \text{ henry.}$$

2. The larger coil.

$$Q = 31.15; \; R = 131.7.$$

With $C = 0.6$ mf., the deflection was $+ 65$ scale parts.
" $C = 0.7$ " " " " $- 15$ " "
To balance, $C = 0.68$ mf.
Therefore,

$$L = 0.00000068 \times 31.15 \times 131.7 = 0.00279 \text{ henry.}$$

The condenser was a microfarad subdivided into .5, .2, .2, .05, .05 mf.

**119. Rimington's Modification of Maxwell's Method.**[1] — In this method one side of the condenser is connected to $F$ (Fig. 118), and the other side to a point $N$, which can be shifted along so as to vary $r$ without any change in the resistance $R$ of that branch. In this arrangement the discharges through the galvanometer, due to the discharge of the condenser and the self-induction of the coil, are in opposite directions and equal, when both balances have been secured.

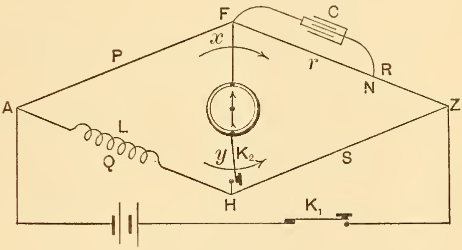


Fig. 118.

Let $y$ be the current flowing in the arms $Q$ and $S$, when it has reached its steady value, and $x$ that in $P$ and $R$.

Let both keys be closed and then let $K_1$ be opened. The quantity of electricity which passes through the galvanometer, due to self-induction in $Q$, is

$$\frac{Ly}{P+Q+\dfrac{G(R+S)}{G+R+S}} \times \frac{R+S}{G+R+S} = \frac{Lya}{P+Q+Ga}.$$

This is the integral of the current between the limits 0 and $y$. The quantity passing through the galvanometer from the discharge of the condenser is

$$Cxr\frac{r}{R+S+\dfrac{G(P+Q)}{G+P+Q}} \times \frac{P+Q}{G+P+Q} = \frac{Cxr^2b}{R+S+Gb}.$$

[1] *Phil. Mag.*, Vol. XXIV., 1887, p. 54.

This discharge passes while the current through $r$ falls from $x$ to zero.

These quantities pass through the galvanometer in opposite directions, and if there is no deflection,

$$\frac{Lya}{P+Q+Ga}=\frac{Cxr^2b}{R+S+Gb}.$$

But
$$\frac{Lya}{P+Q+Ga}=\frac{Ly\,(R+S)}{c},$$

and
$$\frac{Cxr^2b}{R+S+Gb}=\frac{Cxr^2\,(P+Q)}{c}.$$

Hence
$$Ly\,(R+S)=Cxr^2\,(P+Q).$$

And
$$L=Cr^2\,\frac{x}{y}\,.\,\frac{P+Q}{R+S}.$$

Now $\frac{x}{y}=\frac{Q}{P}$. Therefore $\frac{x}{y}\,.\,\frac{P+Q}{R+S}=\frac{Q}{P}\,.\,\frac{P+Q}{R+S}=\frac{Q}{R}$, since $PS=QR$. Hence

$$L=Cr^2\frac{Q}{R}.$$

If $r=R$, we have Maxwell's formula,

$$L=C\,QR.$$

The resistance must be such that $r$ can be adjusted without changing the value of $R$ after a balance has been obtained for steady currents. The double commutator, illustrated in Fig. 47, may be used in this method when suitable adjustments of the two commutators are made.

The condition $L=\frac{Cr^2Q}{R}$ may be obtained directly from Maxwell's equation $L=CRQ$. When no deflec-

tion of the galvanometer is observed on opening the battery circuit, a certain quantity of electricity, coming from the condenser, must pass through the branch $S$. If one of the terminals of the condenser is moved along $R$ to the point $N$, the fraction of the charge passing through $S$ will be decreased in the ratio of $\frac{r}{R}$; and as the total charge will be decreased in the same ratio because of the lower potential to which the condenser is charged, the quantity passing through $S$ on the discharge will be reduced in the ratio of $\frac{r^2}{R^2}$. Consequently, if the same quantity is to pass through $S$ as in Maxwell's method, the capacity of the condenser must be increased in the ratio of $\frac{R^2}{r^2}$. Whence it follows that

$$L = \frac{CQr^2}{R}.$$

**120. Comparison of Two Coefficients of Self-Induction.**[1] — The double commutator of Fig. 47 may be used for this purpose to increase the sensibility. The four points of the bridge (Fig. 119) are connected with the double commutator exactly as in Fig. 49.

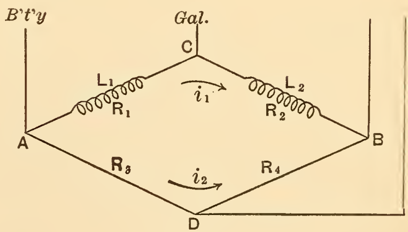


Fig. 119.

Let $R_1$ and $R_2$ be inductive resistances with coefficients $L_1$ and $L_2$, and let $R_3$ and $R_4$ be inductionless resistances. Then if

[1] Maxwell's *Elec. and Mag.*, Vol. II., p. 367.

$R_3$ and $R_4$ be adjusted to give a balance with a steady current, a balance will also be obtained with varying currents when

$$\frac{L_1}{L_2}=\frac{R_3}{R_4}.$$

The rate of rotation of the commutator must not be too great to permit the currents to reach their steady values between consecutive reversals.

The equation may be demonstrated as follows: Let $i_1$ be the current through $AC$ and $i_3$ that through $AD$ (Fig. 119), at the same instant $t$ after closing the circuit, or after reversal. Then, since no current traverses the galvanometer when a balance has been obtained, $i_1$ and $i_3$ are also the currents through $CB$ and $DB$ respectively. The difference of potential between $A$ and $C$ is the same as between $A$ and $D$; also the fall of potential from $C$ to $B$ is the same as from $D$ to $B$. Hence

$$R_1 i_1 + L_1 \frac{di_1}{dt} = R_3 i_3,$$

$$R_2 i_1 + L_2 \frac{di_1}{dt} = R_4 i_3.$$

Whence,

$$R_1 R_4 i_1 i_3 + L_1 R_4 \frac{di}{dt} i_3 = R_2 R_3 i_1 i_3 + L_2 R_3 \frac{di_1}{dt} i_3.$$

But $R_1 R_4 = R_2 R_3$ is the condition of a balance with a steady current. The other condition for a balance with varying currents is therefore

$$L_1 R_4 = L_2 R_3,$$

or

$$\frac{L_1}{L_2}=\frac{R_3}{R_4}.$$

If one of these coefficients, as $L_1$, is a standard of self-

induction, the equation gives the value of the other. Such a standard is shown in Fig. 120. It contains two coils without iron joined in series, one of them fixed and

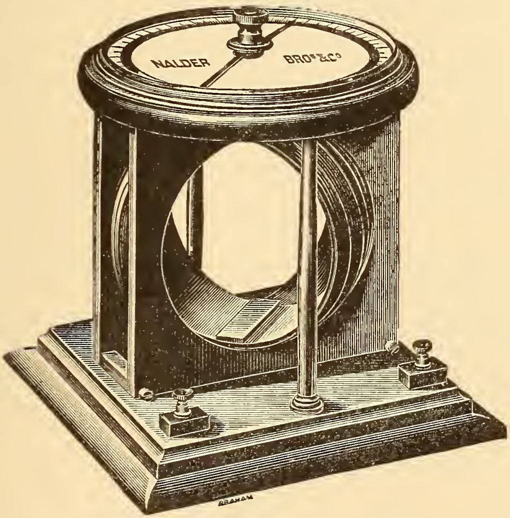



Fig. 120.

the other movable about a vertical axis. The self-induction of the two depends upon their relative position, and the scale at the top is graduated to read in milli-henrys. Since the self-induction of the standard is variable, a balance can often be obtained for variable currents by changing the relative position of its two coils. Its resistance, however, is only about ten ohms; and if the ratio of its smallest inductance to that of the coil to be measured is greater than that of their relative resistances, a balance can be effected only by adding non-inductive resistance in series with the standard.

Incandescent lamps in parallel or in multiple series are convenient for this purpose, since it is not necessary to know their resistance.

Alternating currents and an electrodynamometer may be employed with advantage in this method (Art. 60). The entire current should pass through the field coil, and the suspended coil should take the place of the galvanometer, as in Fig. 50.

**Example.**

*To compare the Two Coils of Art. 118 with the Standard of Inductance.*

The standard was inserted in the arm $R_1$, together with an additional non-inductive resistance, the latter being added in order to increase the ratio $\frac{R_1}{R_2}$, so as to bring the induction in the arm $R_1$ within the limits of the standard.

| Coil. | $R_2$ | $R_1$ | $L_1$ (Standard). | $L_2$ |
|---|---|---|---|---|
| Smaller. | 4.160 | 36.15 | 0.0161 | 0.001853 |
| | | 49.37 | 0.0219 | 0.001845 |
| Larger. | 5.245 | 36.15 | 0.0195 | 0.002829 |
| | | 49.37 | 0.02665 | 0.002832 |
| Two opposed in series. | 9.4 | 141.6 | 0.0152 | 0.00101 |
| | | 167.5 | 0.01795 | 0.00101 |

**121. Niven's Method of comparing Two Self-Inductances.**[1] — The inductance of $R_1$ is to be compared with that of $R_4$ (Fig. 121). First connect $R_1$ in a Wheatstone's bridge with three non-inductive resistances $R_2$, $R_5$, and $R_6$ and obtain a balance for steady currents. Then add the inductive resistance $R_4$ in series with $R_2$ and balance again for steady currents by adding a proportional non-inductive resistance to $R_1$. Finally connect $E$ and $F$ by

[1] *Phil. Mag.*, Sept., 1877.

means of the resistance $R_7$, and vary it till the galvanometer shows no deflection on making and breaking the battery circuit.

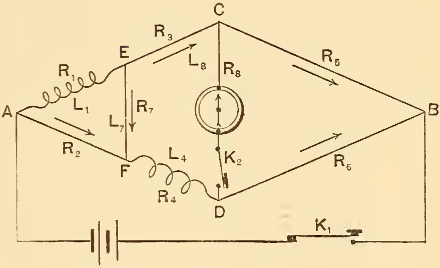


Fig. 121.

Call the quantity of electricity which has passed through each branch of the circuit $q$, with the proper subscript, at the time $t$ after closing the circuit, and let $Q$ with the corresponding subscripts represent the quantities for the several branches when the current has reached a steady state, after an interval $T$, reckoned from the time when the circuit is closed. Then the current is represented by $\frac{dq}{dt}$ and this is zero for each branch when $t$ is zero. It is also zero for the two cross branches $R_7$, $R_8$, when the steady state has been reached.

The potential difference between $C$ and $D$ at any time $t$ is the same by the four paths. Hence

$$R_5\frac{dq_5}{dt} - R_6\frac{dq_6}{dt} = R_8\frac{dq_8}{dt} + L_8\frac{d^2q_8}{dt^2} = R_4\frac{dq_4}{dt} + L_4\frac{d^2q_4}{dt^2} + R_7\frac{dq_7}{dt} + L_7\frac{d^2q_7}{dt^2} - R_3\frac{dq_3}{dt} = R_4\frac{dq_4}{dt} + L_4\frac{d^2q_4}{dt^2} + R_2\frac{dq_2}{dt} - R_1\frac{dq_1}{dt} - L_1\frac{d^2q_1}{dt^2} - R_3\frac{dq_3}{dt}. \qquad (1)$$

Integrating between limits $t = 0$ and $t = T$, we have all the terms zero when $t$ is zero; and putting $I_1$, $I_2$, $I_3$,

etc., for the maximum values of the several currents, corresponding to $t = T$, the equation above becomes

$$R_5Q_5 - R_6Q_6 = R_8Q_8 + L_8I_8 = R_4Q_4 + L_4I_4 + R_7Q_7 + L_7I_7 - R_3Q_3 = R_4Q_4 + L_4I_4 + R_2Q_2 - R_1Q_1 - L_1I_1 - R_3Q_3 \quad (2)$$

But since the galvanometer shows no deflection, both $Q_8$ and $I_8$ are zero, and

$$R_5Q_5 = R_6Q_6 \text{ or } \frac{R_5}{R_6} = \frac{Q}{Q_5}; \quad . \quad . \quad . \quad . \quad (3)$$

and since a balance exists for $R_1$, $R_2$, with $R_5$, $R_6$, also for $R_1 + R_3$ and $R_2 + R_4$ with $R_5$, $R_6$, it follows that

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{R_5}{R_6};$$

also since $Q_6 = Q_4$ and $Q_5 = Q_3$, there being no flow through the galvanometer, we have

$$R_4Q_4 - R_3Q_3 = 0,$$

because by substituting in (3)

$$\frac{R_3}{R_4} = \frac{Q_4}{Q_3}.$$

$I_7$ is also zero. It follows from equation (2) that

$$L_4I_4 + R_7Q_7 = 0 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and $$R_2Q_2 + L_4I_4 - R_1Q_1 - L_1I_1 = 0. \quad . \quad . \quad (5)$$

Further $Q_7 = Q_1 - Q_3$ and $Q_1 + Q_2 = Q_3 + Q_4$,

or $$Q_2 = Q_3 + Q_4 - Q_1.$$

Substituting in (4) and (5)

$$L_4I_4 + R_7Q_1 - R_7Q_3 = 0. \quad . \quad . \quad . \quad (6)$$

$$R_2Q_3 + R_2Q_4 - R_2Q_1 + L_4I_4 - R_1Q_1 - L_1I_1 = 0. \quad (7)$$

Multiplying (6) by $(R_1 + R_2)$ and (7) by $R_7$ and adding,

$$(R_1 + R_2 + R_7)\,L_4I_4 - R_7L_1I_1 - R_7R_1Q_3 + R_2R_7Q_4 = 0. \quad (8)$$

But $R_1Q_3 = R_2Q_4$ and $\frac{I_1}{I_4} = \frac{I_1}{I_2} = \frac{R_2 + R_4 + R_6}{R_1 + R_3 + R_5} = \frac{R_6}{R_5}$.

Therefore from (8)

$$\frac{L_1}{L_4} = \frac{R_1 + R_2 + R_7}{R_7} \cdot \frac{I_4}{I_1} = \frac{R_1 + R_2 + R_7}{R_7} \cdot \frac{R_5}{R_6}.$$

The ratio $\frac{R_5}{R_6}$ may be replaced by $\frac{R_1}{R_2}$ or by $\frac{R_3}{R_4}$.

**Example.**

*Comparison of the Inductances of the Two Coils of the Last Example.*

In the branch $R_1$ was put the larger of the two coils with an additional non-inductive resistance, so that $R_1$ was 31.1 ohms.

Arm $R_2$ was another non-inductive resistance of 25.9 ohms. In $R_4$ was the smaller coil (4.16 ohms), balanced in $R_3$ by a part of the non-inductive resistance of Fig. 80. $R_5$ and $R_6$ were formed by a slide wire bridge, the point $B$ being the sliding contact.

The first balance was obtained by moving the contact at $B$, and the second by adjusting the resistance $R_3$. $R_7$ was a resistance box and was 200 ohms for a balance with variable currents. Then

$$\frac{L_1}{L_4} = \frac{57 + 200}{200} \cdot \frac{31.1}{25.9} = 1.543.$$

From the last experiment,

$$\frac{L_1}{L_4} = \frac{283}{185} = 1.530.$$

**122. Mutual Induction.**[1] — Mutual induction is the induction taking place between adjacent circuits. The coil or circuit in which the inducing current is made to

[1] Nichol's *Laboratory Manual*, Vol. I., p. 242.

vary is called the primary, and the circuit acted on inductively is the secondary circuit.

Let $P$ be the primary and $S$ the secondary (Fig. 122). The primary coil is connected in series with a battery $B$, a variable resistance $R$, and an ammeter $A$. In the

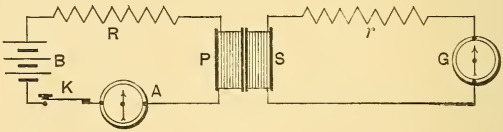



Fig. 122.

circuit of the secondary are connected a resistance $r$ and a ballistic galvanometer $G$.

*First*, observe the throw of the galvanometer needle when $K$ is opened and closed. At the same time measure the steady current flowing through the primary. Then reduce $R$ and repeat the observations, keeping the resistance of the secondary circuit constant. The resistances $R$ and $r$ should be so adjusted that the series of deflections of the ballistic galvanometer may vary from the smallest that can be accurately read to the largest that the scale will allow. The readings may be corrected for proportionality to $\sin\frac{\theta}{2}$.

Finally plot the primary currents as abscissas and the corrected deflections as ordinates. The resulting curve should be a straight line passing through the origin, or

$$Q \propto I, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where $Q$ is the quantity of electricity discharged through the secondary, and $I$ the current in the primary.

*Second*, to determine the relation between the quantity of electricity which flows in the secondary circuit and

the resistance of that circuit, observe the throw of the galvanometer when the primary circuit is closed and opened for several different resistances in the secondary. Then plot the deflections as ordinates and the reciprocals of the resistances as abscissas. The result will be a straight line through the origin. Hence

$$Q \propto \frac{1}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

in which $R$ is the resistance of the secondary circuit.

Combining ($a$) and ($b$), we have

$$Q = M\frac{I}{R}.$$

The constant $M$ is defined as the coefficient of mutual induction, or the mutual inductance, of the two coils. It is the electromotive force induced in the one coil while the current varies in the other at the rate of one ampere per second.

The value of $M$ depends on the geometrical form and winding of the two coils and on their relative position.

*Third*, $Q$ may be measured in coulombs by finding the constant of the ballistic galvanometer, using a condenser of known capacity and a standard cell. If, further, $I$ is measured in amperes and $R$ in ohms, then $M$ in the above equation will be expressed in henrys.

**Example.**

I. The quantity is proportional to the primary current.

The table contains the results of an experiment. In the third column the deflections are corrected so as to be proportional to $2 \sin \frac{1}{2}\theta$.

| Current in Primary. | Observed Deflections. | Corrected Deflections. |
|---|---|---|
| 1.470 | 31.2 | 30.78 |
| 1.295 | 27.5 | 27.22 |
| 1.159 | 24.5 | 24.31 |
| 0.982 | 20.9 | 20.77 |
| 0.722 | 15.2 | 15.15 |
| 0.627 | 13.2 | 13.16 |
| 0.518 | 10.7 | 10.68 |
| 0.420 | 8.8 | 8.78 |
| 0.330 | 6.9 | 6.89 |
| 0.214 | 4.5 | 4.49 |
| 0.113 | 2.3 | 2.3 |

II. The quantity is inversely proportional to the resistance of the secondary circuit.

| Resistance of Secondary. | $\frac{1}{R}$ | Observed Deflections. | Corrected Deflections. |
|---|---|---|---|
| 7000 | .0001429 | 35.6 | 35.15 |
| 17000 | .0000588 | 14.3 | 14.26 |
| 27000 | 370 | 9.1 | 9.08 |
| 37000 | 270 | 6.6 | 6.59 |
| 47000 | 213 | 5.2 | 5.19 |
| 57000 | 175 | 4.25 | 4.24 |
| 87000 | 115 | 2.78 | 2.77 |
| 107000 | 0934 | 2.2 | 2.2 |

The first and third columns of the first table and the second and fourth of the second table are plotted as coördinates in

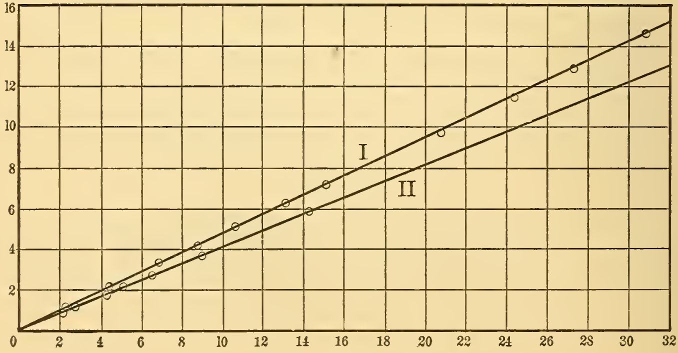



Fig. 123.

Fig. 123. The result in both cases is a straight line through the origin.

**123. Comparison of Two Mutual Inductances.**[1] — Let $A_1$, $A_2$ (Fig. 124) be the two coils whose mutual inductance $M_{12}$ is to be compared with the mutual inductance $M_{34}$ of the coils $A_3$, $A_4$. The coils $A_3$, $A_4$ are placed in the required relation to each other,

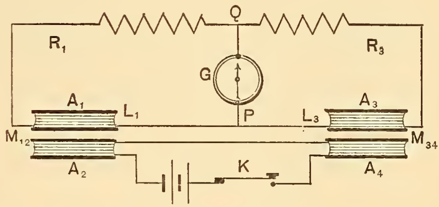


Fig. 124.

while $A_1$ and $A_2$ must be at such a distance from $A_3$ and $A_4$ that there is no mutual inductance between $A_1$ and $A_4$, nor between $A_2$ and $A_3$. The coils are joined in series as shown. Then the resistances of the branches containing $A_1$ and $A_3$ must be varied by the addition of non-inductive resistances till the galvanometer shows no deflection on closing and opening the key $K$. The sensibility will be increased by the use of the double commutator. When a balance has been obtained,

$$\frac{M_{12}}{M_{34}} = \frac{R_1}{R_3}.$$

The theory is as follows: Write the potential difference by the three paths between $P$ and $Q$ and place them equal to one another. Then

$$M_{34}\frac{di_4}{dt} - L_3\frac{di_3}{dt} - R_3 i_3 = M_{12}\frac{di_4}{dt} - L_1\frac{di_1}{dt} - R_1 i_1 = Ri + L\frac{di}{dt}.$$

$R$, $i$, and $L$ are the resistance, current, and inductance

[1] Maxwell's *Elec. and Mag.*, Vol. II., p. 364.

for the circuit through the galvanometer from $P$ to $Q$.

If we integrate this equation from $i_4 = 0$ to $i_4 = I$, or to the time of the establishment of a steady current, then

$$M_{34}I - L_3\int di_3 - R_3\int i_3dt = M_{12}I - L_1\int di_1 - R_1\int i_1dt = R\int idt + L\int di.$$

But $L_3\int di_3$ and $L_1\int di_1$ are both zero, since the currents $i_1$ and $i_3$ are zero when $i_4$ is zero, and when it is $I$, a maximum. Also, since the adjustment of resistances is so made that there is no integrated current through the galvanometer, the last two terms are both zero. Hence

$$M_{34}I - R_3\int i_3dt = M_{12}I - R_1\int i_1dt = 0,$$

and $$M_{34}I = R_3\int i_3dt\,;\ M_{12}I = R_1\int i_1dt.$$

Therefore

$$\frac{M_{12}}{M_{34}} = \frac{R_1}{R_3} \cdot \frac{\int i_1dt}{\int i_3dt}.$$

But since there is no integrated current through the galvanometer

$$\int i_1dt = \int i_3dt.$$

Hence $$\frac{M_{12}}{M_{34}} = \frac{R_1}{R_3}.$$

After a balance has been obtained the resistances $R_1$, $R_3$ may be measured by means of a Wheatstone's bridge. It is assumed in this discussion that $L_1$ and $L_3$ are both constants.

**124. Modification of Maxwell's Method of comparing Mutual Inductances.** — Let the resistance, self-induction, and current through coil $A_1$ (Fig. 125), including the galvanometer, be represented by $R_1$, $L_1$, $i_1$, respectively; and let the same quantities for the coil $A_3$ be denoted by $R_3$, $L_3$, and $i_3$. The resistances are to

be varied till the galvanometer shows no deflection on working the key $K$. Let the currents through $A_2$ and $A_4$ be $i_2$ and $i_4$ and their final steady values $I_2$ and $I_4$. Let $R$ be the resistance of the branch $AB$, and $S$ that of $PQ$. When the currents in $A_2$ and $A_4$ have reached their steady value

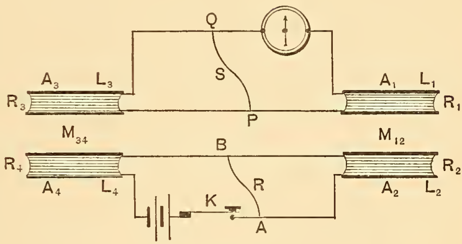



Fig 125.

$$I_4 = I_2 \frac{R + R_2}{R}.$$

Express the potential difference between $P$ and $Q$ by the three paths for any instant, and place their values equal to one another. Then

$$M_{12}\frac{di_2}{dt} - L_1\frac{di_1}{dt} - R_1 i_1 = M_{34}\frac{di_4}{dt} - L_3\frac{di_3}{dt} - R_3 i_3 = Si_1 - Si_3.$$

The electromotive force by the $A_3$ branch is arranged to be opposed to that of the $A_1$ branch.

Integrate from $t = 0$ to $t = T$ when the steady state has been attained in the battery circuit, and

$$M_{12}I_2 - L_1 \int di_1 - R_1 \int i_1 dt = M_{34}I_4 - L_3 \int di_3 - R_3 \int i_3 dt = S\int i_1 dt - S\int i_3 dt.$$

But $L_1 \int di_1$, $R_1 \int i_1 dt$, $L_3 \int di_3$, $S\int i_1 dt$ are all zero when a balance has been obtained, or when the galvanometer shows no integrated current through it when the circuit is opened or closed, or on reversing if the double commutator is used. Since the current is zero when $t = 0$ and

when $t = T$, the sum of the increments $di_1$ equals that of the decrements $-di_1$. Also the galvanometer shows the integrated current $i_1$ to be zero. So also the integral $\int di_3$ is zero because $i_3$ is zero both when $t = 0$ and when the steady state of $i_4$ has been attained. Hence we may write

$$M_{12}I_2 = M_{34}I_4 - R_3\int_0^T i_3 dt = - S\int_0^T i_3 dt.$$

Therefore, $$\int_0^T i_3 dt = -\frac{M_{12}}{S} I_2.$$

Remembering that $$I_4 = I_2\frac{R + R_2}{R},$$

$$M_{12}I_2 = M_{34}I_2\frac{R + R_2}{R} + \frac{R_3 M_{12}}{S} I_2.$$

Hence, $$\frac{M_{12}}{M_{34}} = \frac{R + R_2}{R}\cdot\frac{S}{S - R_3} = \frac{S}{R}\cdot\frac{R + R_2}{S - R_3}.$$

If $S$ is infinite, that is, if the branch $PQ$ is open,

$$\frac{M_{12}}{M_{34}} = \frac{R + R_2}{R}.$$

$R$, $R_2$, $R_3$, and $S$ must all be measured after the adjustment has been made for no deflection of the galvanometer.

**125. Carey Foster's Method of measuring Mutual Inductance.**[1] — The principle of the method is as follows: Let a constant battery be connected in series with one of the coils $P$, a known resistance $R$, and a key $K$. Let a ballistic galvanometer and another resistance $R'$ be connected in series with the other coil $S$. Then if $I$ be the steady current through $P$, $M$ the mutual inductance, and $r_0$ the resistance of the circuit through $S$, $R'$ and the galvanometer, the quantity of electricity

[1] *Phil. Mag.*, Vol. XXIII., p. 121.

passing through the galvanometer on closing or opening the circuit will be $Q = \frac{MI}{r_0}$ (Art. 122).

Next suppose the galvanometer removed from this circuit and put in series with a condenser of capacity $C$, connected as a shunt to the resistance $R$. On closing or opening the battery circuit the quantity of electricity $Q'$ passing through the galvanometer will be $Q' = IRC$. By combining these two equations it is possible to find the relative values of $C$ and $M$. It is better however to connect the apparatus as shown in Fig. 126, so that the charge and discharge of the condenser, and the currents generated at the same time in $S$ by mutual induction are in the same direction through $C$, $R'$, and $S$. If the resistances $R$ and $R'$

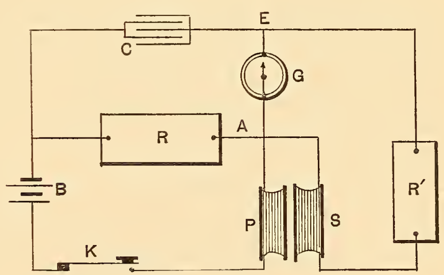



Fig. 126.

and the capacity $C$ are adjusted until there is no deflection of the galvanometer, the time integral of the galvanometer current until the steady current is reached will be zero, and the time integral $Qr$ of the current from $C$ through $R'$ and $S$, multiplied by the resistance $r$ of the same path from $E$ around to $A$, will be exactly equal to the time integral $MI$ of the electromotive force of mutual induction in the coil $S$. The time integral of the electromotive force of self-induction will be zero.

Therefore, $Qr = MI$. But $Q = IRC$.

Hence $$M = CRr.$$

The author of the method says that in order that the galvanometer current may be zero at every instant during the establishment of the steady current, it is essential that the coefficient of self-induction of the coil $S$ should be equal to the coefficient of mutual induction. Under this condition it is possible to replace the galvanometer by a telephone.

**Example.**

*Small Induction Coil.* — No iron core. Resistance of secondary, 194 ohms. Capacity of condenser, 4.926 microfarads. The secondary coil could slide endways remaining coaxial with the primary. The following are the results with the centres of the two coils as nearly coincident as possible:

| $R$. | $r$. | $Rr$ (C.G.S. units). |
|---|---|---|
| 15 | 194 + 217 | $6165 \times 10^{18}$ |
| 14 | + 247 | 6174 |
| 13 | + 282 | 6188 |
| 12 | + 322 | 6192 |
| 11 | + 367 | 6171 |
| 10 | + 423 | 6170 |
| 9 | + 490 | 6156 |
| 8 | + 576 | 6160 |
| 7 | + 688 | 6174 |
| 6 | + 835 | 6174 |

Mean value of $\dfrac{M}{C} = 6172.4 \times 10^{18}$.

Hence

$M = 4.926 \times 10^{-15} \times 6172 \times 10^{18} = 3.0403 \times 10^{7}$, or 0.0304 henrys.

In the same way the values of $M$ were obtained for the same pair of coils with the secondary displaced endways through various distances. The following results are given in Professor Foster's paper:

| Distance between centres of coils. | Value of $M$. | Distance between centres of coils. | Value of $M$. |
|---|---|---|---|
| 0.55 | $304.0 \times 10^5$ | 8.55 | $97.3 \times 10^5$ |
| 1.55 | 294.2 | 9.55 | 71.1 |
| 2.55 | 270.5 | 10.55 | 49.7 |
| 3.55 | 246.4 | 11.55 | 33.0 |
| 4.55 | 215.9 | 12.55 | 23.3 |
| 5.55 | 187.8 | 13.55 | 16.5 |
| 6.55 | 158.4 | 14.55 | 12.35 |
| 7.55 | 127.2 | 15.55 | 9.48 |

These values are represented graphically in the curve of Fig. 127, where the ordinates denote values of $M$ and the abscissas distances between the centres of the coils.

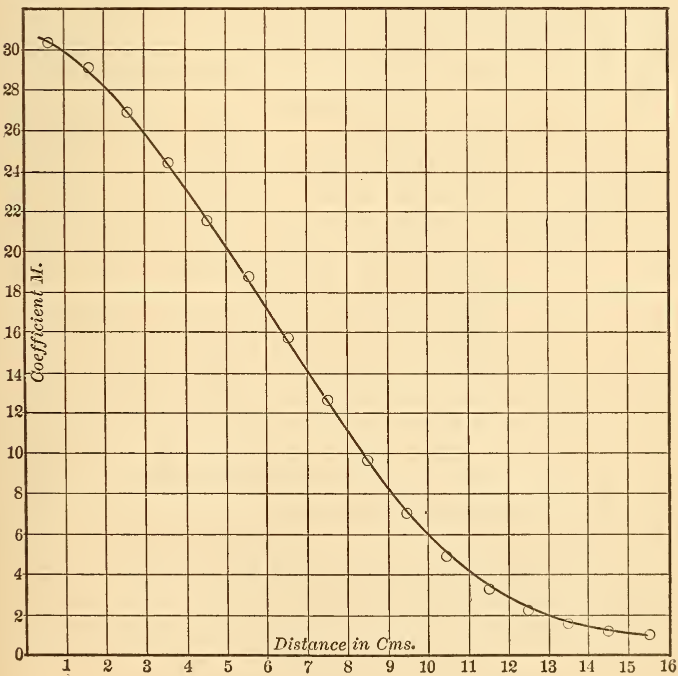



Fig. 127.

It is of interest to note that this curve is of the same form as that of Fig. 57. The mutual induction affecting the coil $S$ depends upon the number of lines of force passing through it at different distances from the primary coil $P$. In the same way the force deflecting the needle of the tangent galvanometer depends upon the magnetic field due to the coil at the several positions of the needle. The tangents of the deflections therefore follow the same law of variation as that of the mutual inductance at different distances.

**126. To compare the Mutual Inductance of Two Coils with the Self-Inductance of One of Them.**[1] — Let the coil of resistance $R_1$ and self-inductance $L$ be included in one branch $AC$ of a Wheatstone's bridge (Fig. 128) whose other branches are non-inductive. The other coil of the pair is put in the battery branch, and is so connected that the current flows in opposite directions through the two coils. The self-inductance of the coil $P$ therefore produces an electromotive force opposite in direction to that due to the mutual induction $M$ between $P$ and $Q$, and the one may be made to balance the other.

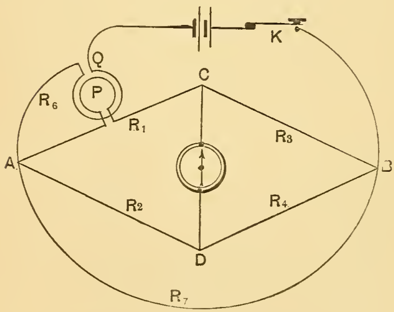


Fig. 128.

The resistances $R_1$, $R_2$, $R_3$, and $R_4$ are to be adjusted till there is a balance for steady currents. Then we may get rid of transient currents through the galvanometer

[1] Maxwell's *Elec. and Mag.*, Vol. II., p. 365.

by altering $R_2$ and $R_4$ in such a way that their ratio remains constant. There will then be neither transient nor permanent currents through the galvanometer.

Let the current from $A$ to $C$ be $i_1$, and that from $A$ to $D$, $i_2$. Then the current through $Q$ will be $i_1 + i_2$. The potential difference between $A$ and $C$ will be

$$R_1 i_1 + L\frac{di_1}{dt} - M\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right). \quad . \quad . \quad (1)$$

The potential difference between $A$ and $D$ is $R_2 i_2$. Since a balance is maintained between $C$ and $D$

$$R_2 i_2 = R_1 i_1 + L\frac{di_1}{dt} - M\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right). \quad . \quad . \quad (2)$$

But if $R_2$, $R_3$, and $R_4$ are inductionless resistances,

$$R_2 i_2 = R_1 i_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence

$$L\frac{di_1}{dt} - M\left(\frac{di_1}{dt} + \frac{di_2}{dt}\right) = 0. \quad . \quad . \quad . \quad . \quad (4)$$

From (3) $$\frac{di_2}{dt} = \frac{R_1}{R_2} \cdot \frac{di_1}{dt}.$$

Therefore from (4) $$L = M\left(1 + \frac{R_1}{R_2}\right). \quad . \quad . \quad (5)$$

The double adjustment of $R_2$ and $R_4$ may be avoided by joining $A$ and $B$ by $R_7$. Beginning with an adjustment in which the electromotive force due to self-induction is slightly in excess of that due to mutual induction, the latter may be augmented by diminishing the resistance $R_7$ till a balance is obtained for transient currents. This addition does not disturb the balance for steady currents.

Then the current through $Q$ will be $i_1 + i_2 + i_7$, and

$$L\frac{di_1}{dt} - M\left(\frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_7}{dt}\right) = 0. \quad . \quad . \quad (6)$$

But $$\frac{i_1}{i_7} = \frac{R_7}{R_1 + R_3} \text{ and } \frac{di_7}{dt} = \frac{R_1 + R_3}{R_7} \cdot \frac{di_1}{dt}.$$

From (6)

$$L\frac{di_1}{dt} = M\left(\frac{di_1}{dt} + \frac{R_1}{R_2}\frac{di_1}{dt} + \frac{R_1 + R_3}{R_7} \cdot \frac{di_1}{dt}\right),$$

or $$L = M\left(1 + \frac{R_1}{R_2} + \frac{R_1 + R_3}{R_7}\right). \quad . \quad . \quad (7)$$

This last method may be further improved by transferring the battery and key to the branch $R_7$. Then

$$L = M\left(\frac{R_1 + R_3}{R_6}\right). \quad . \quad . \quad . \quad . \quad (8)$$

To demonstrate this relation it will be seen that equation (5) is equivalent to

$$L = M\frac{I_6}{I_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This equation is true for all arrangements. In the last arrangement we need only find the ratio $\frac{I_6}{I_1}$. It is $\frac{(R_1 + R_3)}{R_6}$. Substitute in (9) and equation (8) is the result.

# CHAPTER VII.

## MAGNETISM.

**127. General Properties.** — Iron is not the only magnetic substance, for nickel, cobalt, and liquid oxygen are also very conspicuously magnetic; and probably there is no substance which is not susceptible to some extent to magnetic influence. In permanent magnets it has been noticed that there is a certain line through the centre of inertia which always takes a definite direction when the magnet is freely suspended at this point. This line is called the magnetic axis. In most localities this axis takes an approximately north and south direction, in the northern hemisphere the north-seeking end and in the southern hemisphere the south-seeking end pointing downward. In a simple elementary magnet the ends of the magnetic axis are called poles. In larger magnets the poles are not so definitely located. They might be defined as the centres of magnetic action resulting from the actual magnetization. In general they lie on the magnetic axis near its ends.

Until within a few decades the magnetization was considered as residing on the surface of the magnet near the ends, while the middle portion of the magnet was considered to be without influence. Since the time of Faraday the conception of lines of magnetic force and induction has to a considerable extent supplanted that of the poles. These lines of induction are closed curves.

The positive direction along them is by convention from the south-seeking or negative pole to the north-seeking or positive pole within the magnet, and *vice versa* without. Whenever these lines of induction meet a magnet, they tend to enter it by the negative and leave it by the positive pole. Magnetic action, from the point of view of lines of induction, goes on just as though these lines were stretched elastic cords mutually repelling one another. In polar language the same state of affairs is expressed by the law: Like poles repel and unlike poles attract one another with forces proportional directly to the product of the strength of the poles and inversely to the square of the distance separating them.

For certain purposes the conception of polar action at a distance is more convenient; and as the above law does not contradict actual experiment, we may avail ourselves of it, whenever it may be convenient to do so, without invalidating the results.

**128. Strength of Pole and Strength of Field.**—By convention we define as unity, a pole which repels an equal pole at a distance of one centimetre with the force of one dyne.

Strength of field at a point may be defined as the force exerted on a unit pole placed at that point. It is also the flux of magnetic force per square centimetre at that point. If this flux of force is represented by lines of force, the number of lines per square centimetre should equal the numerical value of the flux and of the strength of field. In a uniform field the lines of force are parallel straight lines. If a magnetic pole of strength $m$ be considered as located at a point $O$, the strength of field at all points on the surface of a sphere of unit radius with

$O$ as its centre will be numerically equal to the pole strength, and there will be $m$ lines of magnetic force per square centimetre of surface. There will be therefore in all $4\pi m$ lines from a pole of strength $m$. The letter $\mathcal{H}$ is generally used to designate strength of field.

**129. Intensity of Magnetization.** — When we are dealing with a magnet whose magnetization is solenoidal,[1] all lines of force pass from one end to the other without entering or leaving at the sides. In such cases the poles are at the ends and the intensity of magnetization $\mathcal{J}$ equals the strength of pole $m$ divided by the area of the pole $S$, or $\mathcal{J} = \frac{m}{S}$.

**130. Magnetic Moment.** — If a solenoidal magnet is placed at right angles to a uniform magnetic field of strength $\mathcal{H}$, the moment of the couple tending to turn it into parallelism with the field is $\mathcal{H}ml$; if $\mathcal{H}$ is unity the moment of this couple is called the magnetic moment of the magnet, and is designated by $\mathcal{M}$; or $ml = \mathcal{M}$. As the volume $V$ of the magnet equals $lS$, it follows that $\mathcal{J} = \frac{\mathcal{M}}{V}$; or intensity of magnetization equals magnetic moment per unit of volume. If the magnetization of the magnet is not solenoidal, $\mathcal{J}$ will not be uniform throughout the magnet and the magnetic moment will not be equal to $ml$, unless by $l$ we mean a distance shorter than the length of the magnet so chosen that $\mathcal{M}$ shall equal $ml$. The magnetic mo-

[1] *Solenoidal* is derived from the Greek word meaning pipe-shaped. The idea conveyed by the word is that the flow of magnetic induction is confined within the magnet, just as the flow of water is confined within a water-pipe. In all cases the flow is parallel to the sides of the solenoid.

ment, however, in any case will be equal to the moment of the turning couple, when the magnet is placed across a unit field so that its action on the magnet is greatest.

**131. Strength of Field within a Long Solenoid.** — Suppose an indefinitely long prism (or cylinder) to be uniformly and closely wound with many contiguous turns of insulated wire carrying a current of electricity, the direction of the wire being at every point as nearly as possible perpendicular to the edges of the prism. By reason of symmetry the resultant field within the coil at points indefinitely distant from the ends must be parallel to the edges (or to the axis in the case of a cylinder). Approaching the ends, the resultant field will begin to weaken and there will be a scattering of the lines of force. At points indefinitely distant from the ends all lines of force are therefore parallel, the equipotential surfaces are equidistant planes perpendicular to them, and the field is uniform. Consequently, to find the value of the field for all points it is only necessary to find it for any one. Suppose there are $n$ turns of wire per centimetre of length, each carrying a current $I$; suppose a unit north pole carried one centimetre along the lines of force; each of the $4\pi$ lines of force proceeding from this pole will cut $n$ turns of wire, thus producing an electromotive force in the solenoid of $4\pi n$, and the work done on the pole will be $4\pi nI$; consequently the force opposing the movement of the pole will be equal to the strength of the field, and $\mathcal{H} = 4\pi nI$. $I$ is here in C.G.S. units each equal to 10 amperes; consequently if $I$ is expressed in amperes, $\mathcal{H} = \frac{4\pi nI}{10}$.

If, instead of a prism, the form on which the wire is

wound is described by the revolution of a closed plane curve about an outside axis in its plane, and if the wire wound about it for each turn coincides as nearly as possible with the generatrix, the resultant field will at every point lie along circumferences described about this axis. The intensity of the field at any point within will be $4\pi nI$ as before, $n$ being computed along the corresponding circumference. The field $\mathcal{H}$ will, as a consequence, not be uniform, but it will be absolutely solenoidal, there being no ends to cause a scattering of the flux.

132. **Magnetic Induction.** — Let us suppose a long iron bar placed in a uniform magnetic field of strength $\mathcal{H}$ so as to be parallel to the lines of the field. The portion of the bar which is distant from the ends will have its lines of induction parallel to its axis. Suppose the portion of the iron included between two adjacent planes normal to the axis to be removed. Let the flux of magnetic force passing across this crevasse per square centimetre be $\mathcal{B}$, and let the pole strength per square centimetre of the faces be $\mathcal{J}$. Then $\mathcal{B} = 4\pi\mathcal{J} + \mathcal{H}$, for each unit of pole strength will furnish a flux of $4\pi$ in addition to the preëxistent flux of $\mathcal{H}$ per square centimetre. Even when the above conditions are not fulfilled, the relation $\mathcal{B} = 4\pi\mathcal{J} + \mathcal{H}$ is true in a vector sense. In the cases with which we shall deal, $\mathcal{H}$ will be parallel to $\mathcal{B}$ either in the same or in the opposite direction; then $\mathcal{B} = 4\pi\mathcal{J} \pm \mathcal{H}$.

This flux of force in the crevasse continues as a flux of induction inside the iron. In the crevasse it may be called indifferently a flux of force or of induction. Consequently lines of induction are continuous throughout the magnetic circuit.

Such a uniform field as is premised above may be produced by a long solenoid surrounding the bar.

For practical purposes it is sometimes more convenient to use a ring of iron instead of a bar. In such a case a ring-solenoid is used to produce a field parallel to the circumference of the ring. To avoid errors due to variations in permeability of the iron when in fields of different values, the difference between the outer and inner radii of the iron ring should be small in comparison with either. In such cases the value of $\mathcal{H}$, computed along the mean circumference, may be taken as the mean value for the ring without sensible error.

**133. Magnetic Susceptibility and Permeability.** — The ratio of the intensity of magnetization $\mathcal{J}$ to the strength of the field $\mathcal{H}$ is called the magnetic susceptibility of the substance. It is denoted by the Greek letter $\kappa$. Thus $\kappa = \frac{\mathcal{J}}{\mathcal{H}}$. It follows that $\mathcal{B} = \mathcal{H}\,(1 + 4\pi\kappa)$ and $\kappa = \frac{(\mathcal{B} - \mathcal{H})}{4\pi\mathcal{H}}$.

For many reasons it is more convenient to know the ratio between $\mathcal{B}$ and $\mathcal{H}$, rather than that between $\mathcal{J}$ and $\mathcal{H}$. This ratio is called the magnetic permeability of the substance, and is denoted by the Greek letter $\mu$. Thus $\mu = \frac{\mathcal{B}}{\mathcal{H}}$.

**134. Coercive Force.** — When an iron bar or ring has been magnetized, it has been noticed that a large portion of the magnetization is retained when the magnetizing force has been removed. In a paper by Houston

and Kennelly[1] the theory has been advanced that the residual magnetization is a linear function of the induced magnetization. This theory is based on calculations made from data given in Ewing's *Magnetic Induction in Iron and Other Metals*. Calling $\mathcal{H}$ the intensity of field, $\mathcal{B}$ the resulting magnetic flux, and $\mathcal{B}_0$ the residual magnetic flux, they found for annealed soft iron wire $\mathcal{B}_0 = 0.88\ (\mathcal{B} - 500)$. Other samples of iron and steel give different values for these constants, but in every case the linear relation seems to be true.

The term coercive force has been loosely used to express this tendency to oppose change in the magnetic state. Hopkinson uses the term to denote the intensity of field which will just restore the iron to an apparently neutral condition.

**135. Effect of the Ends of a Bar.** — When an iron bar is magnetized longitudinally in a uniform field, the ends become poles. The effect of these poles is to produce a field at all intermediate points of the bar, whose tendency is to demagnetize it. If the length of the bar is at least fifty times its breadth, it is assumed in practice that the bar is equivalent to a very prolate ellipsoid whose axes correspond to the length and diameter of the bar. The effect of the ends, on this assumption, is given by the following equation, $\mathcal{H} = \mathcal{H}' - N\mathcal{J}$, in which $\mathcal{H}$ is the actual field, $\mathcal{H}'$ the original field, and $N\mathcal{J}$ the effect of the ends. Values for $N$ and $\frac{N}{4\pi}$ are given in the following table:[2]

[1] *Electrical World*, June 1, 1895, p. 631.

[2] Ewing's *Mag. Ind. in Iron and Other Metals*, p. 32.

| Ratio of length to breadth. | $N$. | $\frac{N}{4\pi}$. |
|---|---|---|
| 50 | 0.01817 | 0.001446 |
| 100 | 0.00540 | 0.000430 |
| 200 | 0.00157 | 0.000125 |
| 300 | 0.00075 | 0.000060 |
| 400 | 0.00045 | 0.000037 |
| 500 | 0.00030 | 0.000024 |

By using a ring instead of a rod this correction is avoided.

**136. Magnetic Inclination or Dip by the Dip-Needle.**[1] — Magnetic inclination or dip is the angle which the direction of the earth's magnetic force makes with the horizontal. This direction is given by the magnetic needle if it is movable without friction about an axis at right angles to itself and to the magnetic meridian, if, first, the axis passes through the centre of inertia of the needle, and if, second, its magnetic axis is coincident with its geometric axis. These conditions, however, will in general fail to be satisfied. As a consequence the mode of observation described below is required.

The dip circle is a vertical circle movable about a vertical diameter; the zeros of graduation should be at the extremities of a horizontal diameter. The circle should be in the plane of the magnetic meridian. A long, slender compass-needle may be used in making this adjustment. If the axis of rotation of the circle is vertical, the bubble of a level will not change on turning the circle. The axis of rotation of the needle should pass

[1] Kohlrausch's *Physical Measurements*, 3d English Edition, p. 235. Maxwell's *Elec. and Mag.*, Vol. II., p. 113.

normally through the centre of the graduated circle. Four sets of observations are taken, in each of which both ends of the needle are read and the mean, called the observed angle, is taken: first, $\phi_1$, the original position of the needle; second, $\psi_1$, with the reading and the movable circle turned 180°; third, $\phi_2$, with the needle's magnetization reversed and otherwise as in the first set; fourth, $\psi_2$, with the needle and the movable circle turned 180° again.

If the apparatus is good and the observations carefully made, these four observed angles will be much alike and the angle of dip $\delta$ is expressed as follows:

$$\delta = \frac{\phi_1 + \psi_1 + \phi_2 + \psi_2}{4}.$$

If they differ much, it is possible by grinding the side of the needle to make $\phi_1$ and $\psi_1$ nearly alike and the same for $\phi_2$ and $\psi_2$. Then

$$\tan \delta = \frac{1}{2}\left(\tan \frac{\phi_1 + \psi_1}{2} + \tan \frac{\phi_2 + \psi_2}{2}\right).$$

If this is not done and $\phi_1$ and $\psi_1$ differ considerably, we should write

$$\cot a_1 = \frac{1}{2}(\cot \phi_1 + \cot \psi_1),$$

$$\cot a_2 = \frac{1}{2}(\cot \phi_2 + \cot \psi_2);$$

and finally

$$\tan \delta = \frac{1}{2}(\tan a_1 + \tan a_2).$$

These expressions are obtained by considering the gravitational forces at work resolved into components parallel and perpendicular to the magnetic axis.

Maxwell's method takes into account the relative intensities of magnetization in both cases. Calling these $D_1$ and $D_2$ and using the same notation as above,

$$\delta = \frac{D_1(\phi_1 + \psi_1) + D_2(\phi_2 + \psi_2)}{2(D_1 + D_2)}.$$

For fuller explanations the student is referred to Kohlrausch and Maxwell as cited above.

**137. Magnetic Inclination by Weber's Earth-Inductor.** — When a conductor is moved in a magnetic field so as to cut lines of force, the time integral of the electromotive force generated is equal, in C.G.S. measure, to the number of lines cut. If the conductor is a plane coil of wire of total area $S$ which makes angles $\phi_1$ and $\phi_2$, before and after the movement, with the direction of the lines of force of a magnetic field of intensity $\mathcal{F}$, we obtain the equation $\int edt = S\mathcal{F}(\sin\phi_1 - \sin\phi_2)$. It is necessary to count $\phi$ from 0° to 360°.

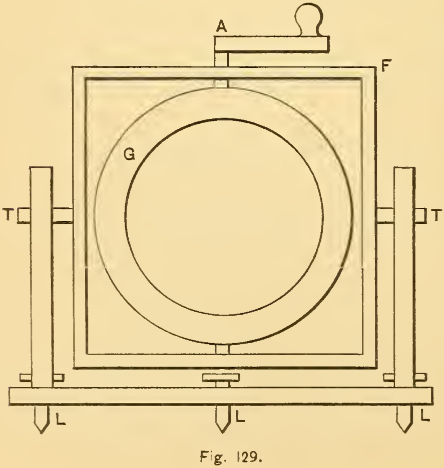


Fig. 129.

In Weber's earth-inductor (Fig. 129) the coil of wire $G$ is usually mounted on an axis $A$ in its plane. This axis is supported by a frame $F$ mounted on two trunnions $T$, whose axis makes

a right angle with the first axis. The trunnions are carried on supports fastened to a platform resting on three levelling screws $L$. For the purposes of this experiment the axis $T$ should be level and in a magnetic east and west line. On the frame are stops which, as generally used, limit the angle through which the coil may be turned to 180°. Some earth-inductors are turned by hand and others are turned by means of springs on the removal of a detent.

The earth-inductor should be joined in series with a ballistic galvanometer of long period of oscillation, and, if need be, with a coil of suitable resistance. On turning the coil through 180° an inductive impulse will be felt in the galvanometer. The sine of one-half the throw of the galvanometer needle will be proportional to the quantity of electricity passing through the circuit. Three methods may be used in producing the deflection. In the first a single reversal of the coil gives a single impulse to the needle. In the second the coil is reversed each time the needle passes through its position of equilibrium, giving it successive impulses until no further increase in its amplitude is obtained. In the third the coil is reversed every second time that the needle reaches its position of equilibrium; as a consequence the impulse causes the needle to recoil, it then reaches its maximum amplitude, then passes through zero to a smaller amplitude, owing to the damping, and on reaching zero recoils, as the coil is reversed, to another maximum amplitude in the opposite direction. This is continued until the arcs of the amplitudes reach constant values $a$ and $b$.

In the first and second methods the quantities of electricity and the time integral of the electromotive force are proportional to the sines of one-half the angles

of deflection; in the third they are proportional to $\sin \frac{a^2 + b^2}{2\sqrt{a \cdot b}}$.[1] For small deflections the scale deflections may be taken proportional to the quantities of electricity passing and to the time integral of the electromotive force. To reduce deflections to sines of one-half the angles, use Table I. in the Appendix.

Several precautions are to be taken in the use of the earth-inductor. As it is assumed that the magnetic field is uniform and of constant direction in the neighborhood of the coil, the presence of masses of iron and particularly of magnets should be avoided. The powerful magnets usually in voltmeters and ammeters will noticeably affect the lines of force for a distance of several metres. A result obtained with an earth-inductor is valuable only in the place in which it is obtained; even in the same room considerable variations may be found. In some cases it may be due to the iron in the red brick walls and foundations for piers. Besides the magnetic disturbances within our control, there are the daily and yearly variations, of which account should be taken in very exact work.

In the determination of magnetic inclination we may make use of a familiar principle, that the direction of a vector or directed quantity is completely defined by the cosines of the angles included between the line of the vector and the three rectangular axes of coördinates passing through the point. The component of the vector along each of the axes is found by multiplying the whole vector by the corresponding cosine. In the present problem the conditions are chosen so that one component is zero, and the vector, the intensity of the earth's field,

---

[1] Kohlrausch's *Phys. Meas.*, 3d English Edition, p. 351.

lies parallel to the plane of the other two. The induction impulses obtained by reversing the coil are then proportional to the vertical and horizontal components $\mathcal{V}$ and $\mathcal{H}$, and, as a consequence, to the cosines of the angles between the lines of force and a plumb line, and a horizontal magnetic north and south line respectively. These last quantities are also the sine and cosine of the inclination, and their ratio, equal to $\frac{\mathcal{V}}{\mathcal{H}}$, is the tangent of the magnetic inclination or dip.

*First Position*, $\mathcal{V}$. Place the earth-inductor so that the plane of the coil is horizontal and the axis $A$ in the magnetic north and south line. An ordinary level and a long, slender compass-needle will suffice to secure these adjustments. The second condition is desired, as it prepares the apparatus for the second position. On reversing the coil the number of lines of force cut is proportional to the vertical component of the earth's field. Observations may be taken in any of the ways mentioned above. These observations should be repeated several times and the mean determined.

*Second Position*, $\mathcal{H}$. Turn the frame $F$ through 90°. The axis of the trunnions should be in a horizontal magnetic east and west line and the axis $A$ vertical. The plane of the coil should now be vertical and at right angles to the magnetic meridian. The coil should be tested for these conditions with the plumb line and the compass needle. On reversing the coil the number of lines of force cut is proportional to the horizontal component of the earth's field. Several sets of observations should be taken as in the first position and the mean determined.

*Calculation of the Ratio of* $\mathcal{V}$ *to* $\mathcal{H}$. Strictly speaking,

the deflections should be reduced to the sines of one-half the angles and their ratio taken. When, however, the deflections are not great, we may use in the place of the ratio of these sines

$$\frac{d_1 - \frac{11}{32}\frac{d_1^3}{a^2}}{d_2 - \frac{11}{32}\frac{d_2^3}{a^2}} = \tan\delta,$$

where $d_1$ and $d_2$ are the scale deflections in the first and second positions, $a$ the distance of the scale from the mirror, and $\delta$ the angle of dip.

**138. Determination of the Horizontal Intensity of the Earth's Magnetic Field.** — The following method of determining $\mathcal{H}$ is due to Gauss. It depends on the measurement of two quantities, viz., the product and the ratio of the horizontal intensity $\mathcal{H}$ of the earth's field and the magnetic moment $\mathcal{M}$ of a particular magnet.

*First, to find the Product of $\mathcal{M}$ and $\mathcal{H}$.* Suppose the magnet $AB$ suspended, in the place where $\mathcal{H}$ is to be measured, by a bundle of long silk fibres; a suitable fine wire may replace the silk fibres. To ensure freedom from torsion a "dummy" magnet of brass, of weight equal to $AB$, may be hung on the fibres and the torsion head turned until the dummy lies in the magnetic meridian. Let the magnet so suspended be made to execute torsional vibrations. Let $T$ be the half period and $K$ be the moment of inertia of the magnet, and let $\theta$ be the ratio of torsion of the fibres; then for small amplitudes

$$T = \pi\sqrt{\frac{K}{\mathcal{M}\mathcal{H}(1+\theta)}}. \quad . \quad . \quad . \quad (1)$$

This value of $T$ should be reduced to the value corre-

sponding to an infinitesimal arc, by Table III. in the Appendix.

The torsional vibrations may be produced by repeatedly presenting first one and then the other pole of a strong magnet at a considerable distance from the suspended magnet. If the change of pole is properly timed, the swing may be greatly multiplied. Conversely if the magnet is swinging, it may be brought to rest by presenting the poles alternately so as to oppose the motion. This magnet should of course be removed to a great distance before the final observations are made.

By $\theta$, the ratio of torsion, is meant the ratio between the restoring forces due to the torsion of the fibres and to the action of the magnetic field respectively, when the magnet is slightly deflected from the magnetic meridian. If the tops of the fibres are held by a graduated torsion head, and the magnet carries a light mirror, to be used in connection with a telescope and scale, the ratio of torsion may be readily measured by turning the torsion head through an angle $a$, thereby turning the magnet and its mirror through an angle $\beta$. To avoid troublesome corrections $\beta$ should be so small that it does not differ materially from its sine. If equilibrium is obtained, $\theta = \dfrac{\beta}{(a-\beta)}$.

To find the value of $\mathcal{M} \cdot \mathcal{H}$ it is necessary to know the moment of inertia $K$ of the magnet, equation (1). If this cannot be calculated from its dimensions, it may be determined experimentally as follows: Take a ring of mass $M$ and outer and inner radii $a_1$ and $a_2$. Its moment of inertia about its axis is $\frac{1}{2}M(a_1^2 + a_2^2) = K'$. Place this ring upon the magnet with its centre in the line of support. Determine $T_1$, the half period of

vibration of the system, and correct to an infinitely small arc. Then

$$T_1 = \pi \sqrt{\frac{K+K'}{\mathfrak{M}\mathfrak{H}(1+\theta)}} . \quad . \quad . \quad . \quad (2)$$

By combining (1) and (2) we obtain

$$\mathfrak{M} \cdot \mathfrak{H} = \frac{\pi^2 K}{(1+\theta)(T_1^2 - T^2)} = P. \quad . \quad . \quad (3)$$

*Second, to find the Quotient of $\mathfrak{M}$ divided by $\mathfrak{H}$.* There are two methods of determining this ratio; in both we combine with the earth's magnetic field at $O$

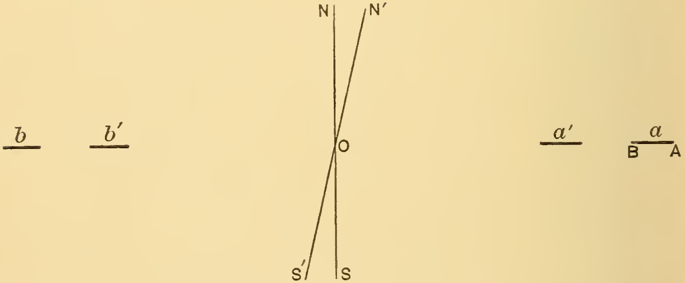



Fig. 130.

(Figs. 130, 131), when $\mathfrak{H}$ is being determined, a field $\mathfrak{F}$ due to the magnet $AB$, which in both cases has its magnetic axis east and west. In the first case the point $O$ is on the prolongation of the magnetic axis of $AB$; in the second it is on the perpendicular to the middle point of this axis. In both cases the field $\mathfrak{F}$ at $O$, due to $AB$, is directed along the magnetic east and west line. The direction of the resultant of $\mathfrak{H}$ and $\mathfrak{F}$ is indicated by $N'S'$. For convenience in deducing the expressions for $\mathfrak{F}$, more detailed sketches of the positions $a$ (Figs. 130,

131) are given in Figs. 132, 133. The point $O$ is at the middle point of $ns$.

*First Method.* Let the magnet $AB$, used in the determination of $\mathfrak{M} \cdot \mathfrak{H}$, be placed with its positive pole to the east and with its centre at a distance $r$ from $O$ (position $a$, Fig. 130). Suppose the magnet $AB$ to produce a certain deflection of the magnet $ns$. Reverse $AB$; the deflection should now be equal and opposite to its first value. Next place the magnet at an equal distance to the west of $O$, and obtain deflections with the positive and negative poles respectively directed toward $O$ (position $b$, Fig. 130). A pair of deflections equal to the first pair should now be obtained. Call the mean of these four deflections $\phi$. Repeat these observations with $AB$ at a distance $r'$ from $O$ (positions $a'$ and $b'$, Fig. 130). Call the mean of the deflections in this position $\phi'$. Kohlrausch says,[1] "In order that the errors of observation may have the least possible influence on the result, it is best that the ratio of the two distances $\frac{r}{r'}$ should equal 1.4;" Gray says[2] 1.32. The distance $r'$ should be at least from three to five times the length of $AB$.


Fig. 131.

Combining these two sets of measurements,

$$\frac{\mathfrak{M}}{\mathfrak{H}} = \frac{1}{2} \cdot \frac{r^5 \tan \phi - r'^5 \tan \phi'}{r^2 - r'^2} = Q_1 . \quad . \quad (4)$$

*Second Method.* Let $AB$ be placed in the position $a$

---

[1] *Phys. Meas.*, 3d English Edition, p. 243.

[2] *Absol. Meas. in Elect. and Mag.*, Vol. II., Part I., p. 93.

(Fig. 131) with its positive pole to the east, and observe the deflection of $ns$; reverse $AB$ and observe the deflection; repeat in position $b$; continue the observations for positions $a'$ and $b'$. Using the same notation as above,

$$\frac{\mathfrak{M}}{\mathcal{H}} = \frac{r^5 \tan\phi - r'^5 \tan\phi'}{r^2 - r'^2} = Q_2. \quad . \quad (5)$$

When the first method is used $\mathcal{H}$ may be obtained by dividing (3) by (4) and extracting the square root. Thus

$$\mathcal{H} = \sqrt{\frac{P}{Q_1}} = \pi\sqrt{\frac{2K(r^2 - r'^2)}{(1+\theta)(T_1^2 - T^2)(r^5\tan\phi - r'^5\tan\phi')}}.$$

When the second method is used

$$\mathcal{H} = \sqrt{\frac{P}{Q_2}} = \pi\sqrt{\frac{K(r^2 - r'^2)}{(1+\theta)(T_1^2 - T^2)(r^5\tan\phi - r'^5\tan\phi')}}.$$

*Proof.* Suppose the magnet $AB$ to have its poles of strength $m_1$ at a distance $l_1$ apart. Find the force acting on a unit pole at a distance $r$ from the centre of $AB$, along its magnetic axis.

*First Method.* Let the negative pole of $AB$ (Fig. 132)

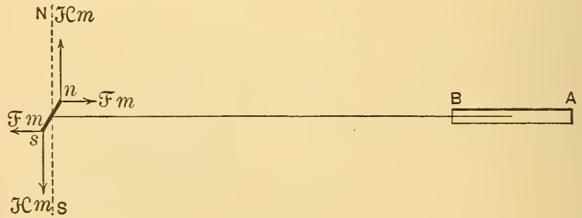



Fig. 132.

be toward the west; the force at $O$ due to it will be $\dfrac{m_1}{(r - \frac{1}{2}l_1)^2}$, directed toward the east. The force due to

the positive pole will be $\frac{m_1}{(r+\frac{1}{2}l_1)^2}$, directed toward the west. The total force on unit pole will then be

$$\mathcal{F} = m_1\left(\frac{1}{(r-\frac{1}{2}l_1)^2} - \frac{1}{(r+\frac{1}{2}l_1)^2}\right) = \frac{2m_1l_1}{r^3\left(1-\frac{l_1^2}{4r^2}\right)^2} =$$

$$\frac{2\mathfrak{M}}{r^3}\left(1+\frac{l_1^2}{2r^2}+\frac{3l_1^4}{16r^4}+\text{etc.}\right)$$

directed toward the east. Neglecting higher powers than the second in the expansion, this may be written

$$\mathcal{F} = \frac{2\mathfrak{M}}{r^3}\left(1+\frac{c}{r^2}\right). \quad . \quad . \quad . \quad (8)$$

*Second Method.* In this method let the positive pole of $AB$ (Fig. 133) be toward the east. Then the force on a unit negative pole at a distance $r$ north from the middle of $AB$, due as before to the negative pole of $AB$, will be $\frac{m_1}{\left(r^2+\frac{l_1^2}{4}\right)}$ directed from $B$. The force due to the positive pole will be the same in magnitude, and will be directed toward $A$. For convenience in drawing Fig. 133, it has been assumed that the poles are at the ends of $AB$. In reality they should be further back. Resolving these forces into south and east and north and east compo-

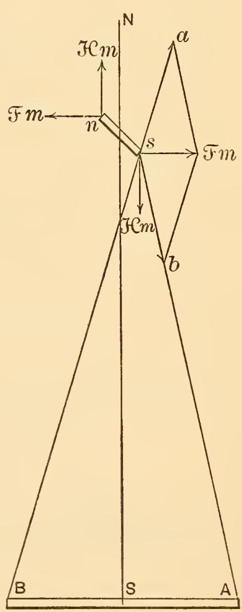



Fig. 133.

nents, we find that the north and south components annul one another, and the east components produce a force on unit negative pole,

$$\mathcal{F} = \frac{m_1 l_1}{\left(r^2 + \frac{l_1^2}{4}\right)^{\frac{3}{2}}} = \frac{\mathcal{M}}{r^3}\left(1 - \frac{3l_1^2}{8r^2} + \text{etc.}\right)$$

directed toward the east. As before, this may be written without sensible error,

$$\mathcal{F} = \frac{\mathcal{M}}{r^3}\left(1 - \frac{c'}{r^2}\right). \quad . \quad . \quad . \quad . \quad (9)$$

Returning now to the first method, we may suppose a short magnet $ns$ (Fig. 132) of length $l$ and pole strength $m$ suspended at $O$. Call the deflection produced by $AB$ $\phi$. Then for equilibrium the moments of the two couples acting on $ns$ must be equal, or

$$\mathcal{H} ml \sin\phi = \mathcal{F} ml \cos\phi.$$

Therefore

$$\mathcal{H} \tan\phi = \mathcal{F} = \frac{2\mathcal{M}}{r^3}\left(1 + \frac{c}{r^2}\right). \quad . \quad . \quad (10)$$

When $AB$ is in position $a'$ we have

$$\mathcal{H} \tan\phi' = \frac{2\mathcal{M}}{r'^3}\left(1 + \frac{c}{r'^2}\right). \quad . \quad . \quad (11)$$

Eliminating $c$ from (10) and (11) we obtain

$$\frac{\mathcal{M}}{\mathcal{H}} = \frac{1}{2}\,\frac{r^5 \tan\phi - r'^5 \tan\phi'}{r^2 - r'^2}.$$

In a similar way for the second method we find equilibrium of the moments of the two couples when

$$\mathcal{H} \tan\phi = \frac{\mathcal{M}}{r^3}\left(1 - \frac{c'}{r^2}\right), \quad . \quad . \quad (12)$$

and $$\mathcal{H} \tan \phi' = \frac{\mathcal{M}}{r'^3}\left(1 - \frac{c'}{r'^2}\right); \quad . \quad . \quad (13)$$

which give on eliminating $c'$

$$\frac{\mathcal{M}}{\mathcal{H}} = \frac{r^5 \tan \phi - r'^5 \tan \phi'}{r^2 - r'^2}.$$

*Correction for Induced Magnetization.* In the measurement of $\mathcal{M} \cdot \mathcal{H}$ the magnet is suspended in the earth's field in such a way that its magnetic moment is increased by induction. In very exact work a correction should be made for this change. This increase may be approximately estimated by the rule that the magnetic moment $\mathcal{M}$ is increased by $\frac{1}{4}\mathcal{H}$ per gramme of steel.[1]

*Precautions.* As the value of $\mathcal{H}$ is constantly changing, and as $\mathcal{M}$ for a magnet is affected by a temperature coefficient, besides being liable to be permanently changed by shocks or blows, or by contact with or even proximity to other magnets or large masses of iron, it is advisable that the whole experiment be performed consecutively. It is unnecessary to add that no iron or other magnetic substance near by should be moved during the experiment. In general the place in which magnetic measurements are made should be free from the presence of unnecessary iron. Iron pipes for water, gas, or steam, iron window weights, iron telescope bases, etc., should be replaced by others made of non-magnetic metals.

**139. Measurement of Intensity of Magnetization, Magnetic Induction, Permeability, and Susceptibility.** — When a magnetic substance is undergoing tests with

[1] Kohlrausch's *Phys. Meas.*, 3d English Edition, p. 245.

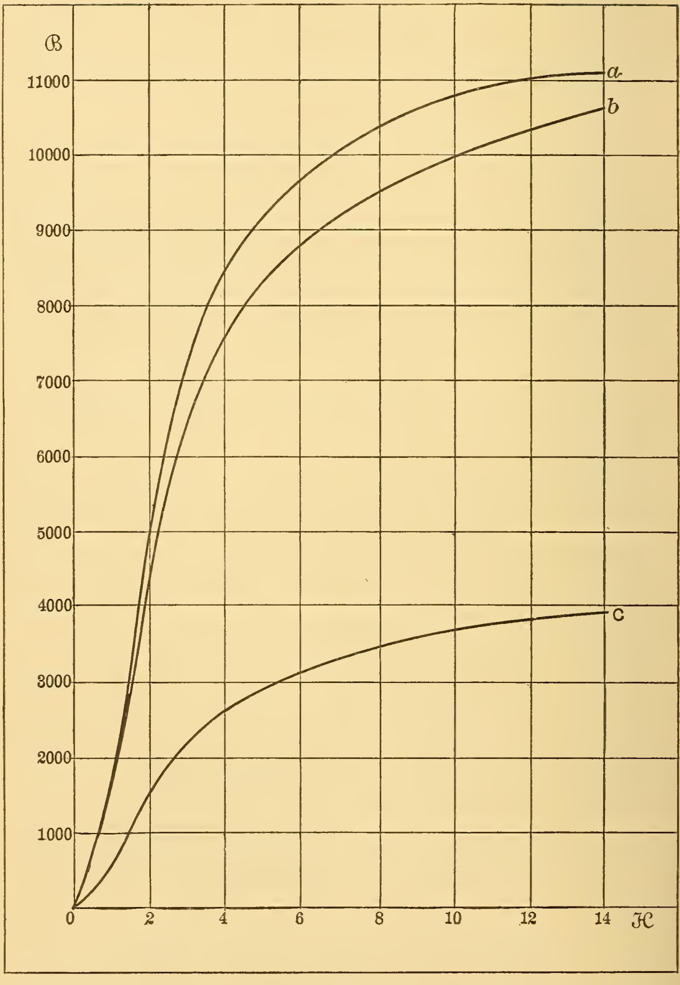


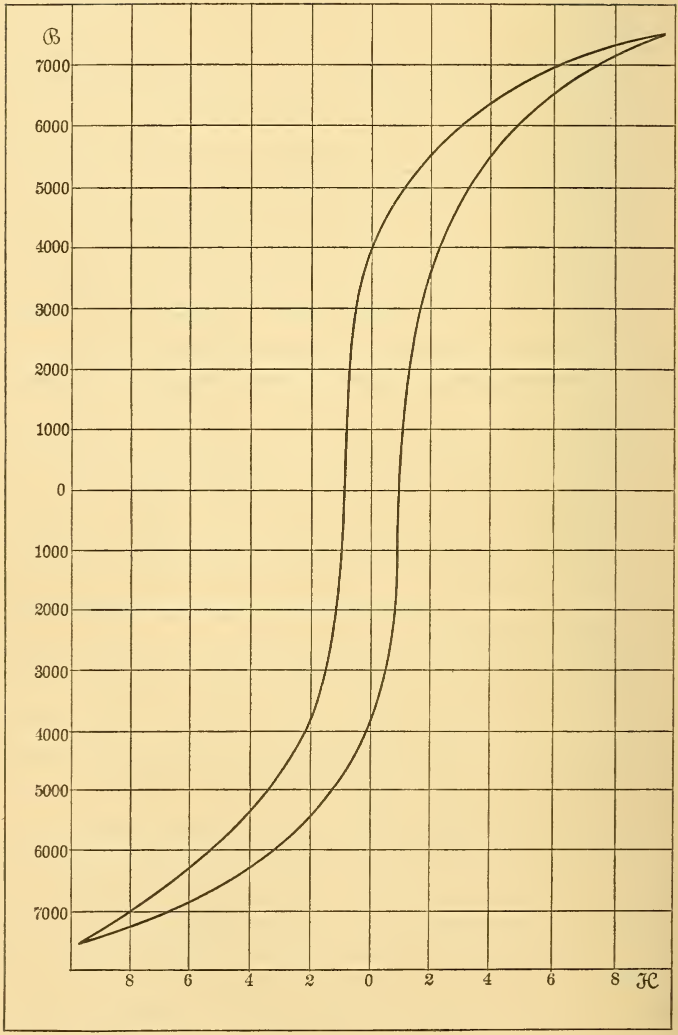

Fig. 134.

reference to its magnetic qualities, it is usual to determine the effect of various magnetic fields in producing

magnetic induction $\mathcal{B}$ in the substance. From the data obtained it is possible to calculate the magnetic permeability $\mu$ of the substance, the intensity of magnetization $\mathcal{J}$, and the magnetic susceptibility $\kappa$. $\mathcal{J}$ and $\kappa$ are little used, but for many reasons the ideas conveyed by these symbols are still useful.

When a piece of previously unmagnetized iron is placed in a magnetic field whose intensity $\mathcal{H}$ is raised uniformly from zero, it is found that the magnetic induction increases at first slowly, then by degrees more rapidly, until a maximum rate of increase is reached; beyond this point the rate decreases toward a constant quantity, which equals the rate of increase of $\mathcal{H}$ as a limit, while $\mathcal{J}$ approaches a maximum. If the piece of iron has been previously magnetized it may be demagnetized by heating to a red heat, or by a process of reversals with gradually decreasing field strength. Curves *a*, *b*, and *c* (Fig. 134) represent the relation of $\mathcal{B}$ to $\mathcal{H}$ under such circumstances for mild steel, wrought iron, and cast iron, respectively. The values of the quantities are in C.G.S. units. The data for these curves were obtained by experiments on rings, using the method of reversals (Art. 145), which does not require the demagnetization to be absolutely complete on starting the tests.

When the intensity of the field is increased by steps from zero to some definite value, decreased from that value to zero, increased in the opposite sense to the same numerical maximum value as before, again decreased to zero and the cycle repeated, the curve representing the relation of $\mathcal{B}$ to $\mathcal{H}$ after the first quarter cycle is similar to that shown in Fig. 135, which was obtained from experiments on a cast-iron ring. The



ℬ
7000
6000
5000
4000
3000
2000
1000
0
1000
2000
3000
4000
5000
6000
7000
8
6
4
2
0
2
4
6
8
ℋ


Fig. 135.

first quarter-cycle, not shown in the figure, might have been represented by a curve similar to those of Fig. 134. It will be noticed that the values of $\mathcal{B}$ corresponding to decreasing values of $\mathcal{H}$ are very much greater than those corresponding to the same values of $\mathcal{H}$ when increasing. This magnetic lag in the values of $\mathcal{B}$, which is due to the tendency of the iron to oppose changes in its magnetic condition, has received the name of magnetic hysteresis. As curves of cyclical magnetization show hysteresis, they are commonly called hysteresis curves. The amount of energy expended in each cubic centimetre of iron per cycle because of hysteresis is

$$W=\frac{1}{4\pi}\int\mathcal{H}d\mathcal{B}=\frac{\text{Area of Curve}}{4\pi}$$

when the curve is plotted to scale.

Four well-known methods have been used to determine the relation of $\mathcal{H}$ to $\mathcal{B}$, $\mathcal{J}$, $\mu$, and $\kappa$; the optical method, used by du Bois,[1] depending upon the phenomenon discovered by Dr. Kerr,[2] that when plane polarized light is reflected by a magnet pole, the plane of polarization is turned through an angle depending upon the intensity of magnetization; the magnetometric method; the tractional method; and, finally, the ballistic method. Of these the first will not be considered here further, inasmuch as it requires a practised experimenter to obtain good results.

**140. The Magnetometric Method.** — This method is applicable to open magnetic circuits only. The theory of the method is similar to that used in the determination of the horizontal component of the earth's field, which

[1] *Phil. Mag.*, March, 1890, April, 1890.

[2] B. A. *Report*, 1876, p. 40; *Phil. Mag.*, May, 1877.

in this section will be designated by $\mathcal{H}_e$. The magnetometer consists essentially of a small magnet suspended by a fibre of little torsion. In order that the deflections may be read, the magnet carries with it a light mirror. In practice the fibre may be attached to a mirror on the back of which several small magnets are cemented. If the larger magnet used in the determination of $\frac{\mathcal{M}}{\mathcal{H}_e}$ be replaced by that on which the experiment is to be made, a single observation will give $\mathcal{M}$ if $\frac{c}{r^2}$ in equation (8) be neglected; for

$$\mathcal{H}_e \tan\phi = \frac{2\mathcal{M}}{r^3}, \text{ and } \mathcal{M} = \frac{1}{2} r^3 \mathcal{H}_e \tan\phi,$$

for the first method; or

$$\mathcal{H}_e \tan\phi = \frac{\mathcal{M}}{r^3}, \text{ and } \mathcal{M} = r^3 \mathcal{H}_e \tan\phi,$$

for the second method.

If $V$ be the volume of the magnet and a solenoidal magnetization be assumed, then $\mathcal{I} = \frac{\mathcal{M}}{V}$.

It is found, however, in practice, that $\frac{c}{r^2}$ should not be neglected. Furthermore, the position of the poles is not at the ends and $\mathcal{I}$ is not uniform or solenoidal. In the case of a bar in the form of a very prolate ellipsoid of revolution, of minor axis $a$ and length $l$, the distance between the poles is $\frac{2l}{3}$, and the following formula is obtained:

$$\mathcal{I} = \frac{3\left(r^2 - \frac{l^2}{9}\right)^2 \mathcal{H}_e \tan\phi}{\pi a^2 l r} \text{ for the first method,}$$

and $$\mathcal{J} = \frac{6\left(r^2 + \frac{l^2}{9}\right)^{\frac{3}{2}} \mathcal{H}_e \tan\phi}{\pi a^2 l}$$ for the second method.

The last formula is frequently applied to long cylindrical bars and leads to little error.

*One-Pole Method.* A better method is to place the bar under test in a vertical position and east or west from the magnetometer. When placed in this position it is found that the bar is affected by the vertical component of the earth's field unless this component is compensated by a solenoid about the bar. The current through the solenoid will, however, affect the magnetometer, unless the horizontal component of the field produced at the magnetometer needle by the solenoid, with the bar removed, is compensated by another solenoid placed with its axis horizontal and in an east and west line passing through the magnetometer needle. The same current should pass through both solenoids, and the relative distances should be arranged so as to annul the effect at the magnetometer. The compensation is then assured with all currents. The current should also pass through an ammeter and an adjustable resistance to insure permanent compensation of $\mathfrak{V}$ at the bar. The magnetizing solenoid also should surround the bar.

The height of the bar should be adjusted until, with a certain magnetization, a maximum effect is obtained on the magnetometer. It is then assumed that one pole is directly behind the magnetometer. If the bar is long the effect of the lower pole is very slight. Assuming that the poles are at equal distances from the ends, the upper one at a horizontal distance $r_1$ from the magnetometer needle and the lower one at a distance $r_2$ along

a line of inclination $\theta$, and calling the distance between the poles $l$ and the cross-section $S$, we have

$$\mathcal{H}_e \tan\phi = \mathcal{J}S\left\{\frac{1}{r_1^2} - \frac{\cos\theta}{r_2^2}\right\} = \frac{\mathcal{J}S}{r_1^2}\left\{1 - \left(\frac{r_1}{r_2}\right)^3\right\},$$

or

$$\mathcal{J} = \frac{\mathcal{H}_e \tan\phi r_1^2}{S\left\{1 - \left(\frac{r_1}{r_2}\right)^3\right\}}.$$

If $\mathcal{H}_e$ is not known, it may be found by comparison with the intensity of the field produced at the magnetometer needle by the second compensating horizontal solenoid.

When the positions of the two compensating solenoids and the bar have been adjusted, the next step to be taken is to demagnetize the bar by reversals. For this there should be introduced into the circuit of the magnetizing solenoid a resistance adjustable by small steps from zero to its full value, and a commutator to reverse the current. The adjustable resistance should be cut down until the magnetization of the bar is as great as any value reached since its last demagnetization. The direction of the current should be continually and rapidly reversed while the adjustable resistance is increased gradually to its highest value, and finally the circuit should be broken. A liquid resistance, such as zinc sulphate solution between zinc plates, whose distance apart may be varied, makes a satisfactory adjustable resistance. If the magnetometer does not return to its zero reading, the current through the compensating solenoids should be changed until it does. As feeble magnetic forces are slow in acting, it is necessary to allow some time for this adjustment.

This method is very valuable for the investigation of

the effects of weak fields on $\mathcal{B}$, $\mathcal{J}$, $\mu$, and $\kappa$. For such work the ballistic method is quite unsatisfactory, owing to the creeping up of the magnetization.

**Example.**[1]

Test of a piece of wrought-iron wire by the magnetometric method. Cross-section of wire, 0.004658 sq. cm.; length of wire, 30.05 cms.; $\mathcal{H}_e$ equalled 0.299 C.G.S. unit.

Distance of millimetre scale, 1 metre; $r_1 = 10$ cms., $r_2 = 31$ cms. Whence $\left(\frac{r_1}{r_2}\right)^3 = 0.0335$.

Deflection of one scale part corresponds to $\tan \phi = 0.0005$.

Value of $\mathcal{J}$ per scale division $= \dfrac{0.299 \times 0.0005 \times 100}{0.004658 \times 0.9665} = 3.32$.

The magnetizing coil contained 69 turns per cm.

Magnetizing force per ampere $= \dfrac{4\pi \,.\, 69}{10} = 86.7$.

| Magnetizing force $\mathcal{H}$ | | Magnetometer readings. | $\mathcal{J}$ | $\kappa = \dfrac{\mathcal{J}}{\mathcal{H}}$ | $\mathcal{B} = 4\pi\mathcal{J} + \mathcal{H}$ | $\mu = \dfrac{\mathcal{B}}{\mathcal{H}}$ |
|---|---|---|---|---|---|---|
| Selenoid alone. | Corrected for ends. | | | | | |
| 0.32 | 0.32 | 1 | 3 | 9 | 40 | 120 |
| 0.85 | 0.84 | 4 | 13 | 15 | 170 | 200 |
| 1.38 | 1.37 | 10 | 33 | 24 | 420 | 310 |
| 2.18 | 2.14 | 28 | 93 | 43 | 1170 | 550 |
| 2.80 | 2.67 | 89 | 295 | 110 | 3710 | 1390 |
| 3.50 | 3.24 | 175 | 581 | 179 | 7300 | 2250 |
| 4.21 | 3.89 | 239 | 793 | 204 | 9970 | 2560 |
| 4.92 | 4.50 | 279 | 926 | 206 | 11640 | 2590 |
| 5.63 | 5.17 | 304 | 1009 | 195 | 12680 | 2450 |
| 6.69 | 6.20 | 327 | 1086 | 175 | 13640 | 2200 |
| 8.46 | 7.94 | 348 | 1155 | 145 | 14510 | 1830 |
| 10.23 | 9.79 | 359 | 1192 | 122 | 14980 | 1530 |
| 12.11 | 11.57 | 365 | 1212 | 105 | 15230 | 1320 |
| 15.61 | 15.06 | 373 | 1238 | 82 | 15570 | 1030 |
| 20.32 | 19.76 | 378 | 1255 | 64 | 15780 | 800 |
| 22.27 | 21.70 | 380 | 1262 | 58 | 15870 | 730 |

**141. The Tractional Method.** — If a longitudinally magnetized bar be cut orthogonally in two, and the parts

[2] Ewing's *Mag. Ind. in Iron*, p. 49.

be separated an infinitesimal distance, both end surfaces will show equal intensities of magnetization $\mathcal{J}$. Call the area of each end surface $S$. The attraction of one surface on the other will be $2\pi\mathcal{J}^2S$, provided the field $\mathcal{H}$ about the bar be negligible. If $\mathcal{H}$ is not negligible and is due to an outside cause, — for example, a magnetizing solenoid not attached to the magnet, — we must add to the above a force $\mathcal{H}\mathcal{J}S$. If the solenoid is in two parts closely wound about the bar, which separate with the parts of the bar, we must add a third term $\frac{\mathcal{H}^2S}{8\pi}$ for the mutual attraction of the two parts of the solenoid, which is assumed to be of the same cross-section as the bar. These forces are in dynes; to reduce to grammes they must be divided by 980. Reducing to a common denominator and substituting the value of $\mathcal{B}$, we obtain for $F$ in grammes under the three conditions,

$$F = \frac{S}{8\pi g}(16\pi^2\mathcal{J}^2) = \frac{\mathcal{B}^2S}{8\pi g} \qquad (a)$$

$$F = \frac{S}{8\pi g}(16\pi^2\mathcal{J}^2 + 8\pi\mathcal{J}\mathcal{H}) = \frac{(\mathcal{B}^2 - \mathcal{H}^2)S}{8\pi g} \qquad (b)$$

$$F = \frac{S}{8\pi g}(16\pi^2\mathcal{J}^2 + 8\pi\mathcal{J}\mathcal{H} + \mathcal{H}^2) = \frac{\mathcal{B}^2S}{8\pi g} \qquad (c)$$

Also
$$\mathcal{B} = \sqrt{\frac{8\pi gF}{S}} = 156.9\sqrt{\frac{F}{S}} \qquad (a \text{ and } c)$$

$$\mathcal{B} = \sqrt{\frac{8\pi gF}{S} + \mathcal{H}^2} \qquad (b)$$

It is evident from the above equations that if $\mathcal{J}$ and $\mathcal{B}$ are not uniform over the whole cross-section of the

magnet, the result obtained will be the square root of the mean square, and not the simple mean. The square root of the mean square is always greater than the mean. It therefore follows that the value here obtained may be slightly larger than that obtained by other methods. Exactly such results were obtained from experiments made with a horse-shoe magnet (Fig. 141). The upper curve of Fig. 142 represents the relation of $\mathcal{B}$ and $\mathcal{H}$, with the values of $\mathcal{B}$ calculated from the force necessary to detach the armature. The lower curve was obtained by the ballistic method (Art. 145), the exploring coil being in the position marked 2. For large values of $\mathcal{H}$ the value of $\mathcal{B}$ tends to become uniform over the whole cross-section, and the curves approach each other.

**142. The Divided Ring.**—Let a divided ring (Fig. 136) of cross-section $S$ sq. cms. be uniformly wound with a magnetizing coil of $n$ turns per cm., measured along the mean circumference; and let the coil be traversed by a current of $I$ units in C.G.S. measure. Then the value of the magnetizing force $\mathcal{H}$ will be $4\pi nI$. Attach the hook on the lower half $C$ to the bottom of a frame, from the top of which the upper half $C'$ is supported by means of a spring balance, hooking into the eye on $C'$, and a turn-buckle to increase the tension. Care should be used in setting up the apparatus so that the line of pull may pass vertically through the centre of the ring and normally to the plane of separation of the two halves.

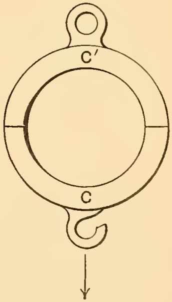


Fig. 136.

Either the balance should be adjusted to zero when supporting $C'$, or else the weight of $C'$ should be subtracted from the observed forces. In calculating $\mathcal{B}$, the value of $S$ should be doubled so as to include both surfaces of contact.

Shelford Bidwell made use of a ring made from a very soft charcoal iron rod. After welding the joint, the ring was turned down in a lathe to a uniform transverse circular cross-section of 0.482 cm. diameter. The outer radius of the ring was 4 cms. and the mean radius 3.76 cms. After the ring was sawed in two, brass collars were fastened to the ends of one half to hold the other half in position, thus insuring freedom from lateral displacement. Both halves were uniformly wound with ten layers of insulated wire 0.07 cm. in diameter. After each turn was in place the radial gaps were filled with paraffin. The half with the collar had 980 turns, part of which were on the collar, and the other half had 949 turns. When the two halves were together, the ring appeared to be uniformly wound without break. The value of $\mathcal{H}$ was carried to 585, when the weight supported by the ring reached 15,905 grammes.

**143. The Divided Rod.** — A rod is a more convenient form for testing than the ring, since it does not need to be bent, welded, and turned true. The apparatus used by Bosanquet[1] for testing rods is shown in Fig. 137. The rod is divided into halves $c$, $c'$, which meet in carefully faced surfaces. Each half has a closely wound solenoid $B$ about it. From the bottom of $c$ a scale pan is sus-

[1] *Phil. Mag.*, Vol. XXII., 1886, p. 535.

pended. The scale pan, the lower half of the rod, and its solenoid are counterbalanced by a lever not shown in the figure, so that when the pan is empty there is no separating force at the junction. An exploring coil $D$, connected with a ballistic galvanometer, surrounds the upper end of $c$. When $c$ and $c'$ separate, $D$ is quickly withdrawn from the field by a spring, thus giving an independent method of measuring $\mathcal{B}$. Bosanquet found a close agreement between the values of $\mathcal{B}$ calculated by the two methods for large values of $\mathcal{H}$. For small values the agreement was not good. He used two cylindrical iron rods 20 cms. long and 0.526 cm. diameter, each wound with 1,096 turns of wire. The maximum weight supported was 20,414 grammes.

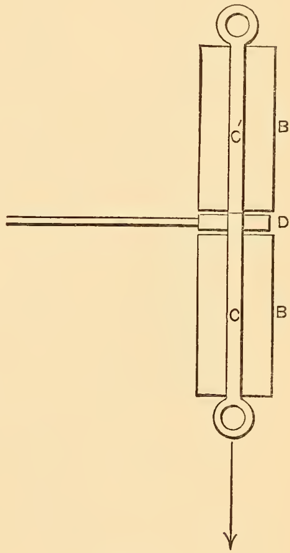


Fig. 137.

**144. Thompson's Permeameter.** — Bosanquet's divided ring method has the disadvantage of having poles at the ends of the divided bar. Allowance must be made for these in computing $\mathcal{H}$. Thompson has avoided this difficulty by slotting out a rectangular block of iron $A$ (Fig. 138) to receive a magnetizing solenoid $B$, through which a coaxial brass tube passes. The sample to be tested is turned into a cylinder just fitting the brass tube, and is inserted from the top. The lower end of

$c$ is carefully surfaced and rests against a part of the yoke, which is also carefully surfaced. The lines of force are assumed to go through the iron only. $\mathcal{B}$ is calculated as before from the pull which will overcome the magnetic attraction at the lower end of the rod. The attraction at the upper end of the rod is at right angles to the rod and has no effect. Thompson gives the formula

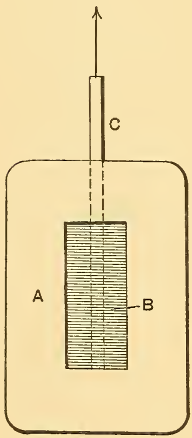


Fig. 138.

$$\mathcal{B} = 1317\sqrt{\text{Pounds} \div \text{sq. in.}} + \mathcal{H} = 156.9\sqrt{\text{Grammes} \div \text{sq. cms.}} + \mathcal{H}$$

to express the relation between $\mathcal{B}$, $\mathcal{H}$, and $F$, which is somewhat different from formula ($b$), Art. 141. Errors of observation will, however, cause larger differences than that between the formulas.

**145. The Ballistic Method.** — This method in its present form is due to Rowland.[1] It depends upon the principle that when the flux of magnetic induction through a coil $S$ (Fig. 139) of $n_1$ turns is changed by a quantity $N$, the time integral of the electromotive force generated in the coil is $n_1N$. If the coil $S$ be in a circuit of resistance $r$, including a ballistic galvanometer $G$ of long period, the quantity of electricity passing through the galvanometer will be $\frac{n_1N}{r}$. All of these quantities are in C.G.S. units.

[1] *Phil. Mag.*, Vol. XLVI., 1873, p. 151.

If the same circuit includes, as part of $r$, an earth-inductor $EI$ of total area $A$ lying horizontally in a place where the vertical component $\mathfrak{V}$ of the earth's magnetic field is known, the constant $k$ of the galvanometer may be determined by a simple reversal of the coil. Let $d_1$ be the deflection (corrected to $\sin \frac{1}{2}$ angle) corresponding to the quantity of electricity $Q$ passing through the galvanometer; then by Art. 137

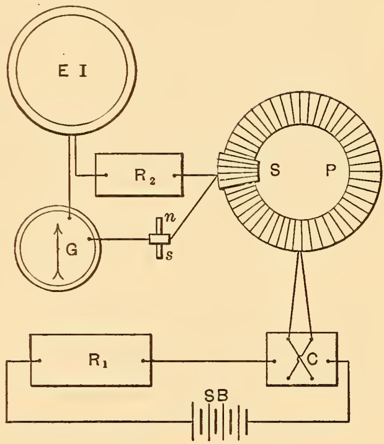


Fig. 139.

$$Q = d_1 k = \frac{2A\mathfrak{V}}{r}, \text{ and } k = \frac{2A\mathfrak{V}}{d_1 r}.$$

Let $d_2$ be the corrected deflection due to the change $N$ in the flux through $S$; then $d_2 k = \frac{n_1 N}{r}$, from which it follows that

$$N = \frac{2A\mathfrak{V}d_2}{n_1 d_1}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

There are several objections to the use of the earth-inductor. In the first place, an error may be made in determining $A$; next, an error may be introduced by a change in $\mathfrak{V}$, due to any one of many causes; and, thirdly, a considerable error may be introduced because

of the large number of observations necessary in determining $\mathcal{V}$. For these reasons it is better to determine $k$ by means of a standard cell and a standard condenser. Let $E$ be the electromotive force of the standard cell, and $C$ the capacity of the condenser. Connect the apparatus as in Fig. 88, p. 188, and charge the condenser and discharge it through the galvanometer. Let $d_3$ be the deflection; then $d_3k = CE$, and

$$N = 100 \frac{d_2CEr}{d_3n_1}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $C$ is in microfarads, $E$ in volts, and $r$ in ohms. The resistance $r$ is the resistance of the entire circuit as connected for the determination of $N$.

As the damping of a ballistic galvanometer may be appreciably different on open and on closed circuits, it is advisable to have the discharge key double so as to close the galvanometer through an external resistance equal to that of the working circuit immediately after the discharge of the condenser.

For this method it is better to use the iron in the form of a ring. It should be wound uniformly all the way around with a primary coil $P$, of $n_2$ turns per cm. measured along the mean circumference. $P$ should be in series with an ammeter, not shown in Fig. 139, a resistance $R_1$ adjustable by small gradations, and a storage battery $SB$, through a commutator $C$. If a current of $I$ C.G.S. units flows through this circuit, the corresponding value of $\mathcal{H}$ will be $4\pi n_2 I$.

To find $\Delta\mathcal{B}$, the change in the value of $\mathcal{B}$, the change of flux in the iron should be divided by the cross-section $A'$ of the iron. To be exact, the part $N'$ included between the iron and the secondary coil $S$,

which is wound outside of the primary, should be subtracted from $N$. But this correction is generally negligible.

*Practice of the Method.* When ready to begin the experiment, the ring if previously used should be demagnetized by reversals, beginning with the highest value of $\mathcal{H}$ employed before. The resistance in $R_1$ should be gradually increased after each reversal of the commutator until its highest value is reached, and the circuit should then be opened.

To obtain a simple magnetization curve by reversals, the value of $R_1$ should be adjusted to give the lowest value of $\mathcal{H}$, the circuit closed, and the value of the current observed. The commutator should then be reversed and the deflection of the galvanometer noted. As the flux through $S$ is only one-half the change in the flux, the value of $\mathcal{B}$ should be calculated from one-half the deflection. To obtain the residual value of $\mathcal{B}$, the circuit should be broken and the deflection again noted. This residual value is proportional to the difference between this deflection and half the previous one. $R_1$ should now be decreased for the next higher value of $\mathcal{H}$, and the observations repeated, and so on. The values of $\mathcal{H}$ when plotted with the corresponding values of $\mathcal{B}$ will give the curves of temporary and residual magnetization.

To obtain a cyclical magnetization or hysteresis curve, the ring should be demagnetized as above. Then, adjusting the value of $R_1$ for the lowest value of $\mathcal{H}$ desired, the circuit should be closed, the deflection of the galvanometer noted, and the ammeter read. The resistance $R_1$ should now be decreased abruptly by suit-

able steps and the corresponding deflections noted. The corresponding values of $\mathcal{B}$ are proportional to the summations of the deflections from the beginning. When the highest value of $\mathcal{H}$ desired has been reached, the resistance $R_1$ is increased by suitable steps, and $\mathcal{H}$ reduced until on breaking the circuit $\mathcal{H}$ is zero again. The commutator is now reversed and $\mathcal{H}$ is carried to corresponding values in the opposite sense, and so on. After the first quarter-cycle the values of $\mathcal{H}$ and $\mathcal{B}$ repeat themselves, and the resulting curve is called a cyclical magnetization or hysteresis curve. The first quarter plots as a simple magnetization curve.

There should be little difference between the outer and inner radii of the iron ring used in this experiment. If a bar is used it should be at least forty diameters in length and the magnetizing solenoid should cover almost the whole length. The secondary coil should be at the centre. To avoid the effects of the earth's field the axis of the ring should be along the lines of force, but it is sufficient if the ring is horizontal and the axis of the secondary coil is east and west. In the case of a bar, the axis should be east and west.

For convenience in bringing the galvanometer needle to rest, a small coil, through which the magnet *ns* may be thrust, is included in the circuit of $G$. Every movement of *ns* produces an induced current which may be so timed as to check the swing of the needle. A solenoid near the galvanometer in circuit with a single cell and a key within reach of the observer may serve the same purpose.

The great fault in the ballistic method as applied to rings is that it takes no account of the gradual changes

in magnetization — the so-called creeping up — which follow any sudden change in $\mathcal{H}$. Hopkinson's bar and yoke method, described in the following article, is to a large extent free from this defect.

### Example.

*The Ballistic Method applied to a Cast-Iron Ring.*

Total area of earth-inductor $A$, 48,600 sq. cms.

Vertical component of the earth's field $\mathcal{V}$, 0.54.

Corrected deflection of the galvanometer for one turn of earth-coil, $d_1$, 75.

Number of turns in $S$, $n_1$ . . . . . . . . 20.

Number of turns in $P$ . . . . . . . . . . 273.

Mean length of magnetic circuit . . . . 39.82 cms.

Number of turns per cm., $n_2$ . . . . . . . 6.86.

Cross-section of ring, $A'$ . . . . . . . . 5.94 sq. cms.

No allowance was made for $N'$. Hence

$$\mathcal{H} = 4\pi n_2 I = 86.2\, I \text{ C.G.S. units.}$$

$$\Delta B = \frac{N}{A'} = \frac{2A\mathcal{V}d_2}{A'n_1d_1} = 5.89\, d_2.$$

$$B = 5.89\, \Sigma d_2\,.$$

The ring had been previously used, and had not been completely demagnetized before the beginning of the test, and as a consequence the values of $\mathcal{B}$ for the first quarter-cycle do not represent changes from a neutral condition. One-half the numerical difference between the extreme observed values of $\mathcal{B}$ will, however, give the real initial value of $\mathcal{B}$. Applying this as a correction, the real values of $\mathcal{B}$ are obtained. The correction in this instance was 1,758.

| $I$ (C.G.S.) | $\mathcal{H}$ | $d_2$ | $\Sigma d_2$ | $\mathcal{B}$ | $\mathcal{B}$ corrected. |
|---|---|---|---|---|---|
| + 0.058 | + 5.00 | + 75.5 | + 75.5 | + 445 | − 1313 |
| 0.079 | 6.81 | 40 | 115.5 | 680 | 1078 |
| 0.126 | 10.86 | 363 | 478.5 | 2818 | + 1060 |
| 0.148 | 12.76 | 165.5 | 644 | 3793 | 2035 |
| 0.176 | 15.17 | 147 | 791 | 4659 | 2901 |
| 0.217 | 18.71 | 150.5 | 941.5 | 5542 | 3784 |
| 0.269 | 23.19 | 138 | 1079.5 | 6358 | 4600 |
| 0.356 | 30.69 | 190.5 | 1270 | 7480 | 5722 |
| 0.490 | 42.24 | 105 | 1375 | 8099 | 6341 |
| 0.669 | 57.67 | 142 | 1517 | 8935 | 7177 |
| 1.117 | 96.29 | 215.5 | 1732.5 | 10201 | 8443 |
| 0.490 | 42.24 | − 210 | 1522.5 | 8968 | 7210 |
| 0.307 | 26.46 | 96.5 | 1426 | 8399 | 6641 |
| 0.127 | 10.95 | 153 | 1273 | 7498 | 5740 |
| 0.056 | 4.83 | 95 | 1178 | 6938 | 5180 |
| 0.000 | 0.00 | 107 | 1071 | 6308 | 4550 |
| − 0.040 | − 3.45 | 127 | 944 | 5560 | 3802 |
| 0.058 | 5.00 | 93 | 851 | 5011 | 3253 |
| 0.077 | 6.64 | 199 | 652 | 3840 | 2082 |
| 0.107 | 9.22 | 395.5 | 256.5 | 1511 | − 247 |
| 0.122 | 10.42 | 152.5 | 104 | 612 | 1146 |
| 0.142 | 12.24 | 169 | − 65 | − 383 | 2141 |
| 0.172 | 14.83 | 156.5 | 221.5 | 1305 | 3063 |
| 0.209 | 18.02 | 155 | 376.5 | 2218 | 3976 |
| 0.267 | 23.02 | 137.5 | 514 | 3027 | 4785 |
| 0.390 | 33.62 | 178 | 692 | 4076 | 5834 |
| 0.508 | 43.79 | 106 | 798 | 4700 | 6458 |
| 0.694 | 59.82 | 134 | 932 | 5489 | 7247 |
| 1.105 | 95.25 | 202.5 | 1134.5 | 6682 | 8440 |
| 0.456 | 39.31 | + 220.5 | 914 | 5383 | 7141 |
| 0.304 | 26.20 | 92 | 822 | 4842 | 6600 |
| 0.127 | 10.95 | 145.5 | 676.5 | 3985 | 5743 |
| 0.056 | 4.83 | 91.5 | 585 | 3446 | 5204 |
| 0.000 | 0.00 | 104 | 481 | 2833 | 4591 |
| + 0.057 | + 4.91 | 188 | 293 | 1726 | 3484 |
| 0.0572 | 4.93 | 33 | 260 | 1531 | 3289 |
| 0.070 | 6.03 | 98.5 | 161.5 | 951 | 2709 |
| 0.086 | 7.41 | 179 | + 17.5 | + 103 | 1655 |
| 0.095 | 8.19 | 130 | 147.5 | 869 | 889 |
| 0.107 | 9.22 | 140.5 | 288 | 1696 | 62 |
| 0.123 | 10.60 | 168.5 | 456.5 | 2689 | + 931 |
| 0.143 | 12.33 | 158.5 | 615 | 3622 | 1864 |
| 0.172 | 14.83 | 146.5 | 761.5 | 4485 | 2727 |
| 0.208 | 17.93 | 156.5 | 918 | 5407 | 3649 |
| 0.264 | 22.76 | 144.5 | 1062.5 | 6258 | 4500 |
| 0.384 | 33.10 | 220 | 1282.5 | 7554 | 5796 |
| 0.491 | 42.33 | 110.5 | 1393 | 8205 | 6447 |
| 0.697 | 60.08 | 138 | 1531 | 9017 | 7259 |
| 1.088 | 93.79 | 200 | 1731 | 10196 | 8438 |

**146. Hopkinson's Bar and Yoke Method.** — This method makes use of a piece of apparatus very much like Thompson's permeameter. The method, however, is a ballistic one. The bar to be tested is divided into

two pieces $c$ and $c'$ (Fig. 140), the former movable in the direction of its length, and the latter fixed during the test. The abutting ends of the two pieces must be carefully surfaced and in close contact. The lateral surfaces of $c$, $c'$ should be in good contact with $A$, which is made of very soft iron. The solenoid $BB$ is divided in halves, and between them is a test coil $D$ wound on an ivory ring. This test coil is connected in circuit with a ballistic galvanometer. When the part $c$ of the bar is abruptly drawn out a short distance by means of the handle, a spring throws the test coil out from the yoke, thereby making it cut the whole flux of induction present when the handle was pulled. The deflection of the galvanometer measures the flux. For cyclical magnetization curves the parts of the bar are not separated, and the apparatus acts in all essential respects like the ring of the previous section.

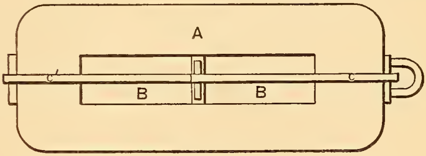


Fig. 140.

Because of the small value of the magnetic reluctance in $A$, it is assumed that the equivalent length of the magnetic circuit is the length of the slot in $A$. It is also assumed that there is no leakage of magnetic induction from the bar. Although these conditions are not exactly fulfilled, and although the reluctance of the joints is not insensible, yet for practical commercial purposes the method is sufficiently exact.

**147. Comparison of the Values of $\mathcal{B}$ by the Ballistic and the Tractional Methods.** — The electromagnet, shown on a scale of one-sixth in Fig. 141, was designed to test the laws of traction, both with the armature in contact and when separated from the pole piece at various distances. In the course of the experiments made upon it $\mathcal{B}$ was measured by both the ballistic and tractional methods. When the armature was in contact the results were exactly what might have been anticipated from theory. With low values of $\mathcal{H}$ there should be a considerable difference between the values of $\mathcal{B}$ on the inner and the outer sides of the magnetic circuit; and as a consequence the mean value of $\mathcal{B}$, determined by the ballistic method, should be less than the square root of the mean square value, determined by traction. For higher values of $\mathcal{H}$ the value of $\mathcal{B}$ tends to become uniform over the whole cross-section, and as a consequence the mean value and the square root of the mean square value tend to become equal, and the results by the two methods should agree closely. Fig. 142 gives $\mathcal{B}\mathcal{H}$ curves calculated by both methods, the upper one by the tractional and the lower by the ballistic method.

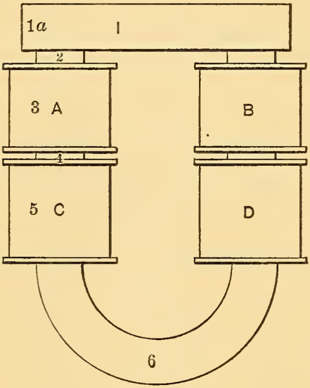


Fig. 141.

The traction was applied at the middle of the armature by means of a spring dynamometer and a lever combined. As the spring dynamometer read no higher

than 50 kilos., it was necessary to have recourse to a lever in addition. The method of operating was to place large weights in the pan attached to the free end

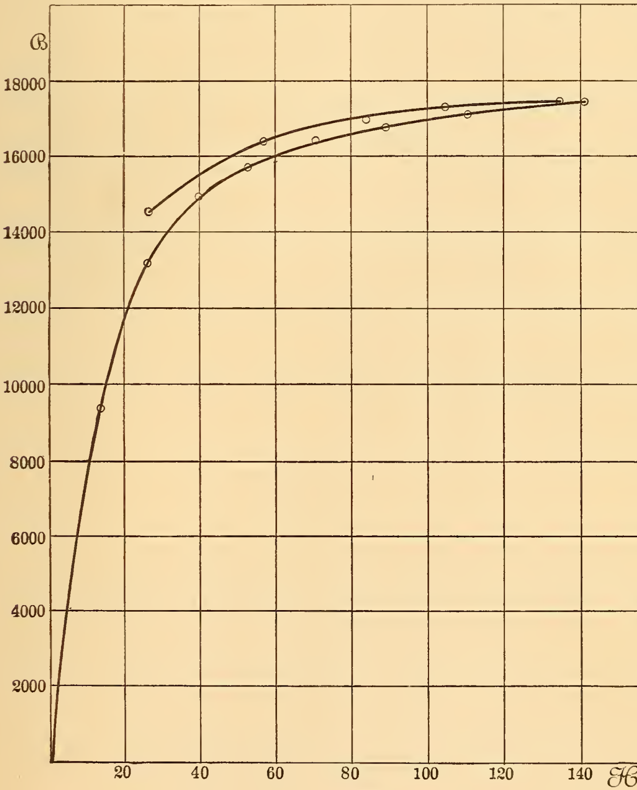



Fig. 142.

of the lever until the proper value was nearly reached; then by means of a turn buckle, the pull was increased by drawing up the dynamometer until the armature was detached. The value of $\mathcal{B}$ was calculated from

the formula, $\mathcal{B}=156.9\sqrt{\frac{F}{2S}}$, where $F$ was the pull in grammes and $S$ the cross-section in sq. cms. In this particular case $S$ was 11.34. Therefore

$$\mathcal{B}=32.94\sqrt{F}.$$

The galvanometer constant was determined by means of a standard Carhart-Clark cell, whose electromotive force was 1.432 volts at 29° C., and an Elliott condenser of 0.5065 mf. capacity. The condenser when charged by the cell and discharged through the galvanometer gave a corrected deflection $d_3$ of 39.4. When the galvanometer was connected with test coil 2 of one turn, $n_1=1$, the resistance of the circuit was 6660 ohms. Calling $d_2$ the deflection caused by reversing the magnetic flux through coil 2, we have by Art. 145,

$$\mathcal{B}=100\,\frac{ECrd_2}{2n_1Sd_3}=541d_2.$$

The total number of turns in $ABCD$ was 3464, and the equivalent length of the magnetic circuit was computed to be 83 cms., making the number of turns per cm. $n_2$ equal to $\frac{3464}{83}$. Consequently, calling the current $I$,

$$\mathcal{H}=4\pi nI=525\,I \text{ C.G.S. units.}$$

The following table gives the results by both methods:

| Tractional Method. | | | | Ballistic Method. | | | |
|---|---|---|---|---|---|---|---|
| Current. | $\mathcal{H}$ | Grammes. | $\mathcal{B}$ | Current. | $\mathcal{H}$ | Deflection. | $\mathcal{B}$ |
| | | | | 0.026 | 13.6 | 17.4 | 9410 |
| 0.050 | 26.2 | 196000 | 14580 | 0.050 | 26.2 | 24.45 | 13230 |
| | | | | 0.076 | 39.9 | 27.6 | 14930 |
| 0.110 | 57.7 | 243000 | 16240 | 0.1012 | 53.1 | 28.9 | 15630 |
| | | | | 0.133 | 69.8 | 30.2 | 16340 |
| 0.160 | 83.9 | 263000 | 16890 | 0.170 | 89.2 | 30.8 | 16660 |
| 0.200 | 105.0 | 270000 | 17120 | 0.210 | 110.0 | 31.6 | 17090 |
| 0.257 | 134.9 | 273000 | 17210 | 0.270 | 141.7 | 32.2 | 17420 |

**148. Magnetic Leakage.** — To determine the value of the magnetic flux with various numbers of ampere-turns and in various parts of the magnetic circuit, six test coils were wound at points designated by the numbers 1 to 6 (Fig. 141). Coil 1 could be moved to the position 1*a*. With the armature in contact with the pole pieces, the value of the flux through the several test coils was determined for various numbers of ampere-turns in *A*, *B*, *C*, and *D*, which were always in series.

With the armature in contact with the pole pieces, the maximum flux was through coil 3, the flux decreasing through the other coils in the order 4, 2, 1, 6. The flux through 5 was not measured. When the armature was separated from the poles by a distance of 0.32 cm., and a smaller number of ampere-turns than 6300 was used, the order was 4, 6, 3, 1, 2; above 6300 ampere-turns 6 and 3 exchanged places. When the armature was in contact with the poles there was leakage from its ends. This was shown by the fact that the deflection produced by coil 1 was reversed in direction when placed at 1*a*. With the armature at 0.32 cm. from the poles, however, the flux through the ends was added to

that through the middle of the armature, the deflections at 1 and 1*a* being of the same sign. At 1.27 cms. coil 5 showed the greatest flux; coils 4 and 6 came next, 6 leading slightly up to 5000 ampere-turns and beyond that coil 4; the others followed in the order 3, 1, 2. The same relative order was maintained when the armature was removed from the poles a distance of 6.35 cms. In every case coil 6 was traversed by a greater flux than coil 4 for the smaller numbers of ampere-turns, the reverse being true for the larger numbers. As the distance of the armature from the poles increased, there was an increase in the number of ampere-turns at which the exchange of relative values between 4 and 6 took place. This exchange is explained by the increased reluctance of the iron portion of the magnetic circuit with the higher values of $\mathcal{B}$.

# APPENDIX A.

## TABLE I.

**Reduction of Deflections observed with Mirror and Scale (Art. 28).**

$d =$ deflection reckoned from the point of rest.

$a =$ distance between mirror and scale.

$\delta = \frac{d}{a}$.

The values of $\theta$, $\tan\theta$, $\sin\theta$, $2\sin\frac{\theta}{2}$ are obtained by multiplying $\frac{\delta}{2}$ by the factor corresponding to the value of $\delta$ in the table. This factor is equal to unity diminished by the value of the expression standing at the head of the column.

| $\delta$ | $\theta$ <br> $\frac{1}{3}\delta^2$ | $\tan\theta$ <br> $\frac{1}{4}\delta^2$ | $\sin\theta$ <br> $\frac{3}{8}\delta^2$ | $2\sin\frac{\theta}{2}$ <br> $\frac{11}{32}\delta^2$ |
|---|---|---|---|---|
| 0.01 | 0.99997 | 0.99998 | 0.99996 | 0.99997 |
| 0.02 | 0.99987 | 0.99990 | 0.99985 | 0.99986 |
| 0.03 | 0.99970 | 0.99978 | 0.99966 | 0.99969 |
| 0.04 | 0.99947 | 0.99960 | 0.99940 | 0.99945 |
| 0.05 | 0.99917 | 0.99938 | 0.99906 | 0.99914 |
| 0.06 | 0.99880 | 0.99910 | 0.99865 | 0.99876 |
| 0.07 | 0.99837 | 0.99878 | 0.99816 | 0.99832 |
| 0.08 | 0.99787 | 0.99840 | 0.99760 | 0.99780 |
| 0.09 | 0.99730 | 0.99798 | 0.99696 | 0.99722 |
| 0.10 | 0.99667 | 0.99750 | 0.99625 | 0.99656 |
| | $\frac{1}{3}\delta^2 + \frac{1}{5}\delta^4$ | $\frac{1}{4}\delta^2 + \frac{1}{8}\delta^4$ | $\frac{3}{8}\delta^2 + \frac{31}{128}\delta^4$ | $\frac{11}{32}\delta^2 + \frac{431}{2048}\delta^4$ |
| 0.11 | 0.99600 | 0.99699 | 0.99550 | 0.99587 |
| 0.12 | 0.99525 | 0.99643 | 0.99465 | 0.99508 |
| 0.13 | 0.99443 | 0.99582 | 0.99372 | 0.99423 |
| 0.14 | 0.99355 | 0.99515 | 0.99272 | 0.99331 |
| 0.15 | 0.99260 | 0.99444 | 0.99166 | 0.99233 |
| 0.16 | 0.99160 | 0.99368 | 0.99054 | 0.99129 |
| 0.17 | 0.99054 | 0.99288 | 0.98935 | 0.99020 |
| 0.18 | 0.98941 | 0.99203 | 0.98809 | 0.98905 |
| 0.19 | 0.98823 | 0.99114 | 0.98677 | 0.98775 |
| 0.20 | 0.98700 | 0.99020 | 0.98539 | 0.98659 |

## TABLE II.

### Reflecting Galvanometer Scale Errors.

(These corrections are to be subtracted from the observed deflections. Art. 28.)

| Distance *a.* / OBSERVED DEFLECTIONS. | 1000 | 1050 | 1100 | 1150 | 1200 | 1250 | 1300 | 1350 | 1400 | 1450 | 1500 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 0.05 | 0.05 | 0.05 | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| 60 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0. |
| 70 | 0.1 | 0.1 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| 80 | 0.15 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.05 | 0.05 | 0.05 | 0.05 |
| 90 | 0.2 | 0.15 | 0.15 | 0.15 | 0.15 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| 100 | 0.25 | 0.2 | 0.2 | 0.2 | 0.2 | 0.15 | 0.15 | 0.15 | 0.15 | 0.1 | 0.1 |
| 110 | 0.35 | 0.3 | 0.25 | 0.25 | 0.25 | 0.2 | 0.2 | 0.2 | 0.2 | 0.15 | 0.15 |
| 120 | 0.45 | 0.4 | 0.35 | 0.3 | 0.3 | 0.3 | 0.25 | 0.25 | 0.25 | 0.2 | 0.2 |
| 130 | 0.55 | 0.5 | 0.45 | 0.4 | 0.4 | 0.35 | 0.3 | 0.3 | 0.3 | 0.25 | 0.25 |
| 140 | 0.7 | 0.65 | 0.55 | 0.5 | 0.5 | 0.45 | 0.4 | 0.35 | 0.35 | 0.35 | 0.3 |
| 150 | 0.85 | 0.8 | 0.7 | 0.65 | 0.6 | 0.55 | 0.5 | 0.45 | 0.4 | 0.4 | 0.35 |
| 160 | 1.0 | 0.95 | 0.85 | 0.8 | 0.7 | 0.65 | 0.6 | 0.55 | 0.5 | 0.5 | 0.45 |
| 170 | 1.2 | 1.1 | 1.0 | 0.95 | 0.85 | 0.8 | 0.7 | 0.65 | 0.6 | 0.6 | 0.55 |
| 180 | 1.4 | 1.3 | 1.2 | 1.1 | 1.0 | 0.95 | 0.85 | 0.8 | 0.7 | 0.7 | 0.65 |
| 190 | 1.65 | 1.55 | 1.4 | 1.3 | 1.2 | 1.1 | 1.0 | 0.95 | 0.85 | 0.8 | 0.75 |
| 200 | 1.95 | 1.8 | 1.65 | 1.5 | 1.4 | 1.25 | 1.15 | 1.1 | 1.0 | 0.95 | 0.9 |
| 210 | 2.25 | 2.05 | 1.9 | 1.7 | 1.6 | 1.45 | 1.35 | 1.25 | 1.15 | 1.1 | 1.05 |
| 220 | 2.6 | 2.35 | 2.15 | 1.95 | 1.8 | 1.65 | 1.55 | 1.45 | 1.3 | 1.25 | 1.2 |
| 230 | 2.95 | 2.7 | 2.45 | 2.25 | 2.05 | 1.9 | 1.75 | 1.65 | 1.5 | 1.45 | 1.35 |
| 240 | 3.3 | 3.05 | 2.8 | 2.55 | 2.35 | 2.15 | 2.0 | 1.85 | 1.75 | 1.65 | 1.5 |
| 250 | 3.75 | 3.45 | 3.15 | 2.9 | 2.65 | 2.45 | 2.25 | 2.1 | 1.95 | 1.85 | 1.7 |
| 260 | 4.25 | 3.85 | 3.5 | 3.25 | 3.0 | 2.75 | 2.55 | 2.35 | 2.2 | 2.05 | 1.9 |
| 270 | 4.75 | 4.3 | 3.95 | 3.6 | 3.35 | 3.1 | 2.85 | 2.65 | 2.45 | 2.3 | 2.15 |
| 280 | 5.3 | 4.8 | 4.4 | 4.0 | 3.7 | 3.45 | 3.15 | 2.95 | 2.75 | 2.55 | 2.4 |
| 290 | 5.85 | 5.3 | 4.85 | 4.45 | 4.1 | 3.8 | 3.5 | 3.25 | 3.05 | 2.85 | 2.65 |
| 300 | 6.45 | 5.85 | 5.35 | 4.9 | 4.5 | 4.2 | 3.9 | 3.6 | 3.35 | 3.15 | 2.95 |
| 310 | 7.1 | 6.45 | 5.9 | 5.4 | 5.0 | 4.6 | 4.3 | 3.95 | 3.7 | 3.45 | 3.25 |
| 320 | 7.8 | 7.1 | 6.5 | 5.95 | 5.5 | 5.05 | 4.7 | 4.35 | 4.05 | 3.8 | 3.55 |
| 330 | 8.5 | 7.75 | 7.1 | 6.5 | 6.0 | 5.55 | 5.15 | 4.75 | 4.45 | 4.15 | 3.9 |
| 340 | 9.3 | 8.45 | 7.75 | 7.1 | 6.55 | 6.05 | 5.6 | 5.2 | 4.85 | 4.55 | 4.25 |
| 350 | 10.1 | 9.2 | 8.4 | 7.75 | 7.15 | 6.6 | 6.1 | 5.65 | 5.3 | 4.95 | 4.6 |
| 360 | 10.95 | 10.00 | 9.15 | 8.4 | 7.75 | 7.15 | 6.65 | 6.15 | 5.75 | 5.35 | 5.0 |
| 370 | 11.85 | 10.8 | 9.9 | 9.1 | 8.4 | 7.75 | 7.2 | 6.65 | 6.2 | 5.8 | 5.45 |
| 380 | 12.8 | 11.7 | 10.7 | 9.85 | 9.05 | 8.4 | 7.8 | 7.2 | 6.7 | 6.3 | 5.9 |
| 390 | 13.8 | 12.6 | 11.5 | 10.6 | 9.75 | 9.05 | 8.4 | 7.8 | 7.25 | 6.8 | 6.35 |
| 400 | 14.85 | 13.55 | 12.4 | 11.4 | 10.5 | 9.75 | 9.05 | 8.4 | 7.85 | 7.3 | 6.85 |
| 410 | 15.9 | 14.5 | 13.3 | 12.25 | 11.3 | 10.45 | 9.7 | 9.05 | 8.4 | 7.85 | 7.4 |
| 420 | 17.05 | 15.55 | 14.25 | 13.15 | 12.1 | 11.2 | 10.4 | 9.7 | 9.05 | 8.45 | 7.95 |
| 430 | 18.2 | 16.65 | 15.25 | 14.05 | 12.95 | 12.0 | 11.15 | 10.4 | 9.7 | 9.05 | 8.5 |
| 440 | 19.45 | 17.75 | 16.3 | 15.0 | 13.85 | 12.85 | 11.9 | 11.1 | 10.35 | 9.7 | 9.05 |
| 450 | 20.7 | 18.95 | 17.35 | 16.0 | 14.75 | 13.7 | 12.7 | 11.85 | 11.05 | 10.35 | 9.7 |
| 460 | 22.05 | 20.15 | 18.5 | 17.05 | 15.75 | 14.6 | 13.5 | 12.65 | 11.8 | 11.0 | 10.35 |
| 470 | 23.45 | 21.4 | 19.65 | 18.10 | 16.8 | 15.5 | 14.4 | 13.45 | 12.55 | 11.7 | 11.0 |
| 480 | 24.85 | 22.7 | 20.9 | 19.25 | 17.85 | 16.45 | 15.3 | 14.25 | 13.3 | 12.4 | 11.7 |
| 490 | 26.35 | 24.05 | 22.2 | 20.4 | 18.9 | 17.5 | 16.25 | 15.15 | 14.15 | 13.15 | 12.4 |
| 500 | 27.9 | 25.45 | 23.55 | 21.55 | 19.95 | 18.55 | 17.2 | 16.05 | 15.0 | 13.95 | 13.2 |

## TABLE II. — *Continued.*

**Reflecting Galvanometer Scale Errors.**

(These corrections are to be subtracted from the observed deflections.)

| 1550 | 1600 | 1650 | 1700 | 1750 | 1800 | 1850 | 1900 | 1950 | 2000 | Distance *a*. / OBSERVED DEFLECTIONS. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | **50** |
| 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | **60** |
| 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0. | 0. | **70** |
| 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | **80** |
| 0.1 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | **90** |
| 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.05 | 0.05 | 0.05 | 0.05 | **100** |
| 0.15 | 0.15 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | **110** |
| 0.2 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.1 | 0.1 | 0.1 | **120** |
| 0.25 | 0.2 | 0.2 | 0.2 | 0.2 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | **130** |
| 0.3 | 0.25 | 0.25 | 0.25 | 0.25 | 0.2 | 0.2 | 0.2 | 0.2 | 0.15 | **140** |
| 0.35 | 0.35 | 0.3 | 0.3 | 0.3 | 0.25 | 0.25 | 0.25 | 0.2 | 0.2 | **150** |
| 0.45 | 0.4 | 0.4 | 0.35 | 0.35 | 0.3 | 0.3 | 0.3 | 0.25 | 0.25 | **160** |
| 0.5 | 0.5 | 0.45 | 0.45 | 0.4 | 0.4 | 0.35 | 0.35 | 0.3 | 0.3 | **170** |
| 0.6 | 0.55 | 0.55 | 0.5 | 0.5 | 0.45 | 0.4 | 0.4 | 0.4 | 0.35 | **180** |
| 0.7 | 0.65 | 0.65 | 0.6 | 0.55 | 0.55 | 0.5 | 0.5 | 0.45 | 0.45 | **190** |
| 0.8 | 0.8 | 0.75 | 0.7 | 0.65 | 0.6 | 0.6 | 0.55 | 0.55 | 0.5 | **200** |
| 0.95 | 0.9 | 0.85 | 0.8 | 0.75 | 0.7 | 0.7 | 0.65 | 0.6 | 0.6 | **210** |
| 1.1 | 1.05 | 1.0 | 0.9 | 0.85 | 0.8 | 0.8 | 0.75 | 0.7 | 0.65 | **220** |
| 1.25 | 1.2 | 1.1 | 1.05 | 1.0 | 0.95 | 0.9 | 0.85 | 0.8 | 0.75 | **230** |
| 1.45 | 1.35 | 1.25 | 1.2 | 1.15 | 1.05 | 1.0 | 0.95 | 0.9 | 0.85 | **240** |
| 1.6 | 1.5 | 1.4 | 1.35 | 1.3 | 1.2 | 1.15 | 1.05 | 1.0 | 1.0 | **250** |
| 1.8 | 1.7 | 1.6 | 1.5 | 1.45 | 1.35 | 1.3 | 1.2 | 1.15 | 1.10 | **260** |
| 2.0 | 1.9 | 1.8 | 1.7 | 1.6 | 1.5 | 1.45 | 1.35 | 1.3 | 1.25 | **270** |
| 2.25 | 2.1 | 2.0 | 1.9 | 1.75 | 1.65 | 1.6 | 1.5 | 1.45 | 1.35 | **280** |
| 2.5 | 2.35 | 2.2 | 2.1 | 1.95 | 1.85 | 1.75 | 1.65 | 1.6 | 1.5 | **290** |
| 2.75 | 2.6 | 2.4 | 2.3 | 2.15 | 2.05 | 1.95 | 1.85 | 1.75 | 1.7 | **300** |
| 3.05 | 2.85 | 2.65 | 2.5 | 2.35 | 2.25 | 2.15 | 2.05 | 1.95 | 1.85 | **310** |
| 3.35 | 3.15 | 2.95 | 2.75 | 2.6 | 2.45 | 2.35 | 2.25 | 2.15 | 2.05 | **320** |
| 3.65 | 3.45 | 3.25 | 3.05 | 2.85 | 2.7 | 2.55 | 2.45 | 2.35 | 2.2 | **330** |
| 4.0 | 3.75 | 3.5 | 3.35 | 3.1 | 2.95 | 2.8 | 2.65 | 2.55 | 2.4 | **340** |
| 4.35 | 4.1 | 3.85 | 3.65 | 3.4 | 3.25 | 3.05 | 2.9 | 2.75 | 2.6 | **350** |
| 4.7 | 4.45 | 4.2 | 3.95 | 3.7 | 3.55 | 3.35 | 3.15 | 3.0 | 2.85 | **360** |
| 5.1 | 4.8 | 4.55 | 4.3 | 4.0 | 3.85 | 3.65 | 3.40 | 3.25 | 3.1 | **370** |
| 5.5 | 5.2 | 4.9 | 4.65 | 4.35 | 4.15 | 3.95 | 3.7 | 3.5 | 3.35 | **380** |
| 5.95 | 5.6 | 5.3 | 5.0 | 4.7 | 4.45 | 4.25 | 4.0 | 3.8 | 3.6 | **390** |
| 6.45 | 6.05 | 5.7 | 5.35 | 5.05 | 4.80 | 4.55 | 4.3 | 4.1 | 3.9 | **400** |
| 6.95 | 6.5 | 6.15 | 5.75 | 5.45 | 5.2 | 4.9 | 4.65 | 4.4 | 4.2 | **410** |
| 7.45 | 7.0 | 6.6 | 6.2 | 5.85 | 5.55 | 5.3 | 5.0 | 4.75 | 4.5 | **420** |
| 7.95 | 7.5 | 7.05 | 6.65 | 6.25 | 5.95 | 5.65 | 5.35 | 5.1 | 4.85 | **430** |
| 8.5 | 8.0 | 7.55 | 7.1 | 6.7 | 6.35 | 6.05 | 5.75 | 5.45 | 5.2 | **440** |
| 9.1 | 8.55 | 8.05 | 7.6 | 7.15 | 6.8 | 6.45 | 6.15 | 5.8 | 5.55 | **450** |
| 9.7 | 9.1 | 8.6 | 8.1 | 7.65 | 7.25 | 6.85 | 6.55 | 6.2 | 5.9 | **460** |
| 10.3 | 9.7 | 9.15 | 8.65 | 8.15 | 7.75 | 7.3 | 6.95 | 6.6 | 6.3 | **470** |
| 10.95 | 10.35 | 9.75 | 9.2 | 8.7 | 8.25 | 7.75 | 7.4 | 7.0 | 6.7 | **480** |
| 11.65 | 11.0 | 10.35 | 9.75 | 9.25 | 8.75 | 8.25 | 7.85 | 7.45 | 7.1 | **490** |
| 12.35 | 11.65 | 10.95 | 10.35 | 9.8 | 9.3 | 8.75 | 8.35 | 7.9 | 7.55 | **500** |

## TABLE III.

**Reduction of the Period to an Infinitely Small Arc.**

If the observed time of oscillation be $T$, with an arc of oscillation of $a$ degrees, $cT$ must be subtracted from the observed value to reduce to an infinitely small arc of oscillation.

| $a$ | $c$ | | $a$ | $c$ | | $a$ | $c$ | | $a$ | $c$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | 0.00000 | | 10° | 0.00048 | | 20° | 0.00190 | | 30° | 0.00428 | |
| 1 | 000 | 0 | 11 | 058 | 10 | 21 | 210 | 20 | 31 | 457 | 29 |
| 2 | 002 | 2 | 12 | 069 | 11 | 22 | 230 | 20 | 32 | 487 | 30 |
| 3 | 004 | 2 | 13 | 080 | 11 | 23 | 251 | 21 | 33 | 518 | 31 |
| 4 | 008 | 4 | 14 | 093 | 13 | 24 | 274 | 23 | 34 | 550 | 32 |
| 5 | 012 | 4 | 15 | 107 | 14 | 25 | 297 | 23 | 35 | 583 | 33 |
| 6 | 017 | 5 | 16 | 122 | 15 | 26 | 322 | 25 | 36 | 616 | 33 |
| 7 | 023 | 6 | 17 | 138 | 16 | 27 | 347 | 25 | 37 | 651 | 35 |
| 8 | 030 | 7 | 18 | 154 | 16 | 28 | 373 | 26 | 38 | 686 | 35 |
| 9 | 039 | 9 | 19 | 172 | 18 | 29 | 400 | 27 | 39 | 723 | 37 |
| 10 | 0.00048 | 9 | 20 | 0.00190 | 18 | 30 | 0.00428 | 28 | 40 | 0.00761 | 38 |

## TABLE IV.

**E.M.F. of Standard Cells at Different Temperatures.**

| Clark Cell. | | | | Carhart-Clark Cell. | | | |
|---|---|---|---|---|---|---|---|
| Temp., C. | E.M.F. | Temp., C. | E.M.F. | Temp., C. | E.M.F. | Temp., C. | E.M.F. |
| 10° | 1.4396 | 20° | 1.4279 | 10° | 1.4428 | 20° | 1.4372 |
| 11 | 1.4385 | 21 | 1.4267 | 11 | 1.4422 | 21 | 1.4367 |
| 12 | 1.4374 | 22 | 1.4254 | 12 | 1.4417 | 22 | 1.4361 |
| 13 | 1.4363 | 23 | 1.4241 | 13 | 1.4411 | 23 | 1.4356 |
| 14 | 1.4352 | 24 | 1.4227 | 14 | 1.4406 | 24 | 1.4350 |
| 15 | 1.4340 | 25 | 1.4214 | 15 | 1.4400 | 25 | 1.4345 |
| 16 | 1.4328 | 26 | 1.4200 | 16 | 1.4394 | 26 | 1.4340 |
| 17 | 1.4316 | 27 | 1.4186 | 17 | 1.4389 | 27 | 1.4334 |
| 18 | 1.4304 | 28 | 1.4172 | 18 | 1.4383 | 28 | 1.4329 |
| 19 | 1.4292 | 29 | 1.4158 | 19 | 1.4378 | 29 | 1.4323 |
| | | 30 | 1.4143 | | | 30 | 1.4318 |

## TABLE V.

### Dimensional Formulas.

#### I. *Mechanical Units.*

| | |
|---|---|
| Area | $L^2$ |
| Volume | $L^3$ |
| Velocity | $LT^{-1}$ |
| Acceleration | $LT^{-2}$ |
| Force | $LMT^{-2}$ |
| Moment of rotation | $L^2MT^{-2}$ |
| Moment of inertia | $L^2M$ |
| Work, energy | $L^2MT^{-2}$ |

#### II. *Electric Units.*

| | |
|---|---|
| Quantity of electricity | $M^{\frac{1}{2}}L^{\frac{1}{2}}$ |
| Electric potential, electromotive force | $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$ |
| Capacity | $L^{-1}T^2$ |
| Current strength | $M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$ |
| Resistance | $LT^{-1}$ |
| Inductance | $L$ |

#### III. *Magnetic Units.*

| | |
|---|---|
| Strength of pole | $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ |
| Magnetic moment | $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$ |
| Intensity of magnetization | $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$ |
| Magnetic force, intensity of field | $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$ |

The above electric and magnetic units are in the electromagnetic system.

## TABLE VI.

**Doubled Square Roots for Kelvin Balances.**

| | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0·000 | 20·00 | 28·28 | 34·64 | 40·00 | 44·72 | 48·99 | 52·92 | 56·57 | 60·00 | 0 |
| 1 | 2·000 | 20·10 | 28·35 | 34·70 | 40·05 | 44·77 | 49·03 | 52·95 | 56·60 | 60·03 | 1 |
| 2 | 2·828 | 20·20 | 28·43 | 34·76 | 40·10 | 44·81 | 49·07 | 52·99 | 56·64 | 60·07 | 2 |
| 3 | 3·464 | 20·30 | 28·50 | 34·81 | 40·15 | 44·86 | 49·11 | 53·03 | 56·67 | 60·10 | 3 |
| 4 | 4·000 | 20·40 | 28·57 | 34·87 | 40·20 | 44·90 | 49·15 | 53·07 | 56·71 | 60·13 | 4 |
| 5 | 4·472 | 20·49 | 28·64 | 34·93 | 40·25 | 44·94 | 49·19 | 53·10 | 56·75 | 60·17 | 5 |
| 6 | 4·899 | 20·59 | 28·71 | 34·99 | 40·30 | 44·99 | 49·23 | 53·14 | 56·78 | 60·20 | 6 |
| 7 | 5·292 | 20·69 | 28·77 | 35·04 | 40·35 | 45·03 | 49·27 | 53·18 | 56·82 | 60·23 | 7 |
| 8 | 5·657 | 20·78 | 28·84 | 35·10 | 40·40 | 45·08 | 49·32 | 53·22 | 56·85 | 60 27 | 8 |
| 9 | 6·000 | 20·88 | 28·91 | 35·16 | 40·45 | 45·12 | 49·36 | 53·25 | 56·89 | 60·30 | 9 |
| 10 | 6·325 | 20·98 | 28·98 | 35·21 | 40·50 | 45·17 | 49·40 | 53·29 | 56·92 | 60·33 | 10 |
| 11 | 6·633 | 21·07 | 29·05 | 35·27 | 40·55 | 45·21 | 49·44 | 53·33 | 56·96 | 60·37 | 11 |
| 12 | 6·928 | 21·17 | 29·12 | 35·33 | 40·60 | 45·25 | 49·48 | 53·37 | 56·99 | 60·40 | 12 |
| 13 | 7·211 | 21·26 | 29·19 | 35·38 | 40·64 | 45·30 | 49·52 | 53·40 | 57·03 | 60·43 | 13 |
| 14 | 7·483 | 21·35 | 29·26 | 35·44 | 40·69 | 45·34 | 49·56 | 53·44 | 57·06 | 60·46 | 14 |
| 15 | 7·746 | 21·45 | 29·33 | 35·50 | 40·74 | 45·39 | 49·60 | 53·48 | 57·10 | 60·50 | 15 |
| 16 | 8·000 | 21·54 | 29·39 | 35·55 | 40·79 | 45·43 | 49·64 | 53·52 | 57·13 | 60·53 | 16 |
| 17 | 8·246 | 21·63 | 29·46 | 35·61 | 40·84 | 45·48 | 49·68 | 53·55 | 57·17 | 60·56 | 17 |
| 18 | 8·485 | 21·73 | 29·53 | 35·67 | 40·89 | 45·52 | 49·72 | 53·59 | 57·20 | 60·60 | 18 |
| 19 | 8·718 | 21·82 | 29·60 | 35·72 | 40·94 | 45·56 | 49·76 | 53·63 | 57·24 | 60·63 | 19 |
| 20 | 8·944 | 21·91 | 29·66 | 35·78 | 40·99 | 45·61 | 49·80 | 53·67 | 57·27 | 60·66 | 20 |
| 21 | 9·165 | 22·00 | 29·73 | 35·83 | 41·04 | 45·65 | 49·84 | 53·70 | 57·31 | 60·70 | 21 |
| 22 | 9·381 | 22·09 | 29·80 | 35·89 | 41·09 | 45·69 | 49·88 | 53·74 | 57·34 | 60·73 | 22 |
| 23 | 9·592 | 22·18 | 29·87 | 35·94 | 41·13 | 45·74 | 49·92 | 53·78 | 57·38 | 60·76 | 23 |
| 24 | 9·798 | 22·27 | 29·93 | 36·00 | 41·18 | 45·78 | 49·96 | 53·81 | 57·41 | 60·79 | 24 |
| 25 | 10·000 | 22·36 | 30·00 | 36·06 | 41·23 | 45·83 | 50·00 | 53·85 | 57·45 | 60·83 | 25 |
| 26 | 10·198 | 22·45 | 30·07 | 36·11 | 41·28 | 45·87 | 50·04 | 53·89 | 57·48 | 60·86 | 26 |
| 27 | 10·392 | 22·54 | 30·13 | 36·17 | 41·33 | 45·91 | 50·08 | 53·93 | 57·52 | 60·89 | 27 |
| 28 | 10·583 | 22·63 | 30·20 | 36·22 | 41·38 | 45·96 | 50·12 | 53·96 | 57·55 | 60·93 | 28 |
| 29 | 10.770 | 22·72 | 30·27 | 36·28 | 41·42 | 46·00 | 50·16 | 54·00 | 57·58 | 60·96 | 29 |
| 30 | 10·954 | 22·80 | 30·33 | 36·33 | 41·47 | 46·04 | 50·20 | 54·04 | 57·62 | 60·99 | 30 |
| 31 | 11·136 | 22·89 | 30·40 | 36·39 | 41·52 | 46·09 | 50·24 | 54·07 | 57·65 | 61·02 | 31 |
| 32 | 11·314 | 22·98 | 30·46 | 36·44 | 41·57 | 46·13 | 50·28 | 54·11 | 57·69 | 61·06 | 32 |
| 33 | 11·489 | 23·07 | 30·53 | 36·50 | 41·62 | 46·17 | 50·32 | 54·15 | 57·72 | 61·09 | 33 |
| 34 | 11·662 | 23·15 | 30·59 | 36·55 | 41·67 | 46·22 | 50·36 | 54·18 | 57·76 | 61·12 | 34 |
| 35 | 11·832 | 23·24 | 30·66 | 36·61 | 41·71 | 46·26 | 50·40 | 54·22 | 57·79 | 61·16 | 35 |
| 36 | 12·000 | 23·32 | 30·72 | 36·66 | 41·76 | 46·30 | 50·44 | 54·26 | 57·83 | 61·19 | 36 |
| 37 | 12·166 | 23·41 | 30·79 | 36·72 | 41·81 | 46·35 | 50·48 | 54·30 | 57·86 | 61·22 | 37 |
| 38 | 12·329 | 23·49 | 30·85 | 36·77 | 41·86 | 46·39 | 50·52 | 54·33 | 57·90 | 61·25 | 38 |
| 39 | 12·490 | 23·58 | 30·92 | 36·82 | 41·90 | 46·43 | 50·56 | 54·37 | 57·93 | 61·29 | 39 |
| 40 | 12·649 | 23·66 | 30·98 | 36·88 | 41·95 | 46·48 | 50·60 | 54·41 | 57·97 | 61·32 | 40 |
| 41 | 12·806 | 23.75 | 31·05 | 36·93 | 42·00 | 46·52 | 50·64 | 54·44 | 58·00 | 61·35 | 41 |
| 42 | 12·961 | 23·83 | 31·11 | 36·99 | 42·05 | 46·56 | 50·68 | 54·48 | 58·03 | 61·38 | 42 |
| 43 | 13·115 | 23·92 | 31·18 | 37.04 | 42·10 | 46·60 | 50·71 | 54·52 | 58·07 | 61·42 | 43 |
| 44 | 13·266 | 24·00 | 31·24 | 37·09 | 42·14 | 46·65 | 50·75 | 54·55 | 58·10 | 61·45 | 44 |
| 45 | 13·416 | 24·08 | 31·30 | 37·15 | 42·19 | 46·69 | 50·79 | 54·59 | 58·14 | 61·48 | 45 |
| 46 | 13·565 | 24·17 | 31·37 | 37·20 | 42·24 | 46·73 | 50·83 | 54·63 | 58·17 | 61·51 | 46 |
| 47 | 13·711 | 24·25 | 31·43 | 37·26 | 42·28 | 46·78 | 50·87 | 54·66 | 58·21 | 61·55 | 47 |
| 48 | 13·856 | 24·33 | 31·50 | 37·31 | 42·33 | 46·82 | 50·91 | 54·70 | 58·24 | 61·58 | 48 |
| 49 | 14·000 | 24·41 | 31·56 | 37·36 | 42·38 | 46·86 | 50·95 | 54·74 | 58·28 | 61·6 | 49 |
| 50 | 14·142 | 24·49 | 31·62 | 37·42 | 42·43 | 46·90 | 50·99 | 54·77 | 58·31 | 61·64 | 50 |

**TABLE VI.** — *Continued.*

**Doubled Square Roots for Kelvin Balances.**

| | 0 | 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 51 | 14·283 | 24·58 | 31·69 | 37·47 | 42·47 | 46·95 | 51·03 | 54·81 | 58·34 | 61·68 | 51 |
| 52 | 14·422 | 24·66 | 31·75 | 37·52 | 42·52 | 46·99 | 51·07 | 54·85 | 58·38 | 61·71 | 52 |
| 53 | 14·560 | 24·74 | 31·81 | 37·58 | 42·57 | 47·03 | 51·11 | 54·88 | 58·41 | 61·74 | 53 |
| 54 | 14·697 | 24·82 | 31·87 | 37·63 | 42·61 | 47·07 | 51·15 | 54·92 | 58·45 | 61·77 | 54 |
| 55 | 14·832 | 24·90 | 31·94 | 37·68 | 42·66 | 47·12 | 51·19 | 54·95 | 58·48 | 61·81 | 55 |
| 56 | 14·967 | 24·98 | 32·00 | 37·74 | 42·71 | 47·16 | 51·22 | 54·99 | 58·51 | 61·84 | 56 |
| 57 | 15·100 | 25·06 | 32·06 | 37·79 | 42·76 | 47·20 | 51·26 | 55·03 | 58·55 | 61·87 | 57 |
| 58 | 15·232 | 25·14 | 32·12 | 37·84 | 42·80 | 47·24 | 51·30 | 55.06 | 58·58 | 61·90 | 58 |
| 59 | 15·362 | 25·22 | 32·19 | 37·89 | 42·85 | 47·29 | 51·34 | 55·10 | 58·62 | 61·94 | 59 |
| 60 | 15·492 | 25·30 | 32·25 | 37·95 | 42·90 | 47·33 | 51·38 | 55·14 | 58·65 | 61·97 | 60 |
| 61 | 15.620 | 25·38 | 32·31 | 38·00 | 42·94 | 47·37 | 51·42 | 55·17 | 58·69 | 62·00 | 61 |
| 62 | 15·748 | 25·46 | 32·37 | 38·05 | 42·99 | 47·41 | 51·46 | 55·21 | 58·72 | 62·03 | 62 |
| 63 | 15·875 | 25·53 | 32·43 | 38·11 | 43·03 | 47·46 | 51·50 | 55·24 | 58·75 | 62·06 | 63 |
| 64 | 16·000 | 25·61 | 32·50 | 38·16 | 43·08 | 47·50 | 51·54 | 55·28 | 58·79 | 62·10 | 64 |
| 65 | 16·125 | 25·69 | 32·56 | 38·21 | 43·13 | 47·54 | 51·58 | 55·32 | 58·82 | 62·13 | 65 |
| 66 | 16·248 | 25·77 | 32·62 | 38·26 | 43·17 | 47·58 | 51·61 | 55·35 | 58·86 | 62·16 | 66 |
| 67 | 16·371 | 25·85 | 32·68 | 38·31 | 43·22 | 47·62 | 51·65 | 55·39 | 58·89 | 62·19 | 67 |
| 68 | 16·492 | 25·92 | 32·74 | 38·37 | 43·27 | 47·67 | 51·69 | 55·43 | 58·92 | 62·23 | 68 |
| 69 | 16·613 | 26·00 | 32·80 | 38·42 | 43·31 | 47·71 | 51·73 | 55·46 | 58·96 | 62·26 | 69 |
| 70 | 16·733 | 26·08 | 32·86 | 38·47 | 43·36 | 47·75 | 51·77 | 55·50 | 58·99 | 62·29 | 70 |
| 71 | 16·852 | 26·15 | 32·92 | 38·52 | 43·41 | 47·79 | 51·81 | 55·53 | 59·03 | 62·32 | 71 |
| 72 | 16·971 | 26·23 | 32·98 | 38·57 | 43·45 | 47·83 | 51·85 | 55·57 | 59·06 | 62·35 | 72 |
| 73 | 17·088 | 26·31 | 33·05 | 38·63 | 43·50 | 47·87 | 51·88 | 55·61 | 59·09 | 62·39 | 73 |
| 74 | 17·205 | 26·38 | 33·11 | 38·68 | 43·54 | 47·92 | 51·92 | 55·64 | 59·13 | 62·42 | 74 |
| 75 | 17·321 | 26·46 | 33·17 | 38·73 | 43·59 | 47·96 | 51·96 | 55·68 | 59·16 | 62·45 | 75 |
| 76 | 17·436 | 26·53 | 33·23 | 38·78 | 43·63 | 48·00 | 52·00 | 55·71 | 59·19 | 62·48 | 76 |
| 77 | 17·550 | 26·61 | 33·29 | 38·83 | 43·68 | 48·04 | 52·04 | 55·75 | 59·23 | 62·51 | 77 |
| 78 | 17·664 | 26·68 | 33·35 | 38·88 | 43·73 | 48·08 | 52·08 | 55·79 | 59·26 | 62·55 | 78 |
| 79 | 17·776 | 26·76 | 33·41 | 38·94 | 43·77 | 48·12 | 52·12 | 55·82 | 59·30 | 62·58 | 79 |
| 80 | 17·889 | 26·83 | 33·47 | 38·99 | 43·82 | 48·17 | 52·15 | 55·86 | 59·33 | 62·61 | 80 |
| 81 | 18.000 | 26·91 | 33·53 | 39·04 | 43·86 | 48·21 | 52·19 | 55·89 | 59·36 | 62·64 | 81 |
| 82 | 18·111 | 26·98 | 33·59 | 39·09 | 43·91 | 48·25 | 52·23 | 55·93 | 59·40 | 62·67 | 82 |
| 83 | 18·221 | 27·06 | 33·65 | 39·14 | 43·95 | 48·29 | 52·27 | 55·96 | 59·43 | 62·71 | 83 |
| 84 | 18·330 | 27·13 | 33·70 | 39·19 | 44·00 | 48·33 | 52·31 | 56·00 | 59·46 | 62·74 | 84 |
| 85 | 18·439 | 27·20 | 33·76 | 39·24 | 44·05 | 48·37 | 52·35 | 56·04 | 59·50 | 62·77 | 85 |
| 86 | 18·547 | 27·28 | 33·82 | 39·29 | 44·09 | 48·41 | 52·38 | 56·07 | 59·53 | 62·80 | 86 |
| 87 | 18·655 | 27·35 | 33·88 | 39·34 | 44·14 | 48·46 | 52·42 | 56·11 | 59.57 | 62·83 | 87 |
| 88 | 18·762 | 27·42 | 33·94 | 39·40 | 44·18 | 48·50 | 52·46 | 56·14 | 59·60 | 62·86 | 88 |
| 89 | 18·868 | 27·50 | 34·00 | 39·45 | 44·23 | 48·54 | 52·50 | 56·18 | 59·63 | 62·90 | 89 |
| 90 | 18·974 | 27·57 | 34·06 | 39·50 | 44·27 | 48·58 | 52·54 | 56·21 | 59·67 | 62·93 | 90 |
| 91 | 19·079 | 27·64 | 34·12 | 39·55 | 44·32 | 48·62 | 52·57 | 56·25 | 59·70 | 62·96 | 91 |
| 92 | 19·183 | 27·71 | 34·18 | 39·60 | 44·36 | 48·66 | 52·61 | 56·28 | 59·73 | 62·99 | 92 |
| 93 | 19·287 | 27·78 | 34·23 | 39·65 | 44·41 | 48·70 | 52·65 | 56·32 | 59·77 | 63·02 | 93 |
| 94 | 19·391 | 27·86 | 34·29 | 36·70 | 44·45 | 48·74 | 52·69 | 56·36 | 59·80 | 63·06 | 94 |
| 95 | 19·494 | 27·93 | 34·35 | 39·75 | 44·50 | 48·79 | 52·73 | 56·39 | 59·83 | 63·09 | 95 |
| 96 | 19·596 | 28·00 | 34·41 | 39·80 | 44·54 | 48·83 | 52·76 | 56·43 | 59·87 | 63·12 | 96 |
| 97 | 19·698 | 28·07 | 34·47 | 39·85 | 44·59 | 48·87 | 52·80 | 56·46 | 59·90 | 63·15 | 97 |
| 98 | 19·799 | 28·14 | 34·53 | 39·90 | 44·63 | 48·91 | 52·84 | 56·50 | 59·93 | 63·18 | 98 |
| 99 | 19·900 | 28·21 | 34·58 | 39·95 | 44·68 | 48·95 | 52·88 | 56·53 | 59·97 | 63·21 | 99 |
| 100 | 20·000 | 28·28 | 34·64 | 40·00 | 44·72 | 48·99 | 52·92 | 56·57 | 60·00 | 63·25 | 100 |

# APPENDIX B.

## SPECIFICATIONS FOR THE PRACTICAL APPLICATION OF THE DEFINITIONS OF THE AMPERE AND VOLT.[1]

### SPECIFICATION A.—The Ampere.

In employing the silver voltameter to measure currents of about one ampere, the following arrangements shall be adopted:

The kathode on which the silver is to be deposited shall take the form of a platinum bowl not less than 10 cms. in diameter, and from 4 to 5 cms. in depth.

The anode shall be a disc or plate of pure silver some 30 sq. cms. in area, and 2 or 3 cms. in thickness.

This shall be supported horizontally in the liquid near the top of the solution by a silver rod riveted through its centre. To prevent the disintegrated silver which is formed on the anode from falling upon the kathode, the anode shall be wrapped around with pure filter paper, secured at the back by suitable folding.

The liquid shall consist of a neutral solution of pure silver nitrate, containing about fifteen parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current some resistance, besides that of the voltameter, should be inserted in the

[1] Legalized by act of Congress, approved July 12, 1894.

circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

**Method of making a Measurement.** — The platinum bowl is to be washed consecutively with nitric acid, distilled water, and absolute alcohol; it is then to be dried at 160° C., and left to cool in a desiccator. When thoroughly cool it is to be weighed carefully.

It is to be nearly filled with the solution and connected to the rest of the circuit by being placed on a clean copper support to which a binding screw is attached.

The anode is then to be immersed in the solution so as to be well covered by it, and supported in that position; the connections to the rest of the circuit are then to be made.

Contact is to be made at the key, noting the time. The current is to be allowed to pass for not less than half an hour, and the time of breaking contact observed.

The solution is now to be removed from the bowl, and the deposit washed with distilled water, and left to soak for at least six hours. It is then to be rinsed successively with distilled water and absolute alcohol, and dried in a hot-air bath at a temperature of about 160° C. After cooling in a desiccator it is to be weighed again. The gain in mass gives the silver deposited.

To find the time average of the current in amperes, this mass, expressed in grammes, must be divided by the number of seconds during which the current has passed and by 0.001118.

In determining the constant of an instrument by this method the current should be kept as nearly uniform as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a

curve from which the reading corresponding to the mean current (time-average of the current) can be found. The current, as calculated from the voltameter results, corresponds to this reading.

The current used in this experiment must be obtained from a battery and not from a dynamo, especially when the instrument to be calibrated is an electrodynamometer.

### SPECIFICATION B.—The Volt.

**Definition and Properties of the Cell.** — The cell has for its positive electrode, mercury, and for its negative electrode, amalgamated zinc; the electrolyte consists of a saturated solution of zinc sulphate and mercurous sulphate. The electromotive force is 1.434 volts at 15° C., and, between 10° C. and 25° C., by the increase of 1° C. in temperature, the electromotive force decreases by .00115 of a volt.

1. **Preparation of the Mercury.** — To secure purity it should be first treated with acid in the usual manner and subsequently distilled *in vacuo*.

2. **Preparation of the Zinc Amalgam.** — The zinc designated in commerce as "commercially pure" can be used without further preparation. For the preparation of the amalgam one part by weight of zinc is to be added to nine (9) parts by weight of mercury, and both are to be heated in a porcelain dish at 100° C. with moderate stirring until the zinc has been fully dissolved in the mercury.

3. **Preparation of the Mercurous Sulphate.** — Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a

bottle; drain off the water and repeat the process at least twice. After the last washing drain off as much of the water as possible. (For further details of purification, see Note A.)

4. **Preparation of the Zinc Sulphate Solution.** — Prepare a neutral saturated solution of pure re-crystallized zinc sulphate, free from iron, by mixing distilled water with nearly twice its weight of crystals of pure zinc sulphate and adding zinc oxide in the proportion of about 2 per cent by weight of the zinc sulphate crystals to neutralize any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised must not exceed 30° C. Mercurous sulphate, treated as described in 3, shall be added in the proportion of about 12 per cent by weight of the zinc sulphate crystals to neutralize the free zinc oxide remaining, and then the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

5. **Preparation of the Mercurous Sulphate and Zinc Sulphate Paste.** — For making the paste, two or three parts by weight of mercurous sulphate are to be added to one by weight of mercury. If the sulphate be dry, it is to be mixed with a paste consisting of zinc sulphate crystals and a concentrated zinc sulphate solution, so that the whole constitutes a stiff mass, which is permeated throughout by zinc sulphate crystals and globules of mercury. If the sulphate, however, be moist, only zinc sulphate crystals are to be added; care must, however, be taken that these occur in excess and are not dissolved after continued standing. The mercury must, in this case also, permeate the paste in little globules. It is advantageous to crush the zinc sulphate

crystals before using, since the paste can then be better manipulated.

**To set up the Cell.** — The containing glass vessel, represented in the accompanying figure,[1] shall consist of two limbs closed at bottom and joined above to a common neck fitted with a ground-glass stopper. The diameter of the limbs should be at least 2 cms. and their length at least 3 cms. The neck should be not less than 1.5 cms. in diameter. At the bottom of each limb a platinum wire of about 0.4 mm. diameter is sealed through the glass.

To set up the cell, place in one limb mercury and in the other hot liquid amalgam, containing 90 parts mercury and 10 parts zinc. The platinum wires at the bottom must be completely covered by the mercury and the amalgam respectively. On the mercury, place a layer one cm. thick of the zinc and mercurous sulphate paste described in 5. Both this paste and the zinc amalgam must then be covered with a layer of the neutral zinc sulphate crystals one cm. thick. The whole vessel must then be filled with the saturated zinc sulphate solution, and the stopper inserted so that it shall just touch it, leaving, however, a small bubble to guard against breakage when the temperature rises.

Before finally inserting the glass stopper, it is to be brushed round its upper edge with a strong alcoholic solution of shellac and pressed firmly in place. (For details of filling the cell, see Note B.)

---

[1] See Fig. 85, page 178.

## NOTES TO THE SPECIFICATIONS.

(A.) **The Mercurous Sulphate.** — The treatment of the mercurous sulphate has for its object the removal of any mercuric sulphate which is often present as an impurity.

Mercuric sulphate decomposes in the presence of water into an acid and a basic sulphate. The latter is a yellow substance — turpeth mineral — practically insoluble in water; its presence, at any rate in moderate quantities, has no effect on the cell. If, however, it be formed, the acid sulphate is also formed. This is soluble in water and the acid produced affects the electromotive force. The object of the washings is to dissolve and remove this acid sulphate, and for this purpose the three washings described in the specification will suffice in nearly all cases. If, however, much of the turpeth mineral be formed, it shows that there is a great deal of the acid sulphate present, and it will then be wiser to obtain a fresh sample of mercurous sulphate, rather than to try by repeated washings to get rid of all the acid.

The free mercury helps in the process of removing the acid, for the acid mercuric sulphate attacks it, forming mercurous sulphate.

Pure mercurous sulphate, when quite free from acid, shows on repeated washing a faint yellow tinge, which is due to the formation of a basic mercurous salt distinct from the turpeth mineral, or basic mercuric sulphate. The appearance of this primrose yellow tint may be taken as an indication that all the acid has been removed; the washing may with advantage be continued until this tint appears.

(B.) **Filling the Cell.** — After thoroughly cleaning and drying the glass vessel, place it in a hot-water bath. Then pass through the neck of the vessel a thin glass tube reaching to the bottom to serve for the introduction of the amalgam. This tube should be as large as the glass vessel will admit. It serves to protect the upper part of the cell from being soiled with the amalgam. To fill in the amalgam, a clean dropping tube about 10 cms. long, drawn out to a fine point, should be used. Its lower end is brought under the surface of the amalgam heated in a porcelain dish, and some of the amalgam is drawn into the tube by means of the rubber bulb. The point is then quickly cleaned of dross with filter paper and is passed through the wider tube to the bottom and emptied by pressing the bulb. The point of the tube must be so fine that the amalgam will come out only on squeezing the bulb. This process is repeated until the limb contains the desired quantity of the amalgam. The vessel is then removed from the water-bath. After cooling, the amalgam must adhere to the glass and must show a clean surface with a metallic lustre.

For insertion of the mercury, a dropping tube with a long stem will be found convenient. The paste may be poured in through a wide tube reaching nearly down to the mercury and having a funnel-shaped top. If the paste does not move down freely it may be pushed down with a small glass rod. The paste and the amalgam are then both covered with the zinc sulphate crystals before the concentrated zinc sulphate solution is poured in. This should be added through a small funnel, so as to leave the neck of the vessel clean and dry.

For convenience and security in handling, the cell

may be mounted in a suitable case so as to be at all times open to inspection.

In using the cell, sudden variations of temperature should, as far as possible, be avoided, since the changes in electromotive force lag behind those of temperature.

# INDEX.

*Numbers refer to pages.*